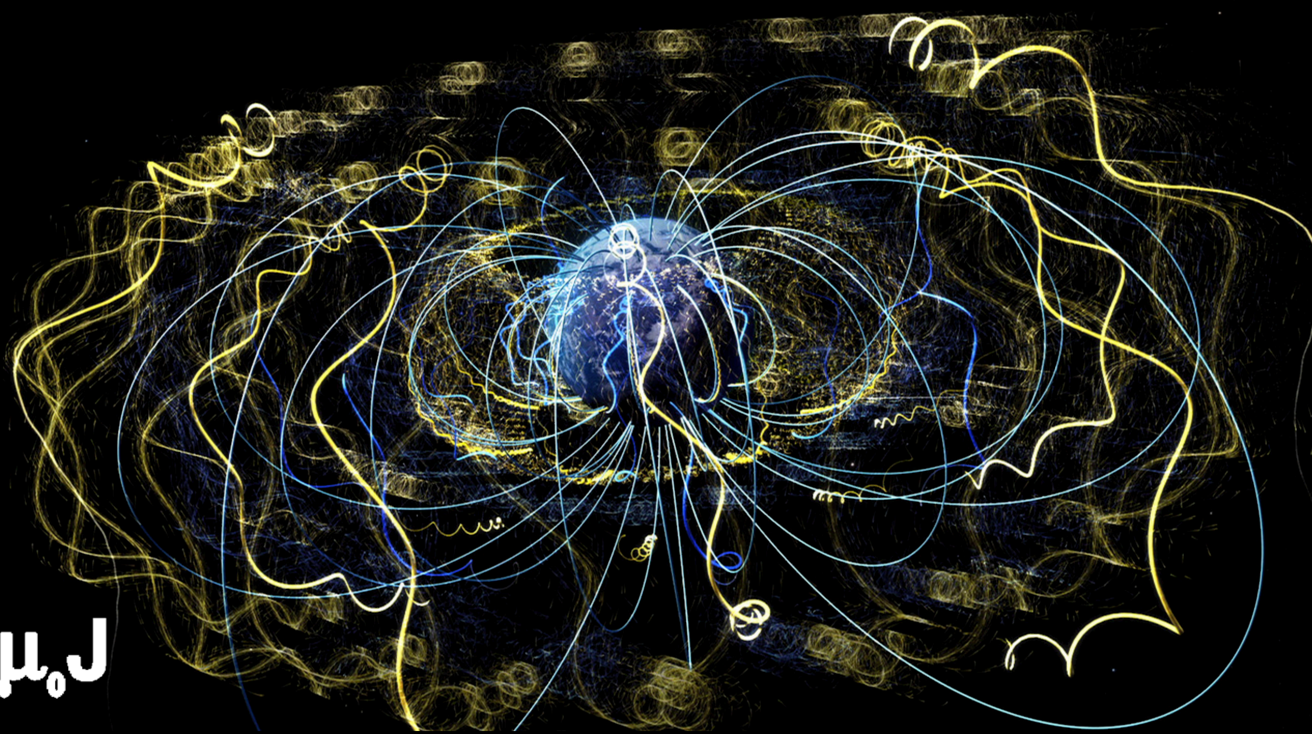
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

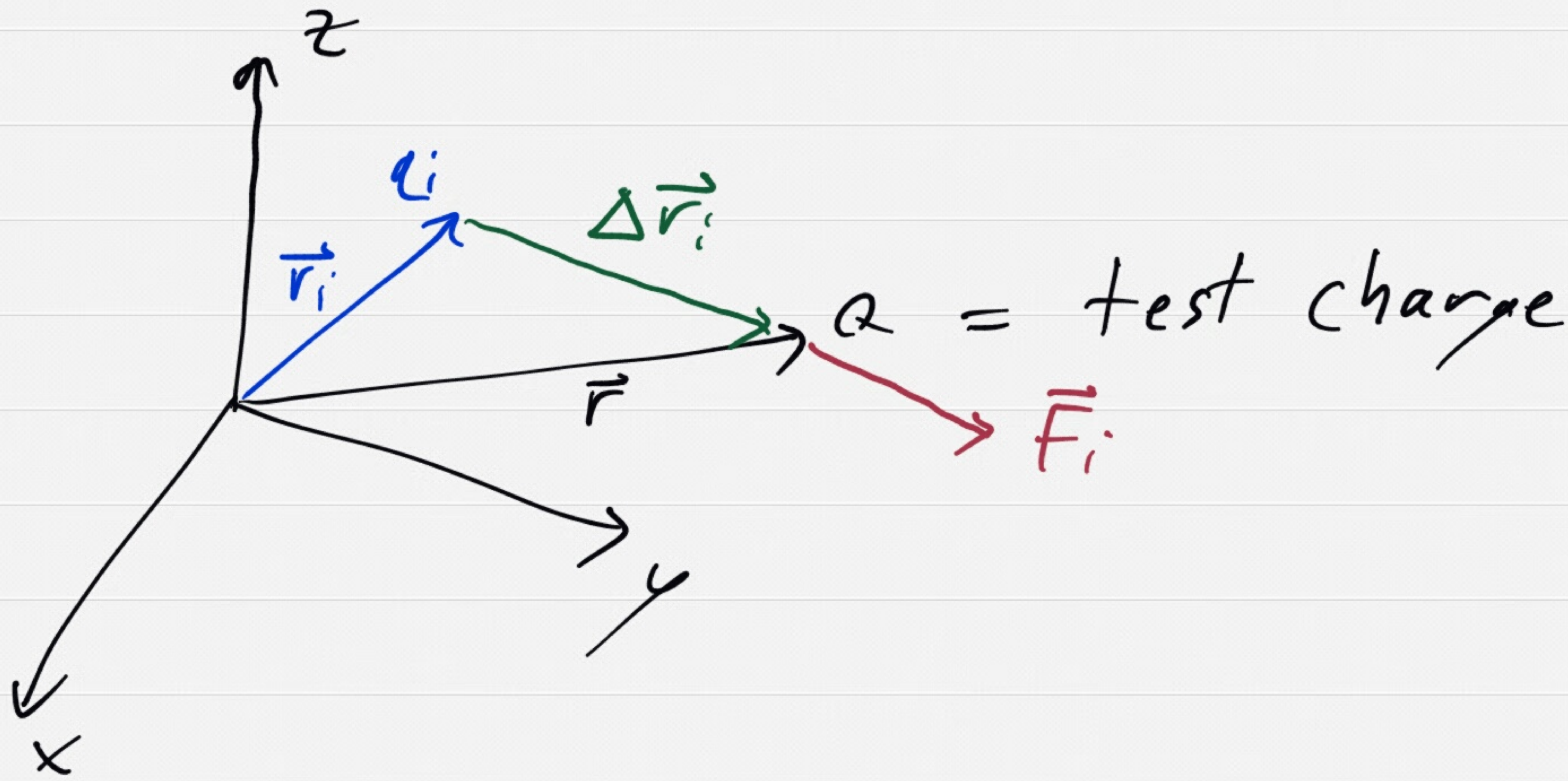
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Electric Field



source charge q_i at
source position $\vec{r}_i$

Coulomb Force:

$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0} \frac{q_i Q}{\Delta r_i^2} \hat{\Delta r}_i$$

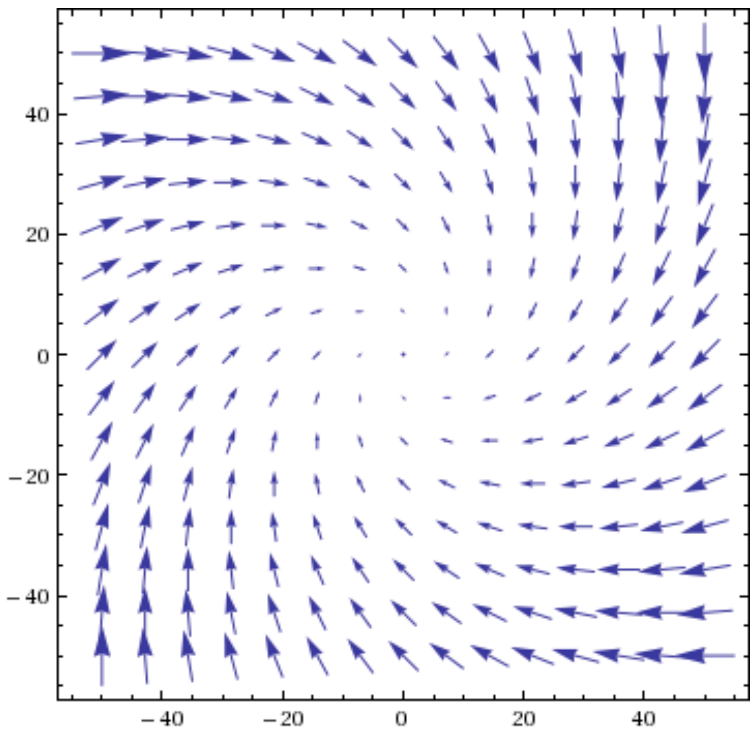
$$\vec{F}_{total} = \sum_i \vec{F}_i$$

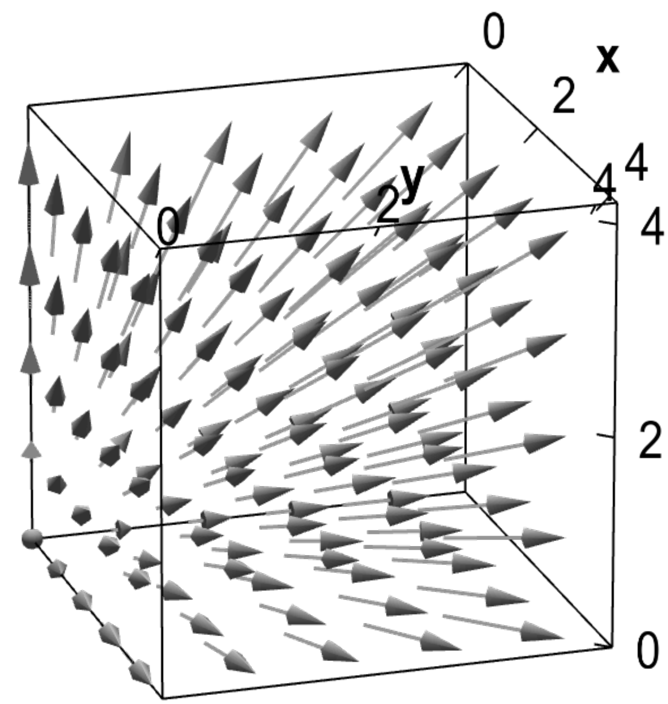
Electric Field $\vec{E} = \vec{F}/Q$

$$\Rightarrow \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{\Delta r_i^2} \hat{\Delta r}_i$$

- All electrostatics is just combinations of q_i, $\vec{r}_i$

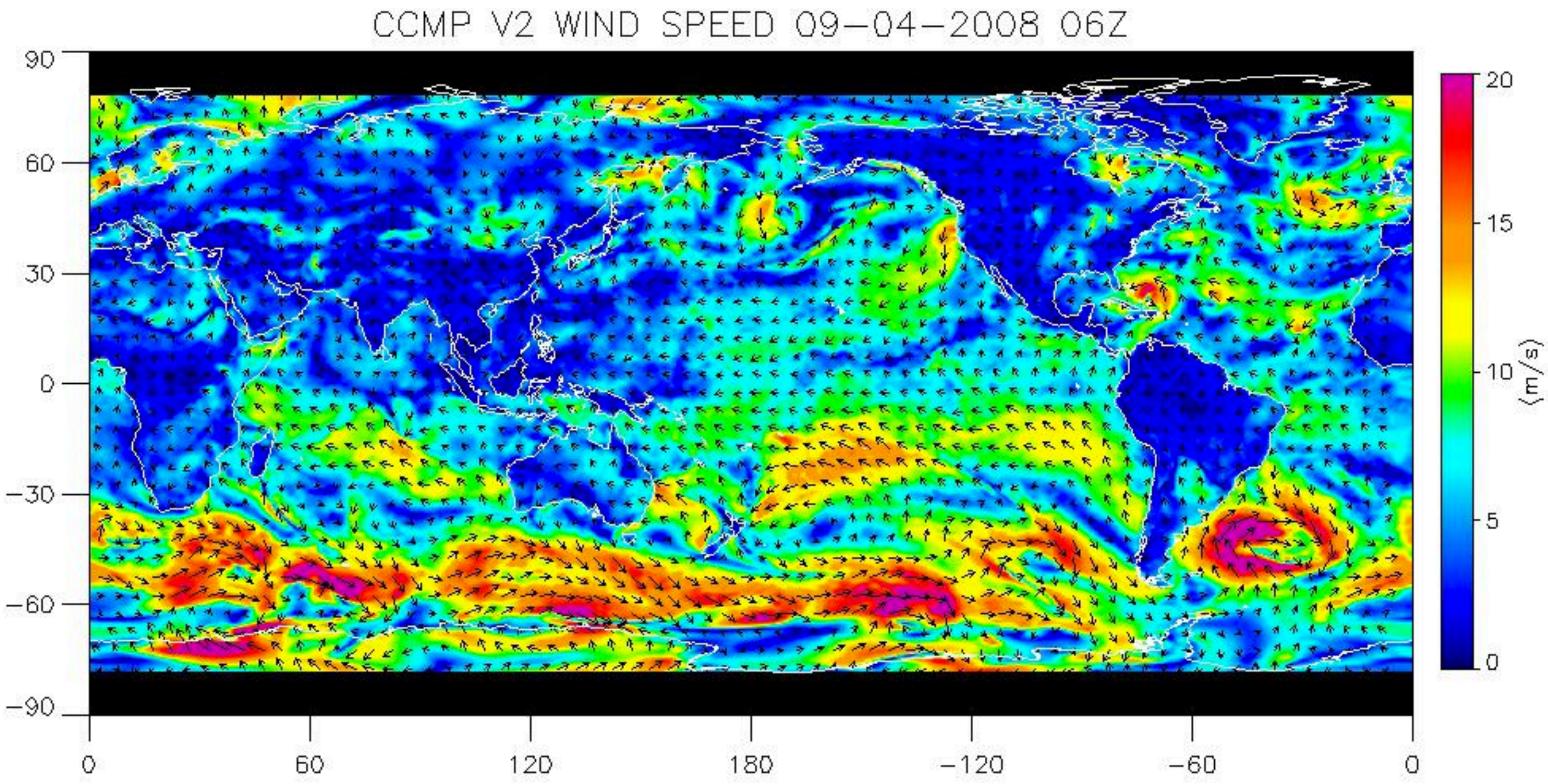
What is a Field?



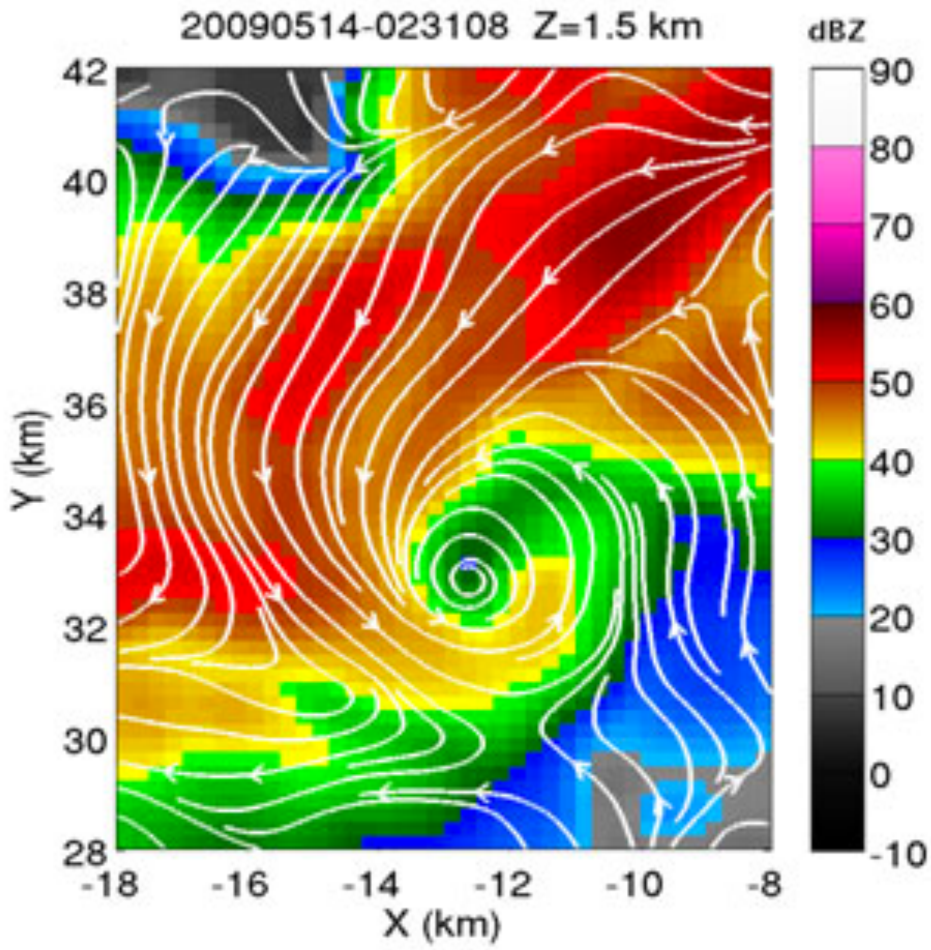


In physics, a field is a physical quantity, represented by a number or tensor, that has a value for each point in space and time. - Wikipedia

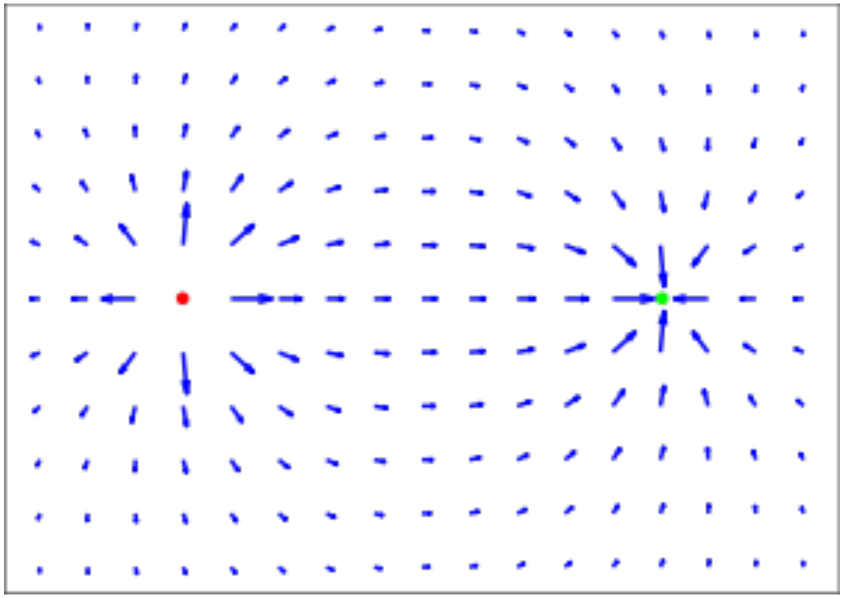
Example Field

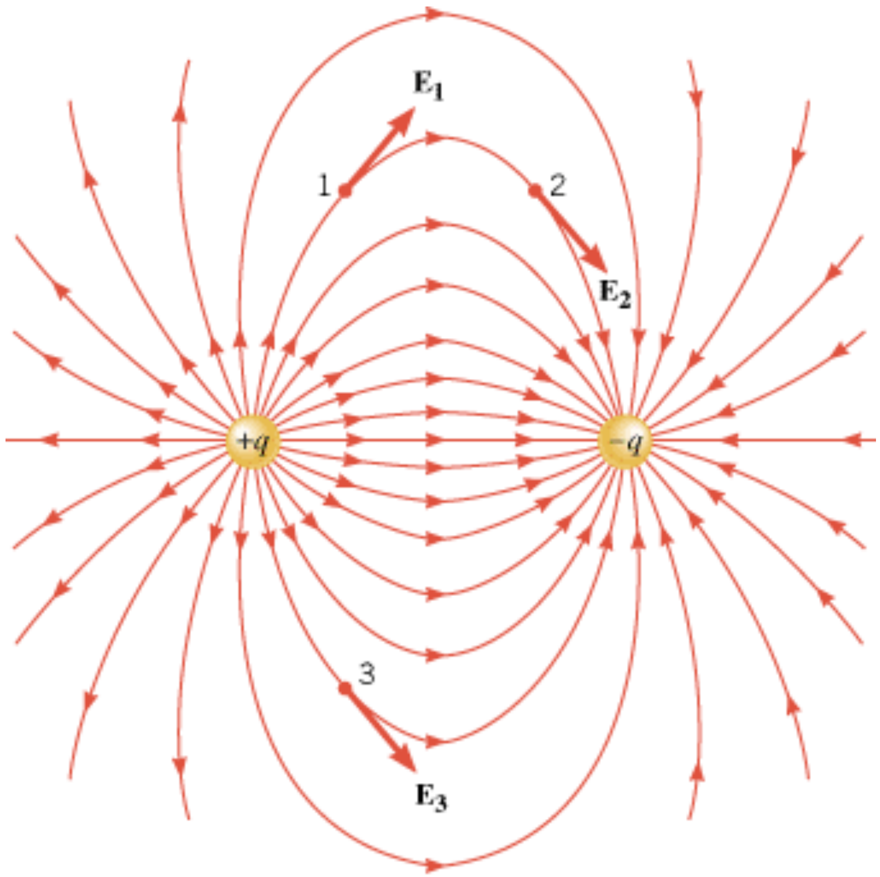


Example Field



Example Field





Seriously, What is an Electric Field?

The Electric Field Has Energy (Even in a Vacuum!)

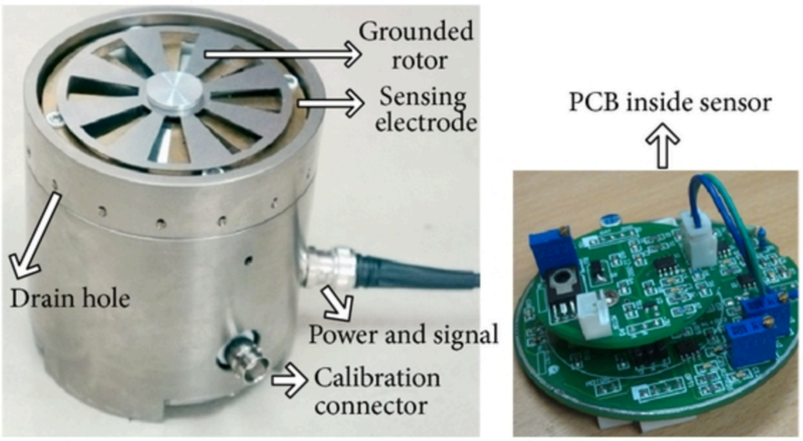
Example: An electromagnetic wave with an electric field amplitude of 1 V/m (pretty big):

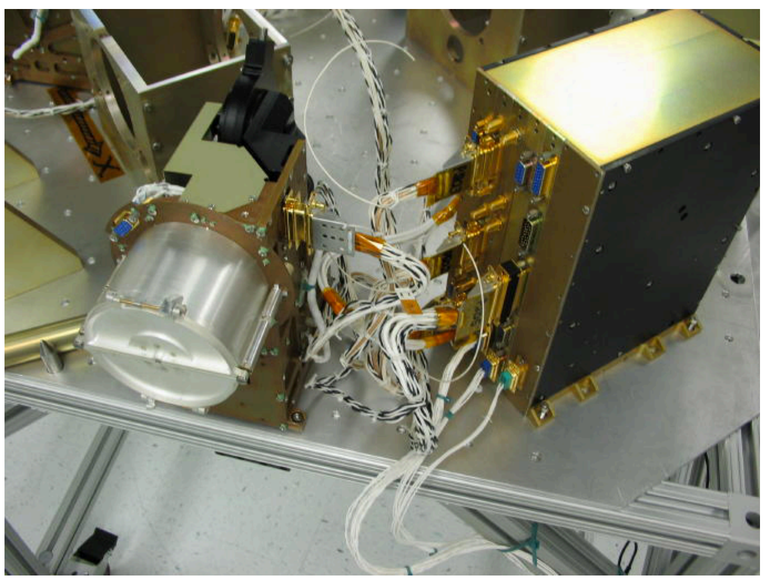
Average energy density of electric field is given by

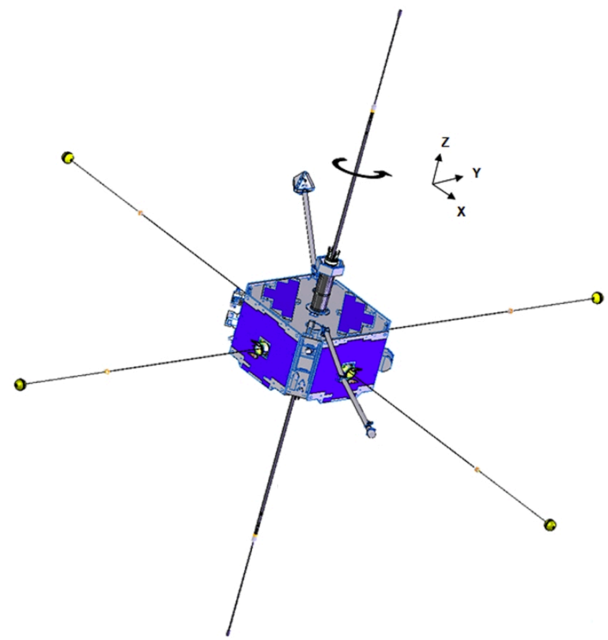
$$u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

$$= \frac{1}{4} \times 8.85 \times 10^{-12} (1)^2 = 2.2 \times 10^{-12}\, J/m^3.$$

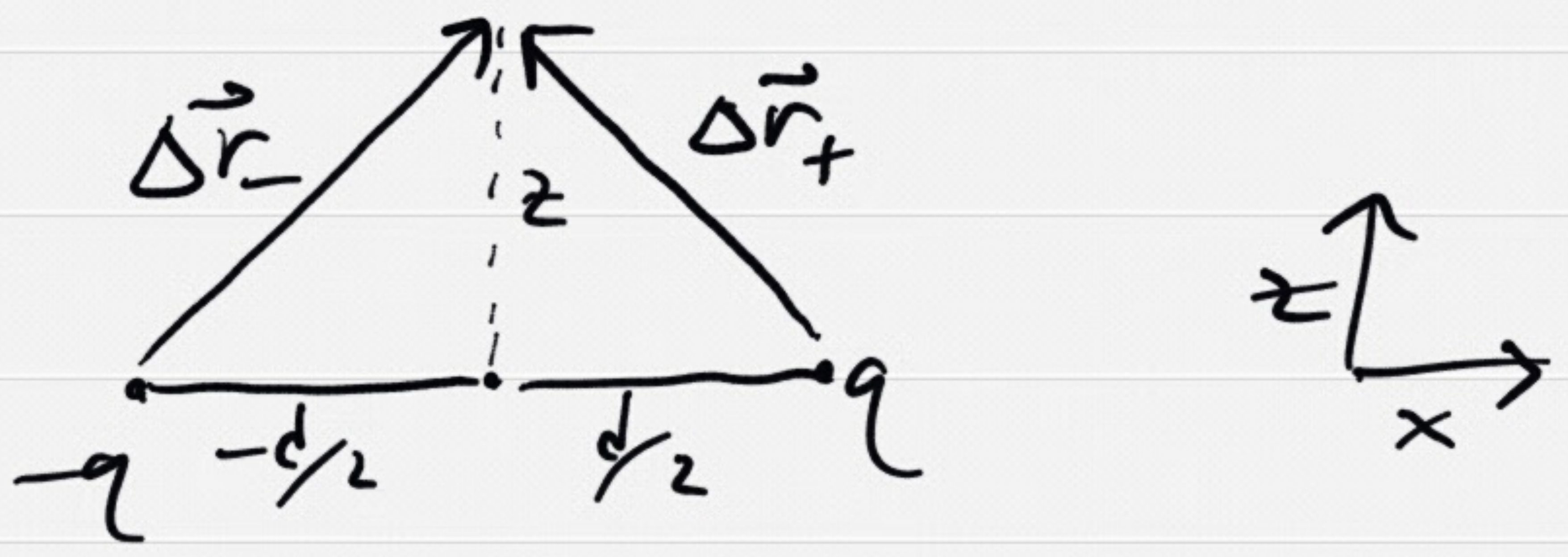
The Electric Field Can Be Measured







Example: Electric Dipole



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\Delta r_+^2}\hat{\Delta r_+} - \frac{q}{\Delta r_-^2}\hat{\Delta r_-}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{\vec{\Delta r_+}}{\Delta r_+^3} - \frac{\vec{\Delta r_-}}{\Delta r_-^3}\right)$$

@ $x = 0$ (on z-axis)

$$\vec{\Delta r_+} = -d/2\,\hat{x} + z\hat{z}$$

$$\vec{\Delta r_-} = d/2\,\hat{x} + z\hat{z}$$

$$\Delta r_+ = \Delta r_- = \sqrt{(d/2)^2 + z^2}$$

$$\vec{E}(z) = \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{((d/2)^2+z^2)^{3/2}}\left(-d/2\,\hat{x} + z\hat{z} - (d/2\,\hat{x} + z\hat{z})\right)$$

$$= \frac{q}{4\pi\epsilon_0}\cdot\frac{-d\hat{x}}{((d/2)^2+z^2)^{3/2}}$$

$$= \frac{-p\hat{x}}{4\pi\epsilon_0((d/2)^2+z^2)^{3/2}} \quad \text{w/ } p = qd$$

$$\rightarrow \frac{-p\hat{x}}{4\pi\epsilon_0 z^3} \quad \text{for } d \ll z$$

Continuous Charge Distributions

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}\,')}{\Delta r^2} \hat{\Delta r}\, d\tau'$$

E.g. $\rho(\vec{r}\,') = q_i \delta^3(\vec{r}\,' - \vec{r}_i)$

$$\Rightarrow \vec{E}_i(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{q_i \delta^3(\vec{r}\,' - \vec{r}_i)}{\Delta r^2} \hat{\Delta r}\, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_i \hat{\Delta r}_i}{\Delta r_i^2}$$

recovers formula for pt. charge

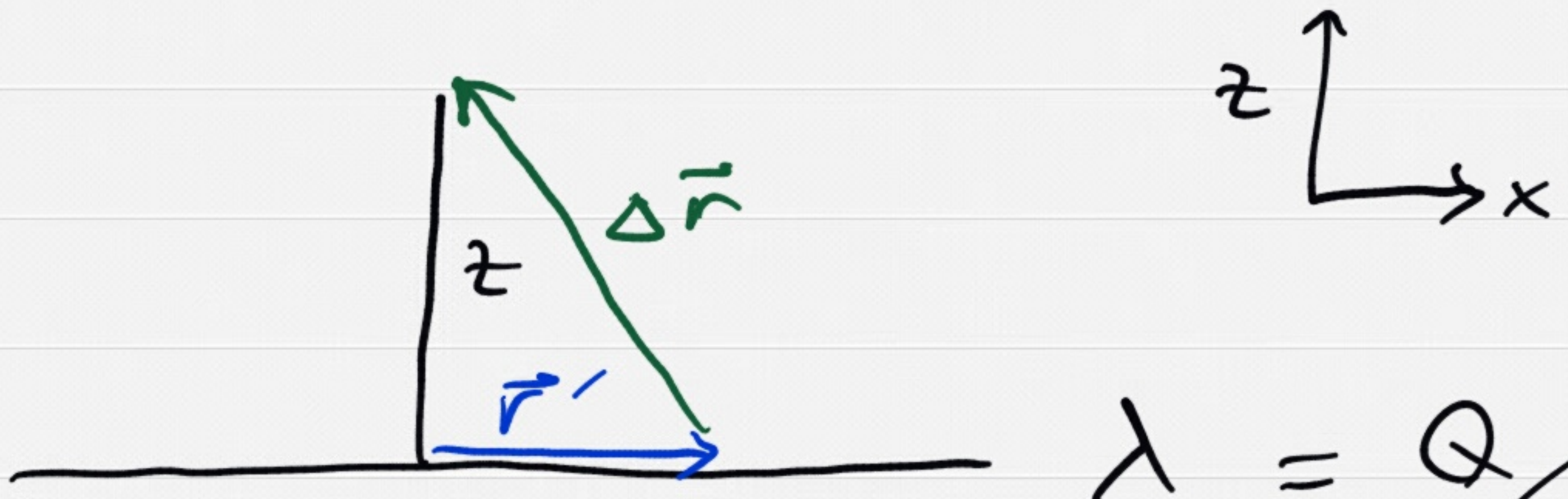
Special Cases

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(\vec{r}\,')}{\Delta r^2} \hat{\Delta r}\, da' \qquad \text{2-d source}$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\vec{r}\,')}{\Delta r^2} \hat{\Delta r}\, dl' \qquad \text{1-d source}$$

Note: $\vec{\Delta r} = \vec{r} - \vec{r}\,'$ so can't be taken out of integral over source coordinates $\vec{r}\,'$

Example: Infinite Line



$$\lambda = Q/L$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r}\,')}{\Delta r^2}\,\Delta\hat{r}\,dl'$$

$$\lambda(\vec{r}\,') = \lambda \qquad dl' = dx'$$

$$\Delta\vec{r} = -x'\hat{x} + z\hat{z}$$

$$\Delta r = \sqrt{x'^2 + z^2}$$

$$\vec{E}(z) = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{-x'\hat{x} + z\hat{z}}{(x'^2 + z^2)^{3/2}}\,dx'$$

$$\int \frac{x'}{\Delta r^3}\,dx' = 0 \quad \text{by symmetry}$$

$$\vec{E}(z) = \frac{\lambda z\hat{z}}{4\pi\varepsilon_0}\underbrace{\int_{-\infty}^{\infty} \frac{1}{(x'^2 + z^2)^{3/2}}\,dx'}_{2/z^2 \text{ from integral table}}$$

$$\boxed{\vec{E}(z) = \frac{\lambda\,\hat{z}}{2\pi\varepsilon_0 z}}$$