Electricity and Magnetism II [3812] Final Exam
Thursday May 9, 2019

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

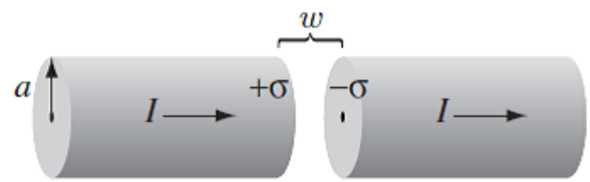
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Question 1 (25 points): A wire of radius *a,* with a constant current *I* distributed uniformly over its cross section, has a narrow gap removed (width $w << a$) so that charge builds up on the two sides of the gap. The resulting uniform electric field in the gap is $\vec{E}(t) = \frac{It}{\pi\varepsilon_0 a^2}\hat{z}$.



1a (8 points): Find the magnetic field magnitude and direction in the gap, as a function of the radial distance *s* from the axis of the wire.

1c (9 points): Find the Poynting vector magnitude and direction in the gap, as a function of time *t*, and use your result to determine the rate at which the total electromagnetic energy U_{EM} in the gap is increasing or decreasing.

1c (8 points): Find the total electromagnetic energy U_{EM} in the gap, as a function of time t. Take its time derivative and verify that this matches the answer from 1b.

Question 2 (20 points): Consider an electromagnetic plane wave with angular frequency ω and electric field amplitude E_0, propagating in the +z direction and polarized in the +y direction.

2a (5 points): Assuming the wave propagates through vacuum, explicitly write the complex vector forms of E and B for the parameters given. You may assume the phase $\delta = 0$.

2b (5 points): Utilize the real part of the solutions from 2a to derive the electric and magnetic field portions of the electromagnetic energy density u_{EM} as a function of z and t (i.e., not the averaged values). You may assume the phase $\delta = 0$. What is the ratio between the electric and magnetic field energy density?

2c (5 points): Now, assume the wave instead propagates through a linear dielectric with permittivity ε and permeability μ. How do the answers from 2a and 2b change?

2d (5 points): If the wave in 2a travels at normal incidence from vacuum to the dielectric of 2c, use the continuity of the magnetic field to derive a relationship between the complex electric field amplitudes of the incident, reflected, and transmitted waves.

Question 3 (20 points): The Liénard-Wiechert potentials for a particle traveling along the x-axis with constant velocity *v* can be written for positions on the x-axis as:

$V(x,t) = \frac{q}{4\pi\varepsilon_0}\frac{1}{\Delta r(1-\frac{v}{c})}$ $\quad\quad$ $\vec{A}(x,t) = \frac{q}{4\pi\varepsilon_0 c^2}\frac{v\hat{x}}{\Delta r(1-\frac{v}{c})}$

In these expressions, $\Delta r = x - vt_r$ is the separation distance from the retarded position of the particle.

3a (8 points): Express the retarded time t_r in terms of Δr, solve for the value of Δr, and substitute the result in order to rewrite these potentials in terms of the quantity $x - vt$, the separation distance from the current position of the particle.

3b (6 points): Explicitly show that these potentials are in the Lorenz gauge.

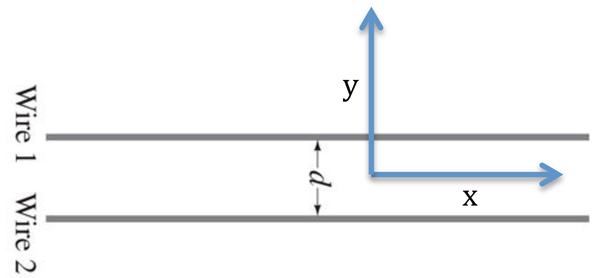
3c (6 points): Use these potentials to compute the electric field along the x-axis.

Question 4 (10 points): Imagine a charged particle with mass m and charge q with an initial velocity $v_0 << c$, which undergoes a constant acceleration a_0 opposite to v_0 until it comes to a stop (in other words, it decelerates for a time $t = v_0/a_0$).

4a (5 points): Find the ratio of the total energy lost to radiation during the deceleration to the kinetic energy $\frac{1}{2}mv_0^2$ lost by the particle.

4b (5 points): How would you solve this problem if the initial velocity <u>was</u> a significant fraction of the speed of light? Don't actually solve any equations, but describe what you would do, including the equations you would use.

Question 5 (25 points): Imagine two long parallel charged wires, each at rest, and each carrying the same positive charge per unit length λ. The two wires are both in the *x-y* plane, and lie parallel to the x-axis, at *y = +d/2*, and *y = -d/2*.



In the frame *S* where the wires are at rest, they each produce an electric field $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}$, where *s* is the radial distance from the wire. The (repulsive) force per unit length between the two wires is then $F/L = \frac{\lambda^2}{2\pi\varepsilon_0 d}$.

Now, imagine that the two wires move with a velocity *v* to the right (+x direction). Analyze this situation by using a Lorentz transformation from *S* to a frame *S'* moving to the left with respect to the wires (i.e. *u = -v*).

5a (5 points): What is the charge density λ' in the frame *S'* where the wires are moving? Use this to compute the electric field due to each wire in *S'*.

5b (5 points): Use the Lorentz transformation of the fields to compute the electric and magnetic fields of the wires in frame S', in the *x'-y'* plane. Verify that your answer for the electric field matches your answer from 5a.

5c (10 points): Calculate the total force per unit length F'/L' between the wires in S', using the charge density from 6a and the electric and magnetic fields from 5b.

5d (5 points): Use the Lorentz transformations of the Minkowski force K^{μ} between the two wires and the length L to verify your answer from 5c.

Note: No Questions on Relativistic Forces/Dynamics on the Final this Year (2020)