Electricity and Magnetism II [3812] Practice Midterm 1

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

This test is designed to be gender and race neutral.

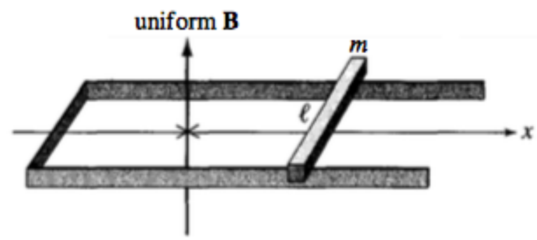
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name________________________________

Print Your Name________________________________

Question 1 (25 points): A bar with resistance *R*, length *l*, and mass *m* is placed on two conducting frictionless rails, which complete the circuit, as shown in the figure. A constant and uniform magnetic field points perpendicular to the plane of the rails as shown.



1a (15 points): Using *F = ma*, derive a differential equation for the motion of the bar, in terms of the bar's velocity *v* and its derivative *dv/dt*, the magnetic field *B*, and the properties of the bar.

1b (10 points): If the initial velocity of the bar is v_0, how much total energy is dissipated in the resistor by the time the bar comes to a complete halt (assuming it doesn't run off the rails)?

Question 2 (35 points): A long solenoid of length l and radius R, with N loops of wire, with vacuum inside, has an approximately uniform axial magnetic field $\vec{B} = \mu_0 \frac{N}{l} I\hat{z}$ within the loops. Now, imagine that the current through this solenoid is increased at a constant rate $\frac{dI}{dt} = K$.

2a (10 points): Find the direction and magnitude of the induced electric field as a function of position inside the solenoid directly from Maxwell's equations. Integrate this electric field around the N loops of wire to get a total induced EMF, as a function of *K*.

2b (10 points): Compute the self-inductance *L* of the solenoid, and from this compute the total induced EMF, as a function of *K*.

2c (15 points): Compute the magnitude and direction of the Poynting vector $\vec{S}$ at the surface of the solenoid and from this find the total electromagnetic energy per unit time transported into the solenoid, as a function of *I* and *K*.

Question 3 (15 points): Compute the elements of the Maxwell stress tensor above and below an infinite sheet of charge lying in the x-y plane with uniform charge density σ (the corresponding electric field is $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$, with $\hat{n}$ the normal to the plane on each side). Is there any electromagnetic force being exerted anywhere in the system? If so, where?

Question 4 (25 points): An electromagnetic wave with angular frequency ω and amplitude E_0 is polarized in the x direction ($\hat{n} = \hat{x}$) and propagating in the +z direction ($\vec{k} = k\hat{z}$).

4a (5 points): Write the time-dependent real vector electric and magnetic fields as a function of position and time.

4b (20 points): Compute the electromagnetic energy density u_{EM} and the Poynting vector $\vec{S}$, and explicitly show that the wave satisfies the energy continuity equation $\frac{\partial u_{EM}}{\partial t} = -\nabla \cdot \vec{S}$.