Electricity and Magnetism II [3812] Midterm 2
Wednesday April 22, 2020 - 9:30am

Directions:

The exam will be posted on the course web page by 9:30am. You must submit your answers to me by e-mail by 11:30am. The exam is intended to take roughly one hour – the extra hour is grace period to check your work, scan it, and submit it.

This exam is open book and open notes. However, it is not open internet, and it is not open solutions manual, or open classmate. Please do not utilize solutions, online or otherwise, to solve the problems. I trust you all not to abuse this unique situation.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Make sure to clearly indicate your final answer.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

Please ask if you have any questions, including clarification about the instructions, during the exam. The class Zoom meeting will be open during the exam.

This test is designed to be gender and race neutral.

Good luck!

Problem 1 (30 points): Derive the Fresnel equations for the complex amplitude of the reflected and transmitted electric fields $\tilde{E}_{0R}$ and $\tilde{E}_{0T}$, in terms of the amplitude of the incident electric field $\tilde{E}_{0I}$, for a plane wave incident at an oblique angle on an interface between two dielectrics, when the polarization of the incident wave is perpendicular to the plane of incidence (s-polarized). Express the answer in terms of the parameters $\alpha = \frac{cos\theta_T}{cos\theta_I}$ and $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$.

Problem 2 (20 points): Consider the following cylindrically symmetric potentials:

$$V(\vec{r}, t) = 0 \qquad \vec{A}(\vec{r}, t) = -\frac{\lambda_0 t}{2\pi\varepsilon_0 s}\hat{s}$$

2a (10 points). Find the corresponding electric and magnetic fields $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$.

2b (10 points). Find the scalar gauge function $\lambda(\vec{r}, t)$ that transforms these potentials to a more familiar form where only the scalar potential is non-zero. *Note that this λ is not the same as the λ_0 that shows up in the potentials*.

Problem 3 (20 points): Two point charges moving along the z-axis with equal but oppositely directed velocities of magnitude *v* pass each other at the origin.

3a (10 points): Find the total electric and magnetic fields $\vec{E}(\vec{r})$ and $\vec{B}(\vec{r})$ at the instant the particles pass each other, if both particles have charges *+q*.

3a (10 points): Find the total fields at the instant the particles pass each other, if the particle moving toward +z has charge *+q* and the particle moving toward -z has *-q.*

Problem 4 (30 points): Compute the radiation from a point charge in circular motion with low velocity (*v* << *c*) in the x-y plane, by modeling it as a rotating electric dipole. Decompose the rotating dipole into two oscillating electric dipoles, one oriented along the x-direction, and one oriented along the y-direction. Use this model to calculate the instantaneous and time averaged Poynting flux $\vec{S}(\vec{r}, t)$ and $\langle\vec{S}(\vec{r}, t)\rangle$ emitted from a point charge *q* in circular motion around the origin with radius *R* and angular frequency *ω*. Express the answers in spherical coordinates, in terms of the radial coordinate *r*, the angle *θ* of the observation point from the z-axis (the axis perpendicular to the circular motion), and the azimuthal angle *ϕ*.