1a. $\vec{E} = \frac{It}{\pi \varepsilon_0 a^2}\hat{z}$

$$d\vec{E}/dt = \frac{I}{\pi \varepsilon_0 a^2}\hat{z}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \int d\vec{E}/dt \cdot d\vec{a} + \cancel{\mu_0 I_{enc}}$$

$$B\cdot 2\pi s = \mu_0 \varepsilon_0 \frac{I}{\pi \varepsilon_0 a^2}\cdot \pi s^2$$

$$\vec{B} = \frac{\mu_0 I s}{2\pi a^2}\hat{\varphi}$$

b. $\vec{S} = (\vec{E}\times\vec{B})/\mu_0$

$$= \frac{I^2 t s}{2\pi^2 \varepsilon_0 a^4}\hat{z}\times\hat{\varphi}$$

$$= -\frac{I^2 t s}{2\pi^2 \varepsilon_0 a^4}\hat{s}$$ inward

$$dU_{EM}/dt = -\oint \vec{S}\cdot d\vec{a}$$

$$= -SA = -S\cdot 2\pi a w$$

$$= \frac{I^2 t w}{\pi \varepsilon_0 a^2}$$

c. $U_{EM} = \int \left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}\right) d\tau$

$$= \int \left(\frac{I^2 t^2}{2\pi^2 \varepsilon_0 a^4} + \frac{\mu_0 I^2 s^2}{8\pi^2 a^4}\right)\cdot s\, ds\, d\varphi\, dz$$

$$= 2\pi w \int_0^a \left(\frac{I^2 t^2}{2\pi^2 \varepsilon_0 a^4}s + \frac{\mu_0 I^2}{8\pi^2 a^4}s^3\right) ds$$

$$= 2\pi w \left(\frac{I^2 t^2}{4\pi^2 \varepsilon_0 a^2} + \frac{\mu_0 I^2}{32\pi^2}\right)$$

$$= \frac{I^2 t^2 w}{2\pi \varepsilon_0 a^2} + \frac{\mu_0 I^2 w}{16\pi}$$

$$dU_{EM}/dt = \frac{I^2 t w}{\pi \varepsilon_0 a^2}$$

2.a. $\vec{E} = \tilde{E}_0 e^{i(kz-\omega t)} \hat{n}$

$\delta = 0 \Rightarrow \tilde{E}_0 = E_0$

$\vec{E} = E_0 e^{i(kz-\omega t)} \hat{y}$

$\vec{B} = \frac{k}{\omega} E_0 e^{i(kz-\omega t)} \cdot \hat{z} \times \hat{y}$

$= -E_0/c \, e^{i(kz-\omega t)} \hat{x}$

b. $u_E = 1/2 \, \epsilon_0 E^2$

$= 1/2 \, \epsilon_0 E_0^2 \cos^2(kz-\omega t)$

$u_B = B^2/2\mu_0 = \frac{E_0^2}{2\mu_0 c^2} \cos^2(kz-\omega t)$

$u_E/u_B = \frac{1/2 \, \epsilon_0 E_0^2}{E_0^2/(2\mu_0 c^2)} = \mu_0 \epsilon_0 c^2 = 1$

c. $\vec{E} = E_0 e^{i(kz-\omega t)} \hat{y}$

$\vec{B} = -E_0/v \, e^{i(kz-\omega t)} \hat{x}$ w/ $v = 1/\sqrt{\mu\epsilon}$

$u_E = 1/2 \, \epsilon E^2$

$= 1/2 \, \epsilon E_0^2 \cos^2(kz-\omega t)$

$u_B = B^2/2\mu$

$= E_0^2/(2\mu v^2) \cos^2(kz-\omega t)$

$u_E/u_B = \mu\epsilon v^2 = 1$

d. $\Delta B_\perp = 0$, $\Delta \vec{H}_\parallel = 0$

Normal incidence, so $B_\perp = 0$

$\Delta \vec{H}_\parallel = 0 \Rightarrow \Delta(\vec{B}_\parallel/\mu) = 0 \Rightarrow \Delta(\vec{B}/\mu) = 0$

$\frac{\tilde{B}_{0I} - \tilde{B}_{0R}}{\mu_0} = \frac{\tilde{B}_{0T}}{\mu} \Rightarrow \frac{\tilde{E}_{0I} - \tilde{E}_{0R}}{\mu_0 c} = \frac{\tilde{E}_{0T}}{\mu v}$

3.a. $t_r = t - \Delta r/c$

$$\Rightarrow \Delta r = x - v(t - \Delta r/c)$$
$$= x - vt + v\Delta r/c$$
$$\Rightarrow \Delta r(1 - v/c) = x - vt$$
$$\Rightarrow \Delta r = (x - vt)/(1 - v/c)$$

$$\boxed{V(x,t) = \frac{q}{4\pi\varepsilon_0 (x - vt)}, \quad \vec{A}(x,t) = \frac{qv\hat{x}}{4\pi\varepsilon_0 c^2 (x - vt)}}$$

b. $\nabla \cdot \vec{A} = \partial A_x/\partial x$

$$= \frac{-qv}{4\pi\varepsilon_0 c^2 (x - vt)^2}$$

$$\partial V/\partial t = \frac{qv}{4\pi\varepsilon_0 (x - vt)^2}$$

$$\Rightarrow \boxed{\nabla \cdot \vec{A} = -\frac{1}{c^2}\partial V/\partial t = -\mu_0\varepsilon_0 \partial V/\partial t}$$

c. $\vec{E} = -\nabla V - \partial \vec{A}/\partial t$

$$= -\partial V/\partial x\, \hat{x} - \partial \vec{A}/\partial t$$

$$= \frac{q\hat{x}}{4\pi\varepsilon_0 (x - vt)^2} - \frac{qv^2\hat{x}}{4\pi\varepsilon_0 c^2 (x - vt)^2}$$

$$= \boxed{\frac{q(1 - v^2/c^2)\hat{x}}{4\pi\varepsilon_0 (x - vt)^2}}$$

4. a. $P = \frac{\mu_0 q^2 a^2}{6\pi c} = \left.\frac{dW}{dt}\right|_{rad}$

$$\Delta W_{rad} = P \cdot t = \frac{\mu_0 q^2 a_0^2}{6\pi c} \cdot \frac{v_0}{a_0} = \frac{\mu_0 q^2 a_0 v_0}{6\pi c}$$

$$\Delta W_{rad} / \Delta W_{kin} = \frac{\mu_0 q^2 a_0 v_0}{6\pi c \cdot \frac{1}{2} m v_0^2} = \frac{\mu_0 q^2 a_0}{3\pi c m v_0}$$

b. use Larmor-Liénard formula

$$P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left(a^2 - \left|\frac{\vec{v} \times \vec{a}}{c}\right|\right)^2$$

$$= \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c} \text{ for } \vec{v} \times \vec{a} = 0$$

since γ contains v, you'd have to integrate this equation to get ΔW_{rad}

5.a. Length is contracted $L' = L/\gamma$

$\Rightarrow$ higher charge density

$\lambda' = \gamma \lambda$

$$\Rightarrow \vec{E}' = \frac{\gamma \lambda \hat{s}}{2\pi\varepsilon_0 s}$$

b. $E_y' = \gamma E_y = \dfrac{\gamma \lambda}{2\pi\varepsilon_0 s}$ matches 5a

$$B_z' = -\frac{\gamma u}{c^2} E_y = \frac{\gamma v}{c^2} E_y = \frac{\gamma \lambda v}{2\pi\varepsilon_0 c^2 s}$$

c. $F' = q(\vec{E}' + \vec{v} \times \vec{B}')$

$$= \lambda' L' E_y' - \lambda' L' v B_z$$

$$= \lambda' L' \left(\frac{\gamma \lambda}{2\pi\varepsilon_0 d} - \frac{\gamma \lambda v^2}{2\pi\varepsilon_0 c^2 d} \right)$$

$$= \lambda' L' \frac{\gamma \lambda}{2\pi\varepsilon_0 d} \left(1 - v^2/c^2\right)$$

$$= L' \frac{\gamma^2 \lambda^2}{2\pi\varepsilon_0 d} \frac{1}{\gamma^2} = L' \cdot \frac{\lambda^2}{2\pi\varepsilon_0 d}$$

$$\Rightarrow F'/L' = \lambda^2/2\pi\varepsilon_0 d = F/L$$

- Length contraction of both wires balances change in force

d. $K^\mu = \left(0, 0, \dfrac{\lambda^2 L}{2\pi\varepsilon_0 d}, 0\right)$

$K^{\mu'} = K^\mu$ since y-component unchanged

$K^{2'} = \gamma F_y' \Rightarrow F_y' = \dfrac{\lambda^2 L}{2\pi\varepsilon_0 d} \cdot \dfrac{1}{\gamma}$

$L' = L/\gamma$

$$\Rightarrow \boxed{F'/L' = \lambda^2/2\pi\varepsilon_0 d} \quad //$$