

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



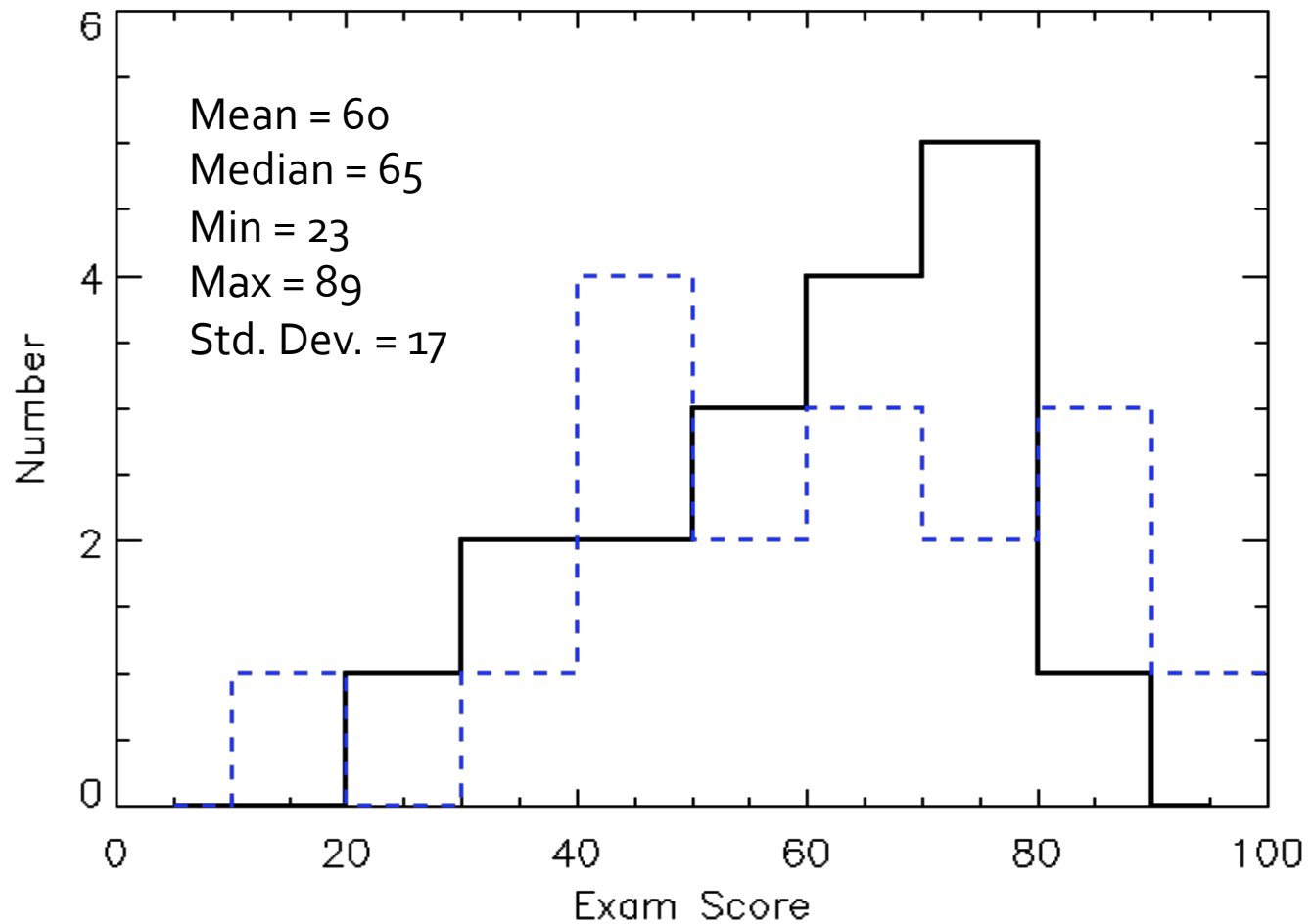
Electricity and Magnetism II: 3812

Professor Jasper Halekas
Virtual by Zoom!
MWF 9:30-10:20 Lecture

Exam 2 Scores

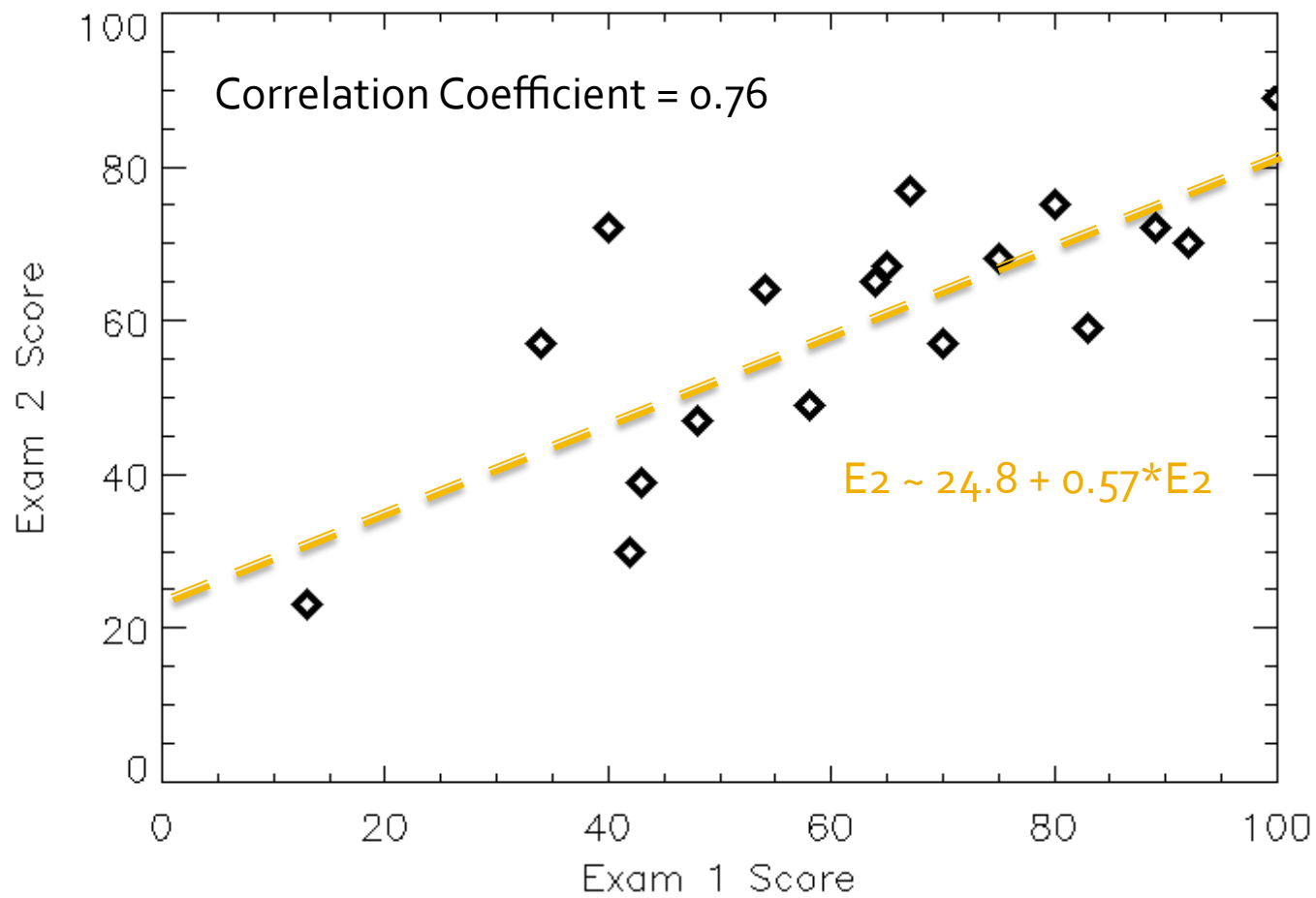


IDL 0



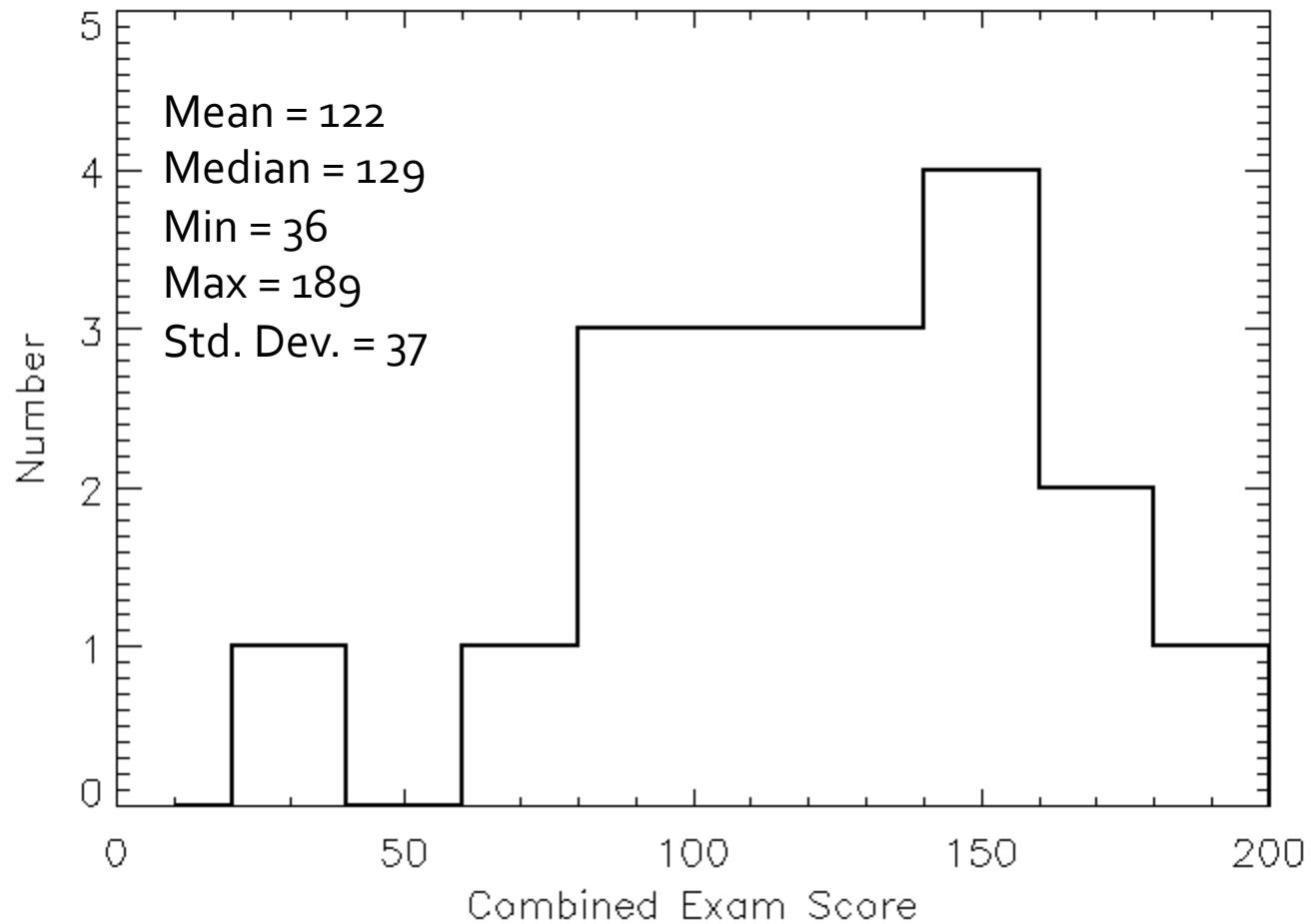
Exam 2 vs. Exam 1

○ ○ ○ IDL 0

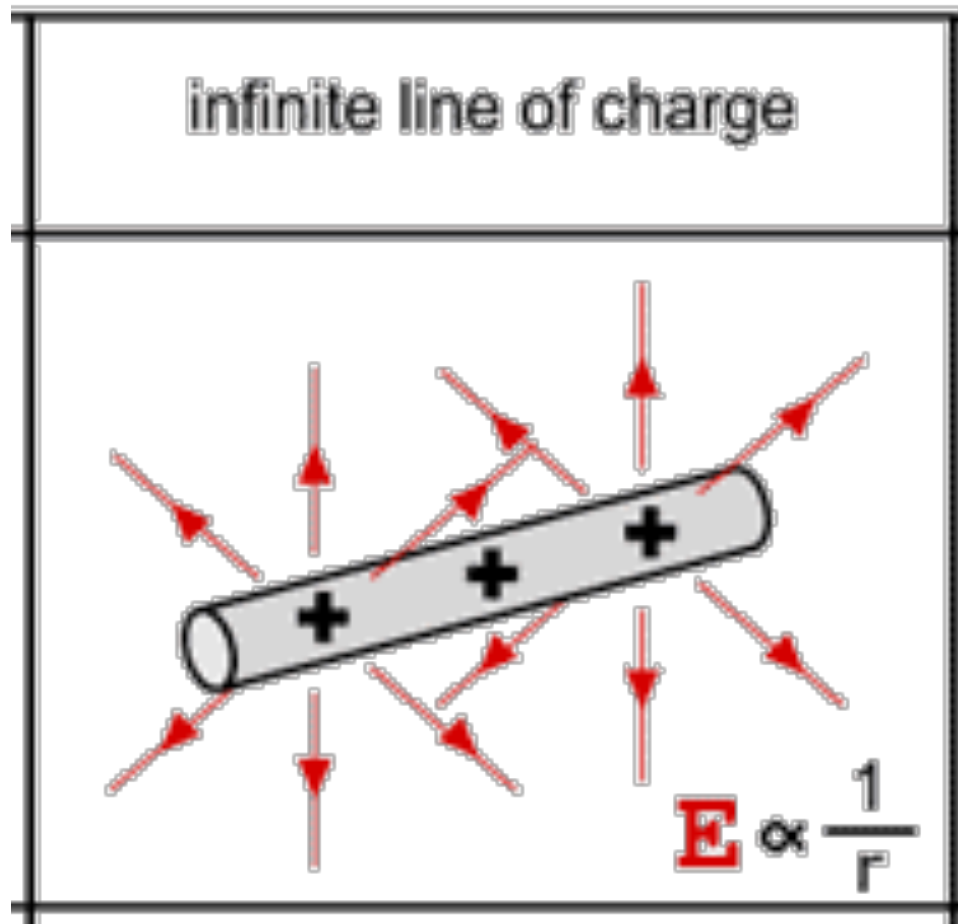


Combined Score Distribution

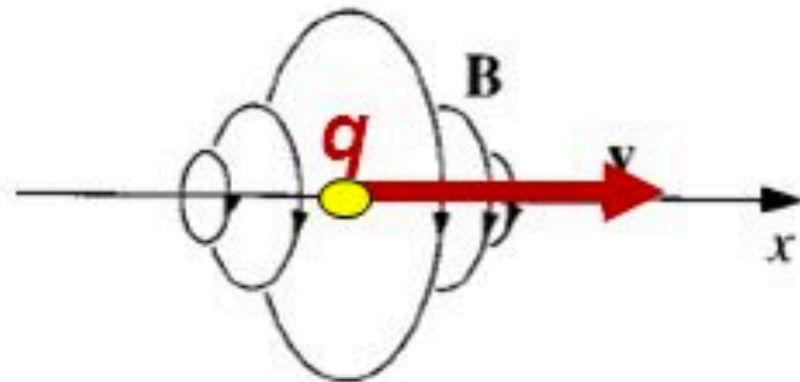
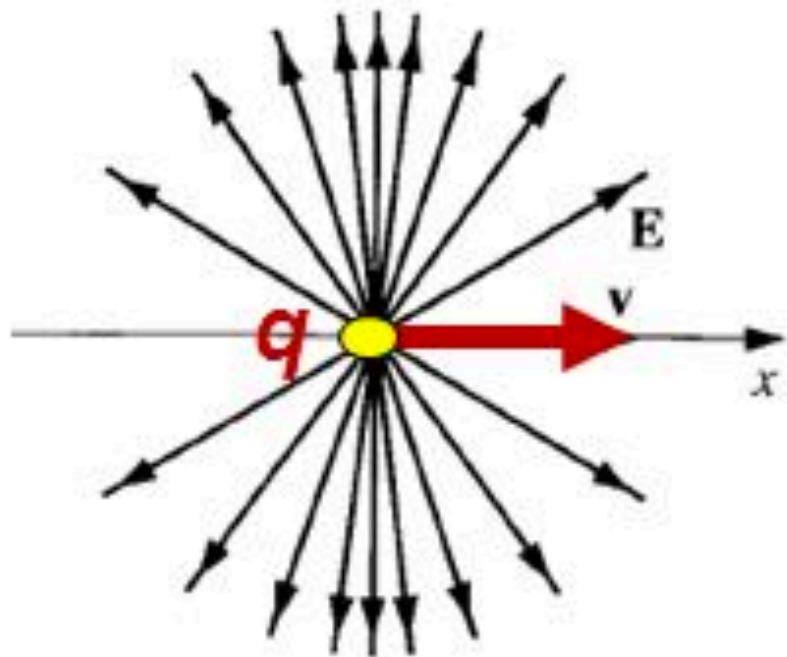
○ ○ ○ IDL 0



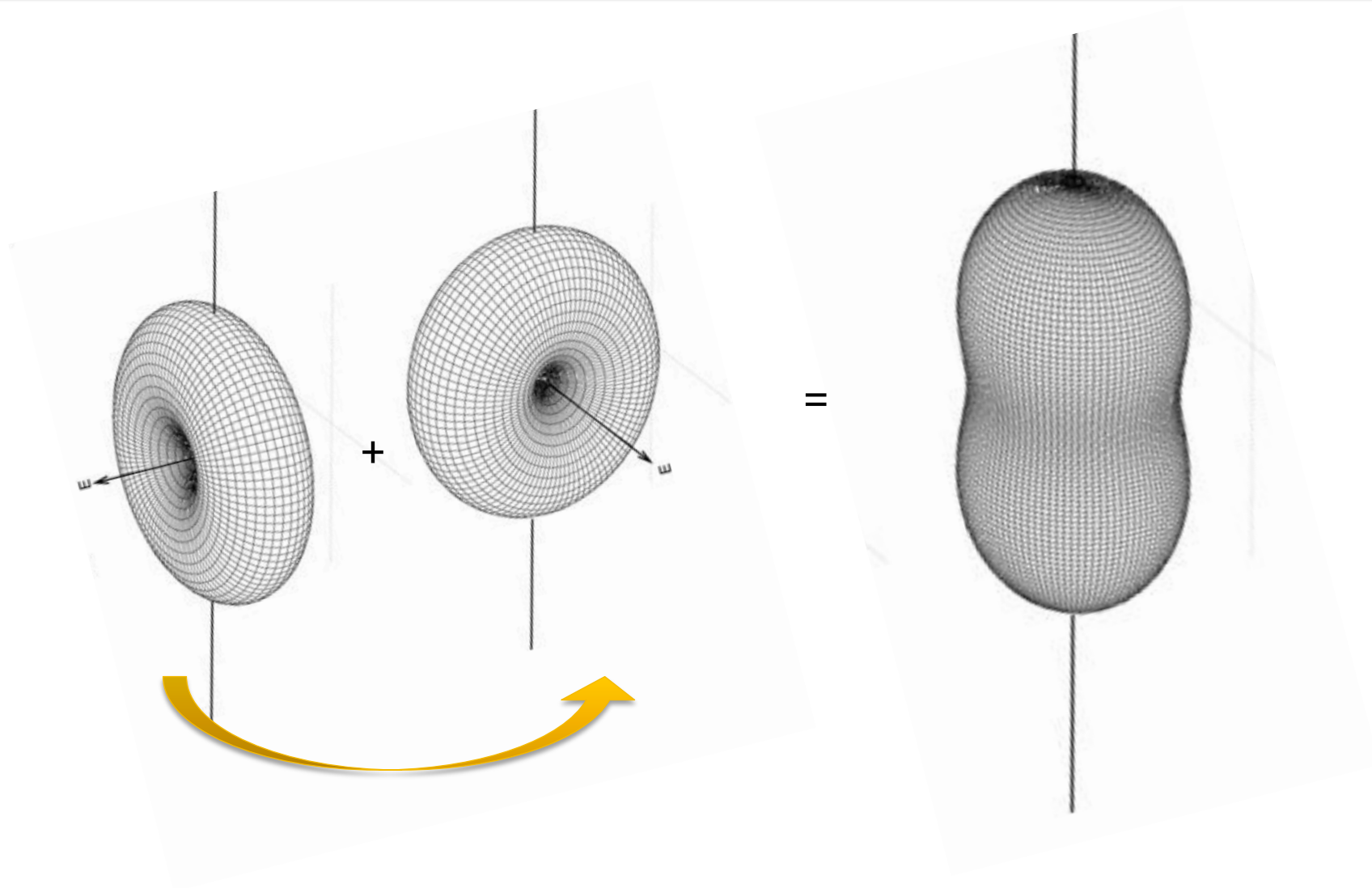
Problem 2



Problem 3



Problem 4



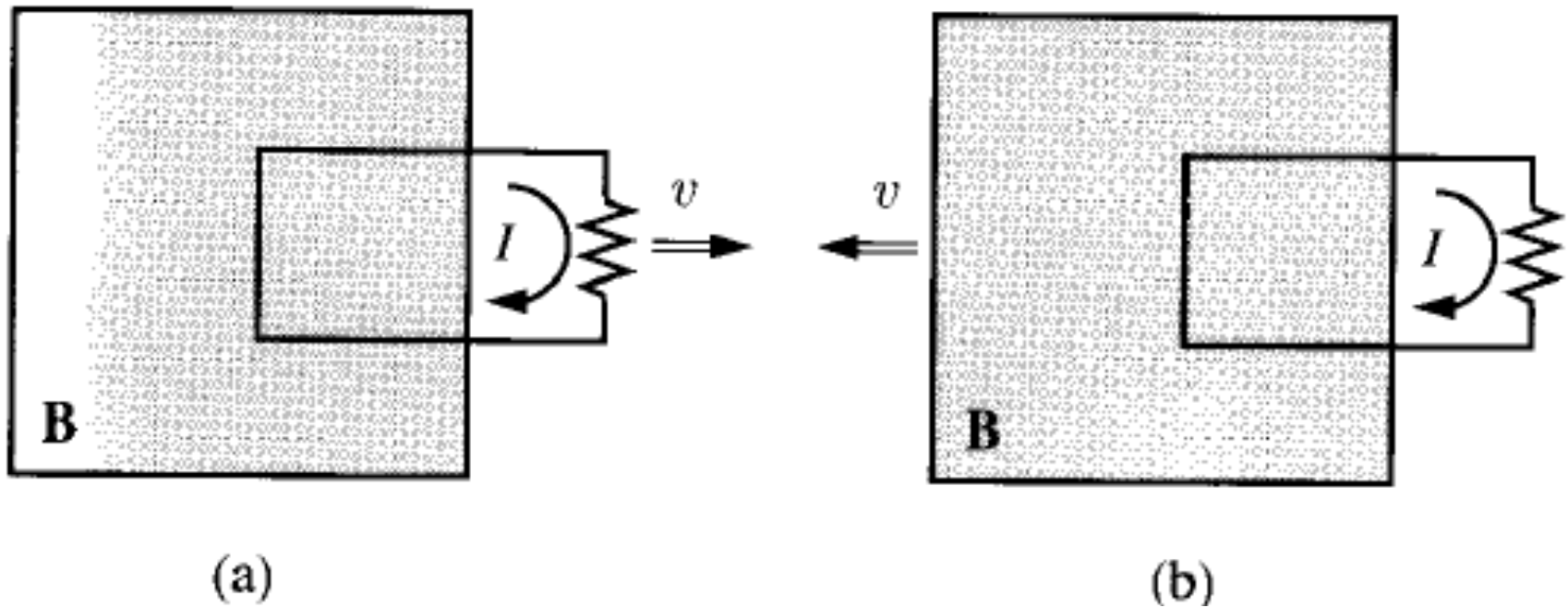
Einstein's Postulates

The laws of physics are the same in all inertial frames of reference.

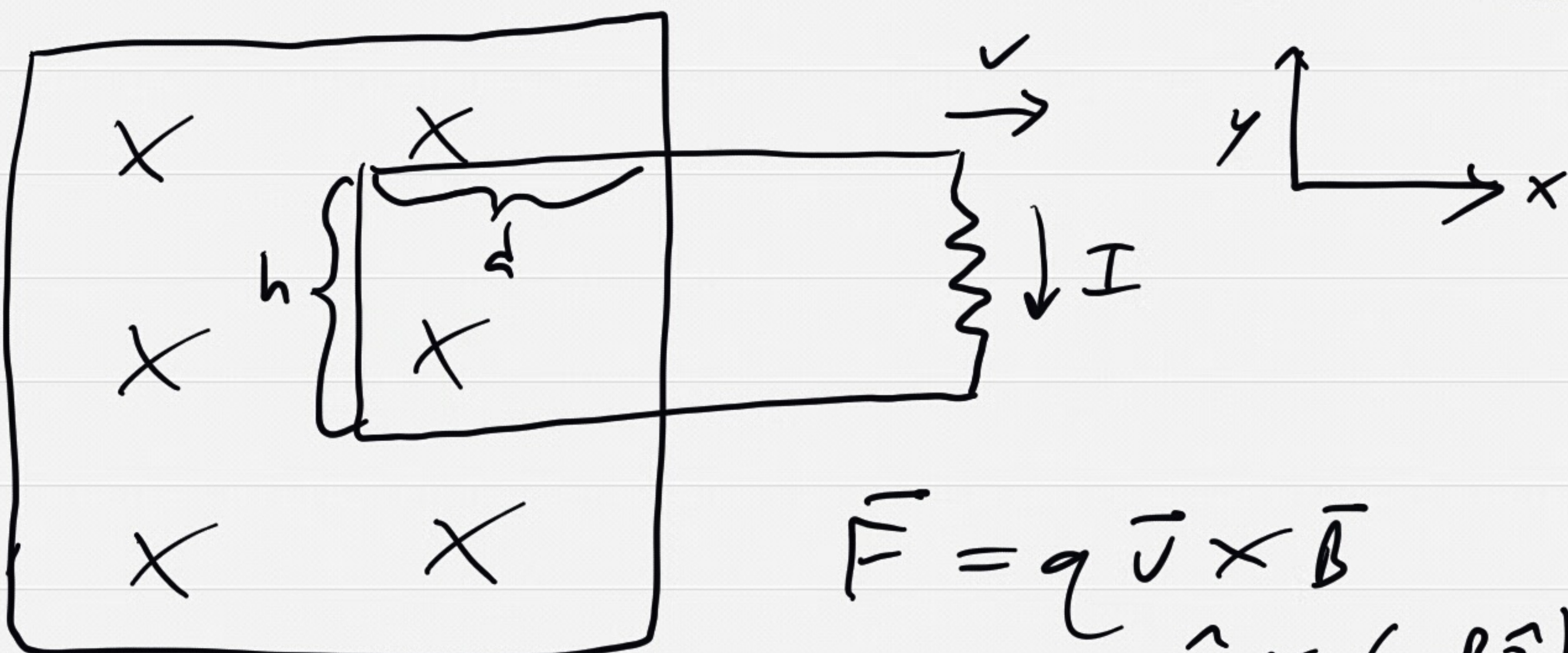
The speed of light is the same in all inertial frames of reference.



Does Relativity Apply to E&M?



Ch. 12: Electrodynamics & Relativity

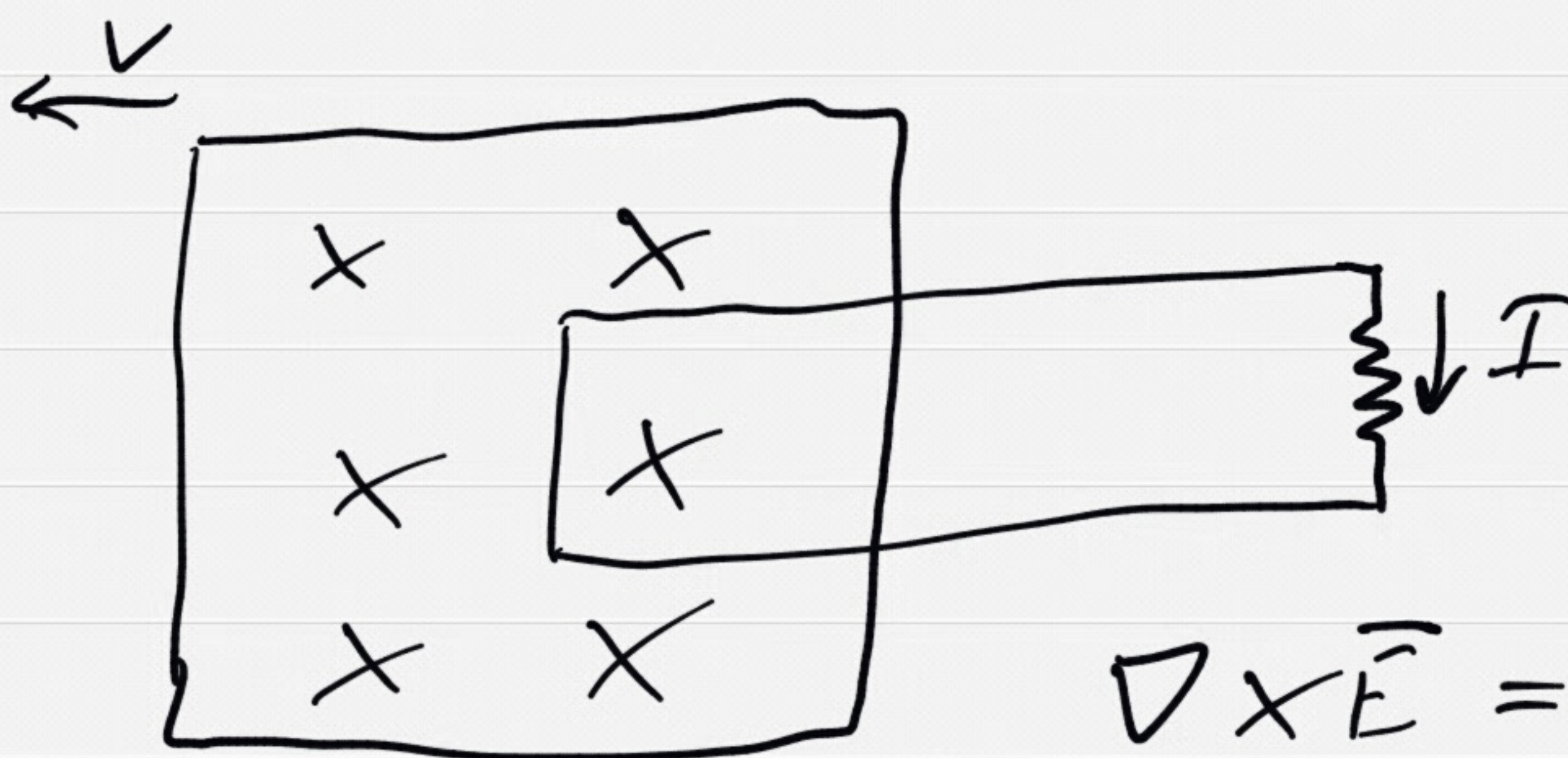


$$\begin{aligned}\vec{F} &= q \vec{v} \times \vec{B} \\ &= q v \hat{x} \times (-B \hat{z}) \\ &= q v B \hat{y}\end{aligned}$$

$$\mathcal{E} = \oint \vec{F} \cdot d\vec{\ell} = \oint \frac{\vec{F}}{q} \cdot d\vec{\ell}$$

$$= \oint v B \hat{y} \cdot d\vec{\ell} = -v B h$$

causes $I = \frac{v B h}{R}$ CW



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\mathcal{E} = -\frac{d}{dt} (-B h d)$$

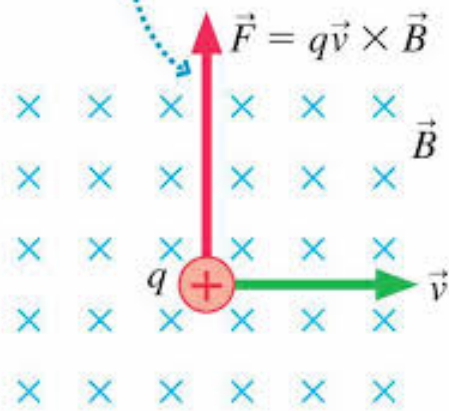
$$= -B h v$$

$$\Rightarrow I = B h v / R \quad \text{CW}$$

EMF from \vec{B} in one frame,
from \vec{E} in the other!!

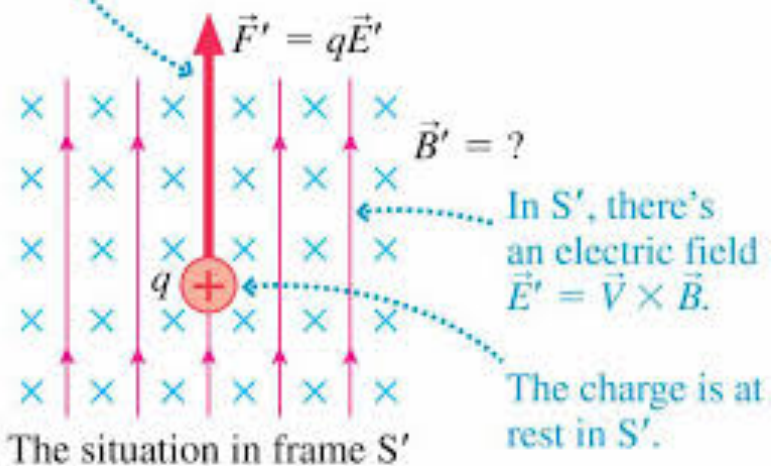
Does Relativity Apply to E&M?

In S, the force on q is due to a magnetic field.

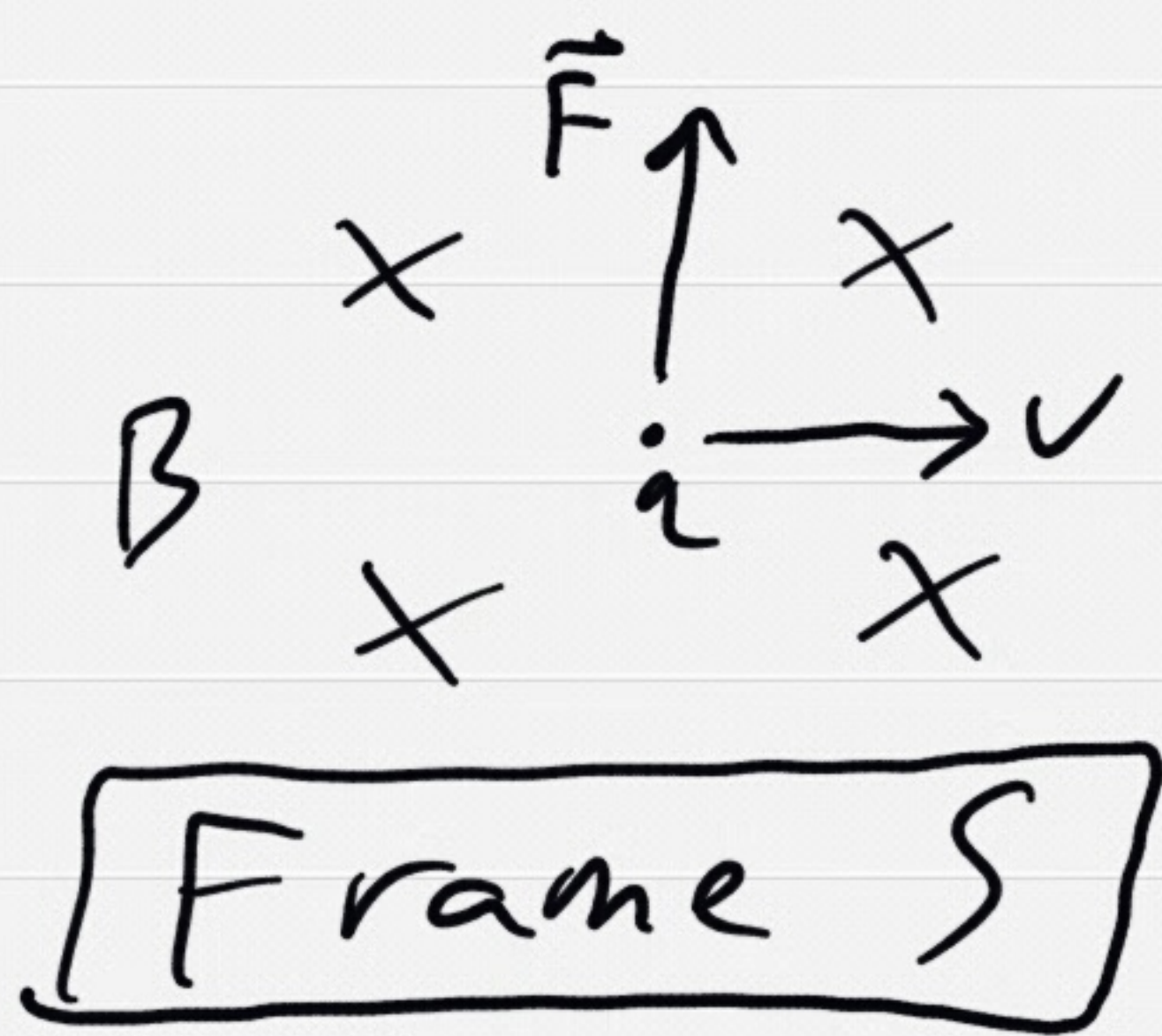


The situation in frame S

In S' , the force on q is due to an electric field.



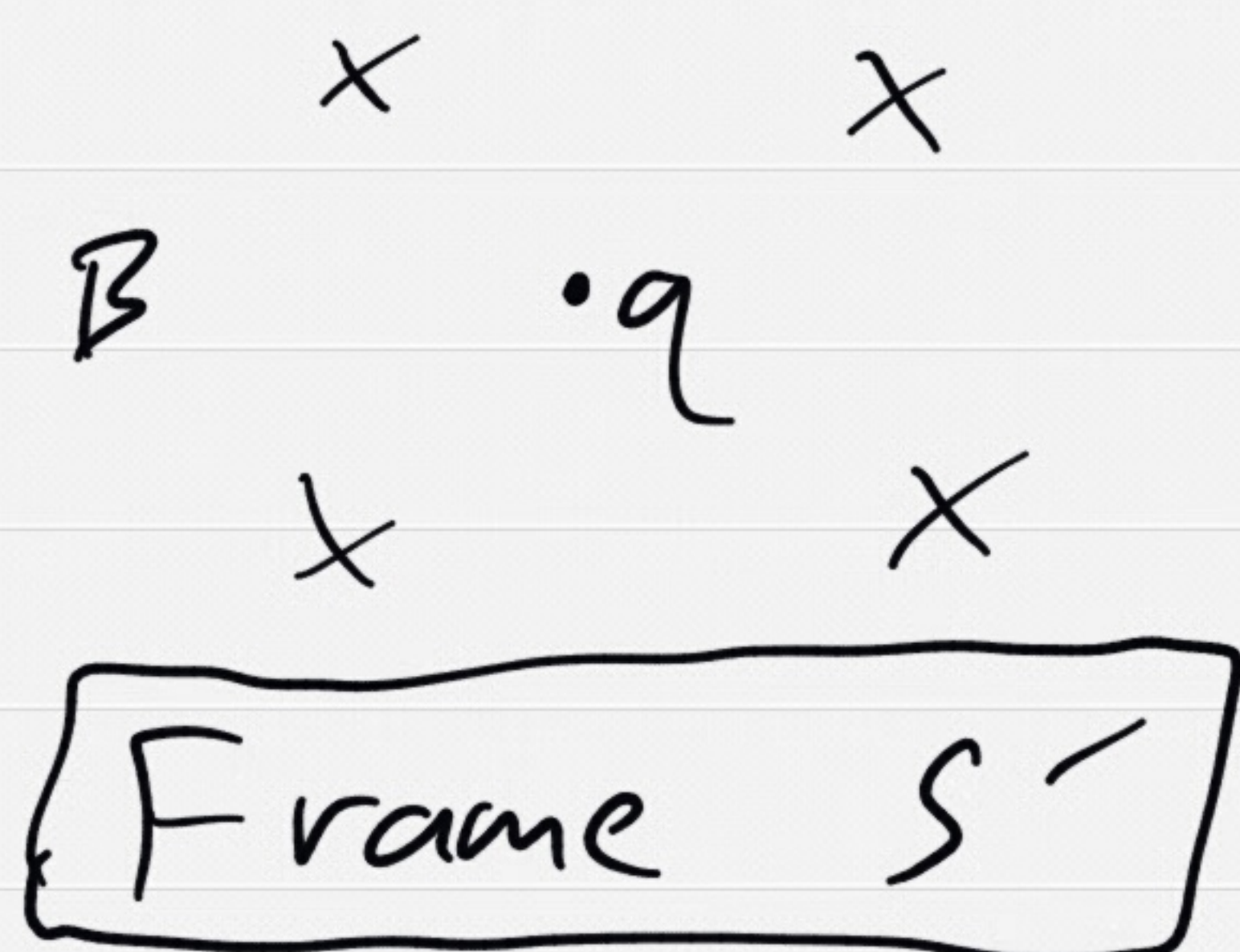
The situation in frame S'



$$\vec{F} = \vec{F}_0 = q \vec{v} \times \vec{B}$$

$$= q v \hat{x} \times (-B \hat{z})$$

$$= q v B \hat{y}$$



$$\vec{F}_0 = q \vec{v} \times \vec{B}$$

$$= 0$$

- In S' , no magnetic Lorentz force

- The only possibility is an electric Lorentz force

- Must have:

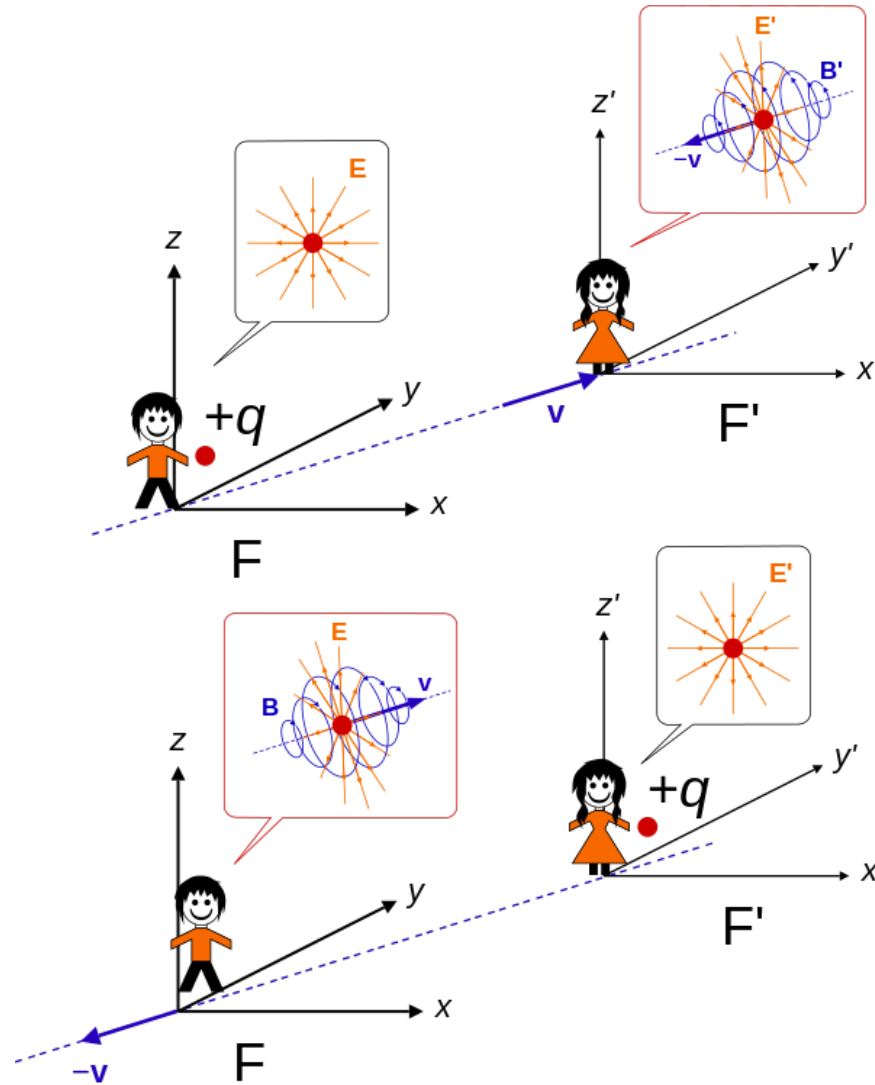
$$\vec{E}' = v B \hat{y} = \vec{v} \times \vec{B}$$

$$\text{so that } \vec{F}' = q \vec{E}' = \vec{F}$$

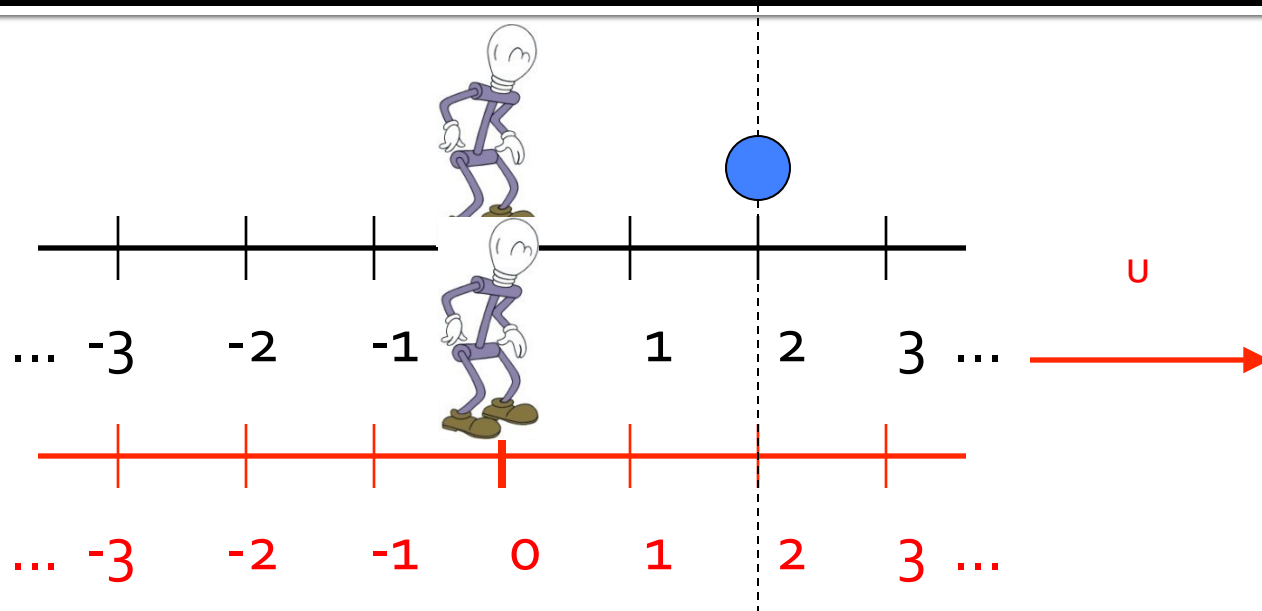
- Galilean transformation

$$\vec{E}' = q \vec{v} \times \vec{B}, \quad \vec{B}' = \vec{B}$$

Relativity and E&M



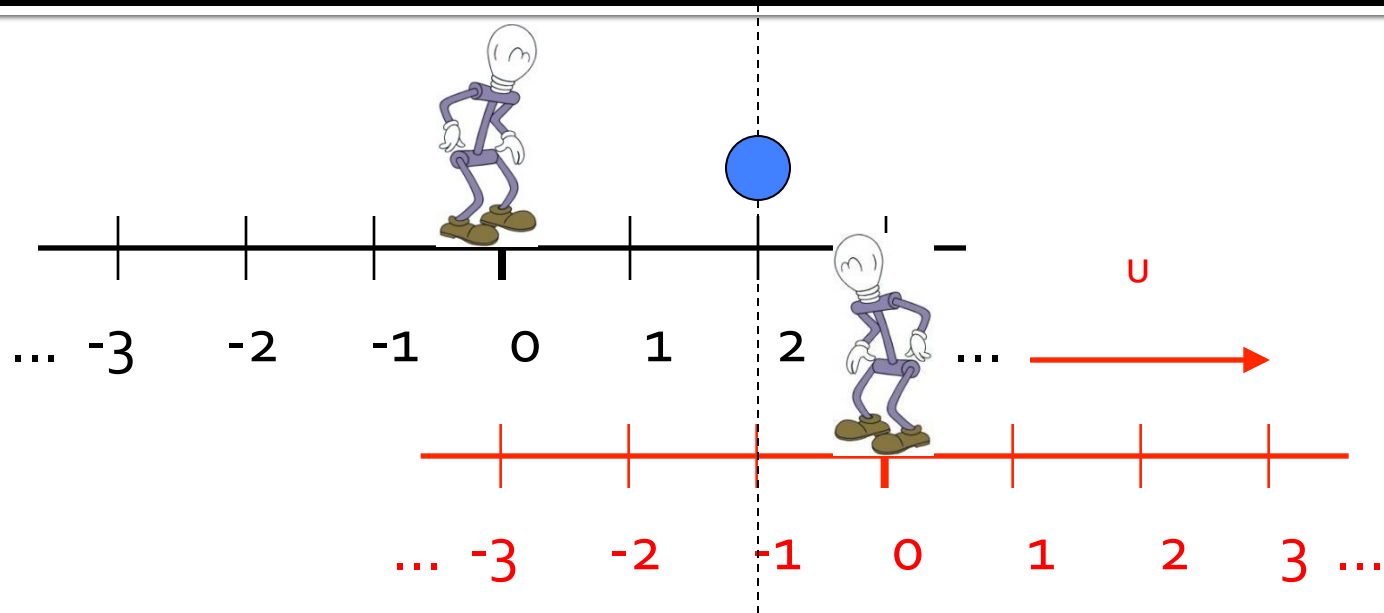
Comparing inertial frames



At time $t = 0$, the two frames coincide. A ball is at rest in frame S. Its position is

- $x = 2$ m in S
- $x' = 2$ m in S'

Comparing inertial frames



Frame S' is moving to the right (relative to S) at $u=1$ m/s. At time $t = 3$ sec, the position of the ball is

- $x = 2$ m in S
- $x' = -1$ m in S'

Galilean Transformation

$$x' = x - ut$$

$$y' = y$$

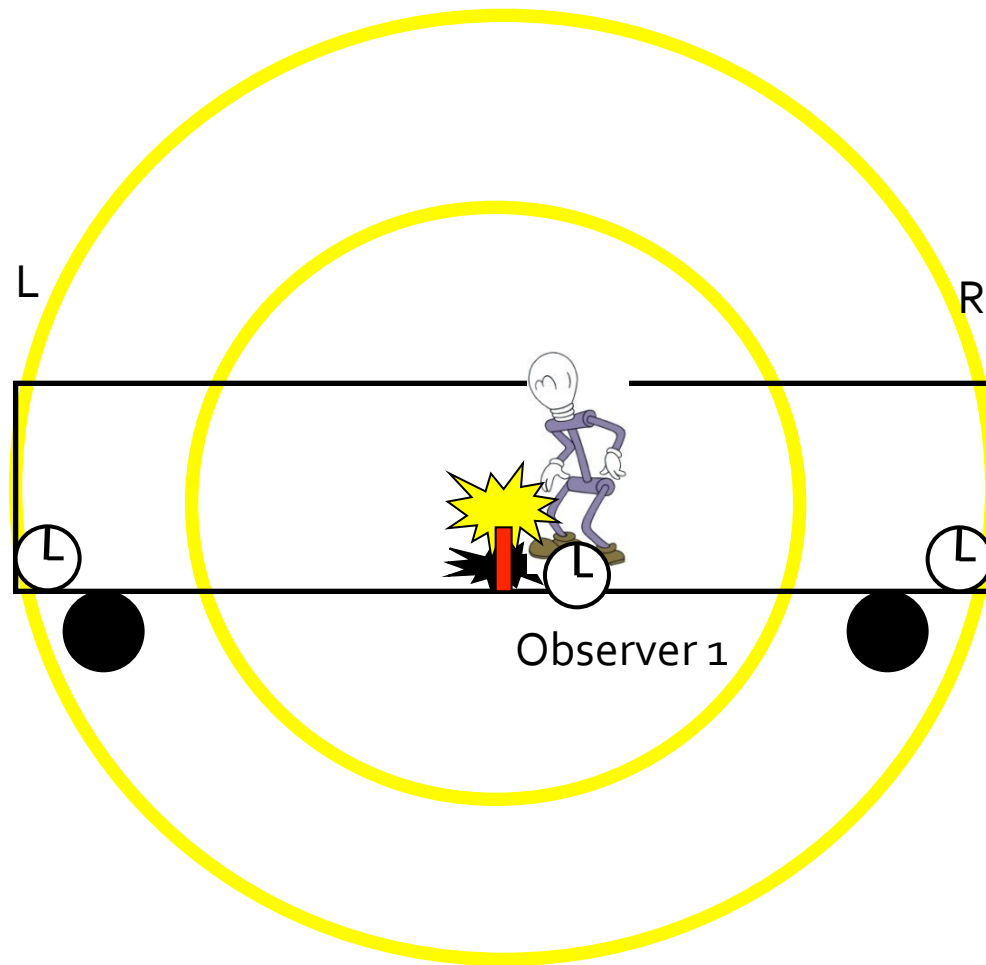
$$z' = z$$

$$t' = t$$

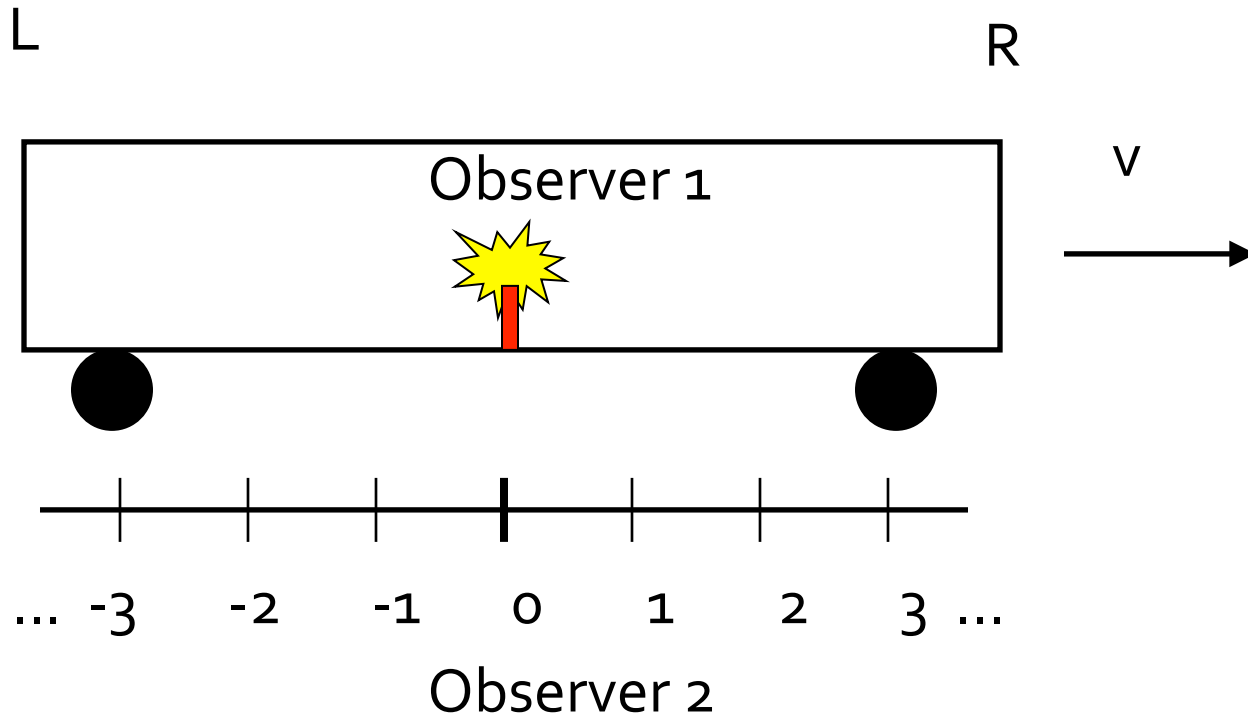
Note:
Assumes frames
aligned at $t = 0$

$$v_x' = \frac{dx'(t)}{dt} = \frac{d}{dt}(x(t) - ut) = \frac{dx(t)}{dt} - u = v_x - u$$

Observer 1 Frame

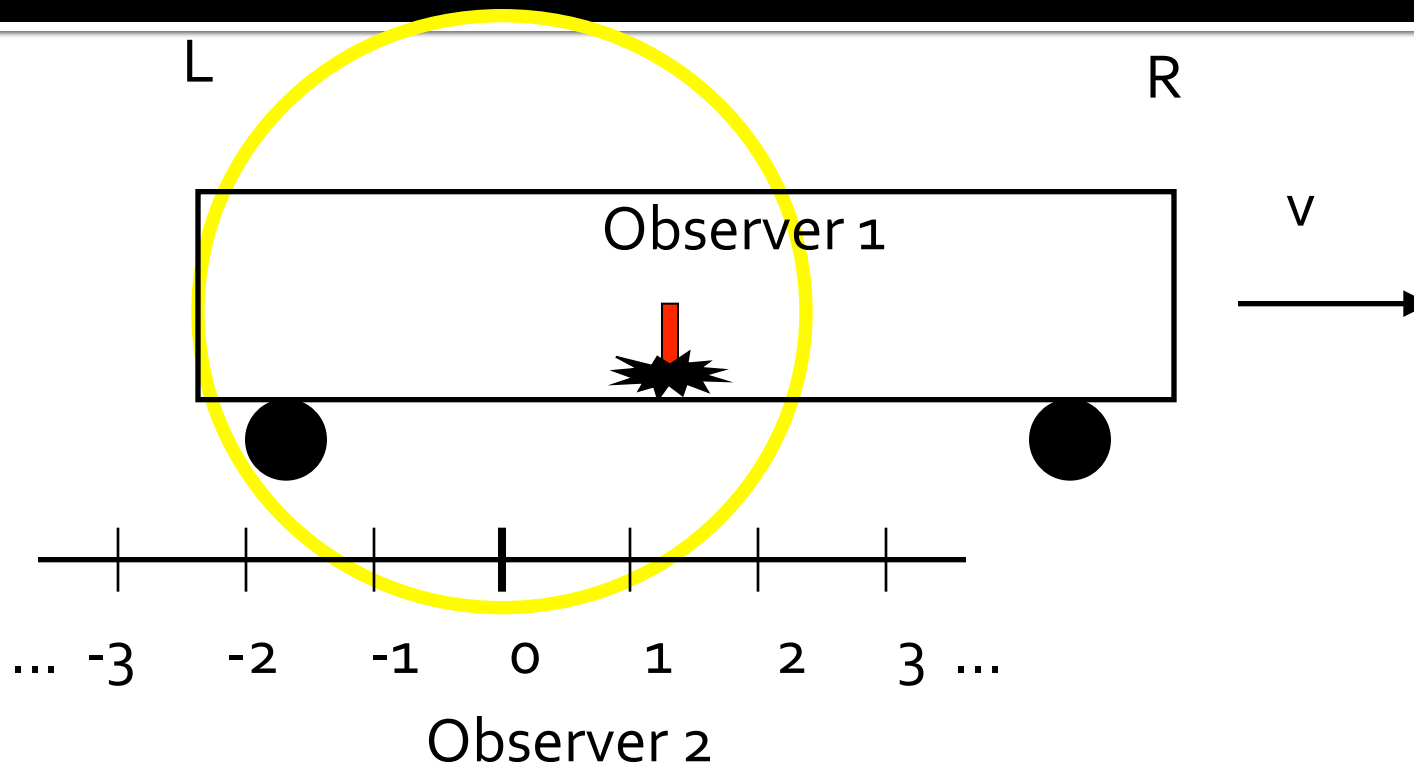


Observer 2 Frame



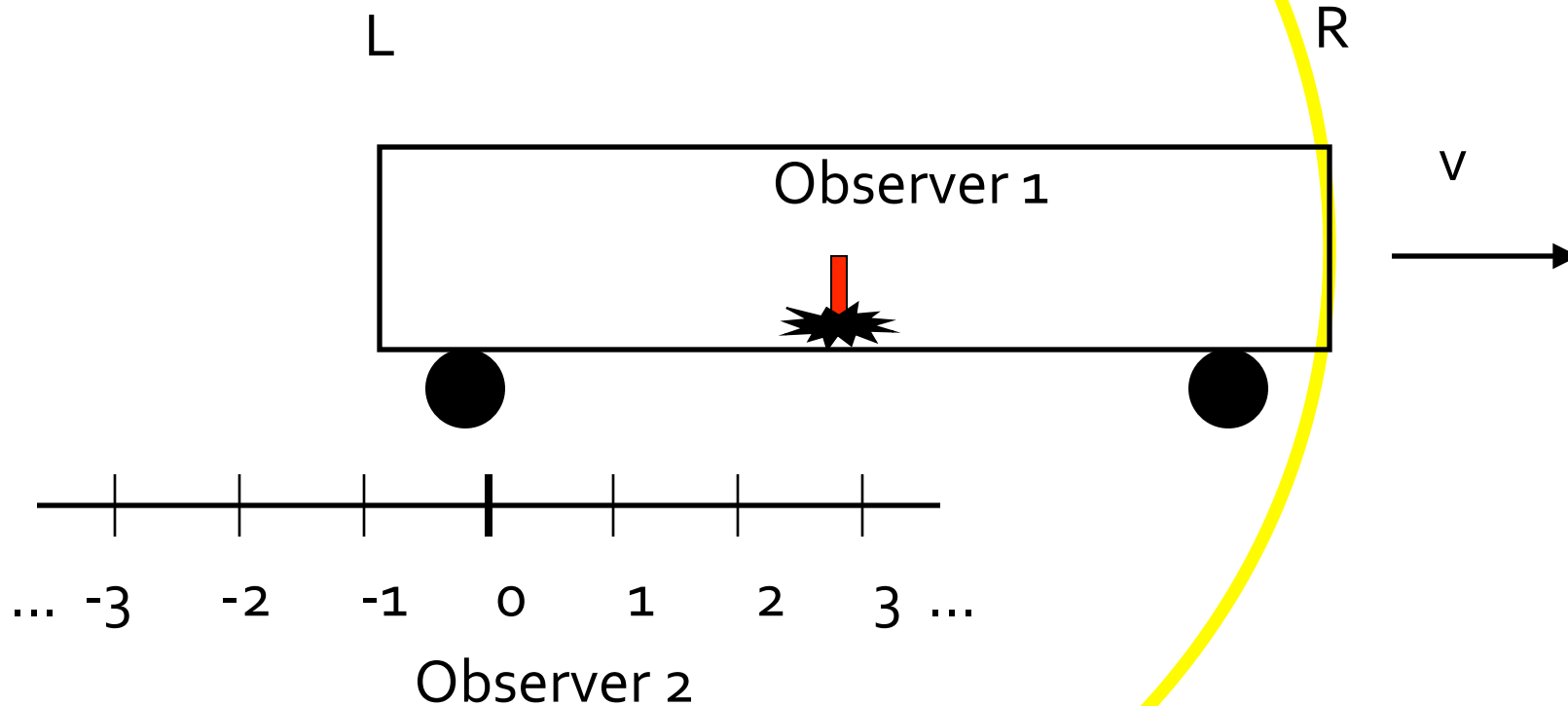
Suppose Observer 1's firecracker explodes at the origin of Observer 2's reference frame.

Observer 2 Frame



The light spreads out in Observer 2's frame from the point where they saw it explode. Because the train car is moving, the light in Observer 2's frame arrives at the left end first.

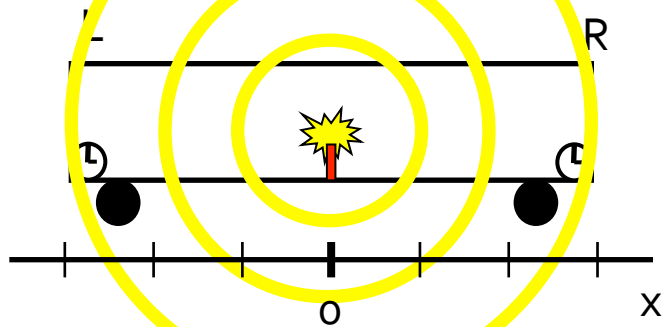
Observer 2 Frame



Sometime later, in Observer 2's frame, the light catches up to the right end of the train.

Simultaneity is Relative

Observer 1: in the train

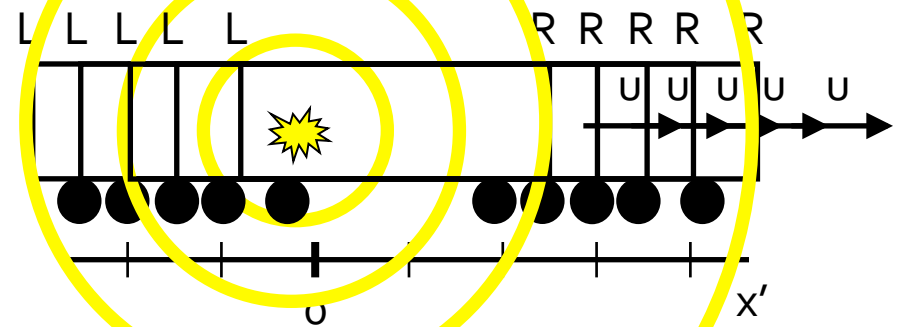


Event L:
($x=-3, t=3s$)

Event R:
($x=+3, t=3s$)

Observer 1 says: 'Simultaneous!'

Observer 2: on the platform

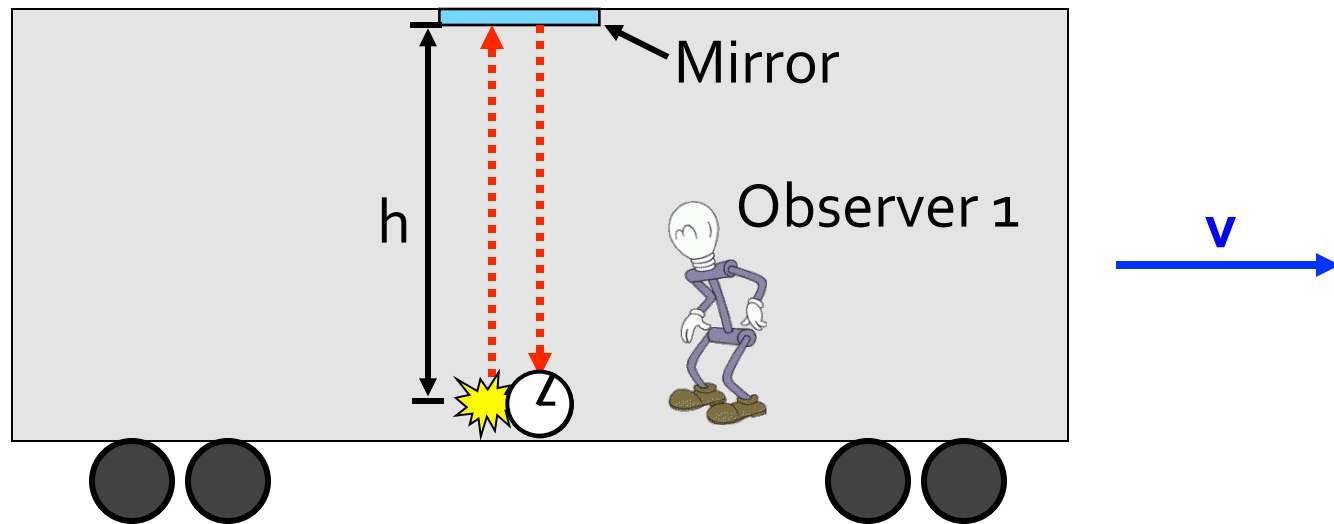


Event L':
($x'=-2, t'=2s$)

Event R':
($x'=+5, t'=4s$)

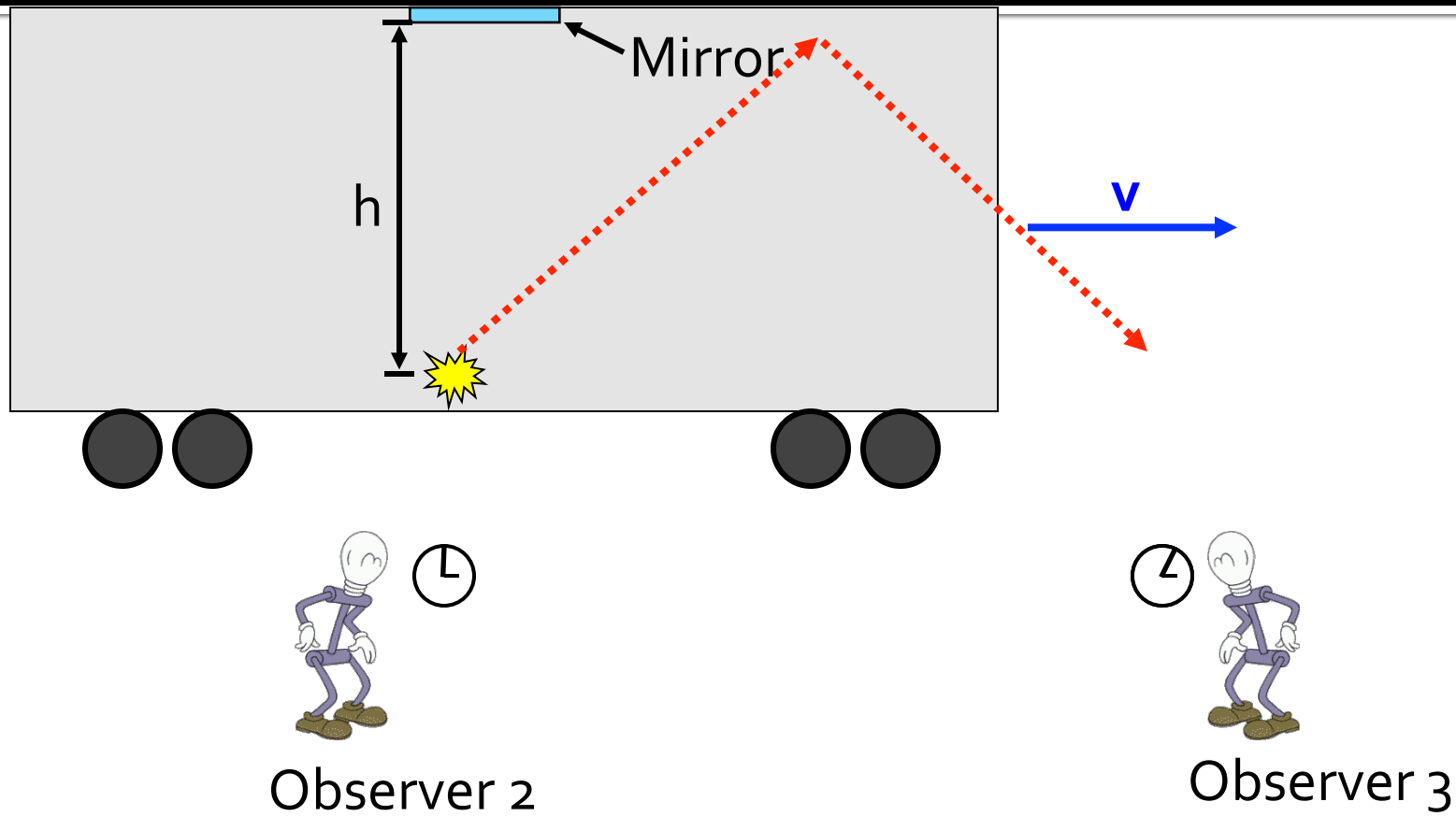
Observer 2 says: 'Not simultaneous!'

Time Dilation



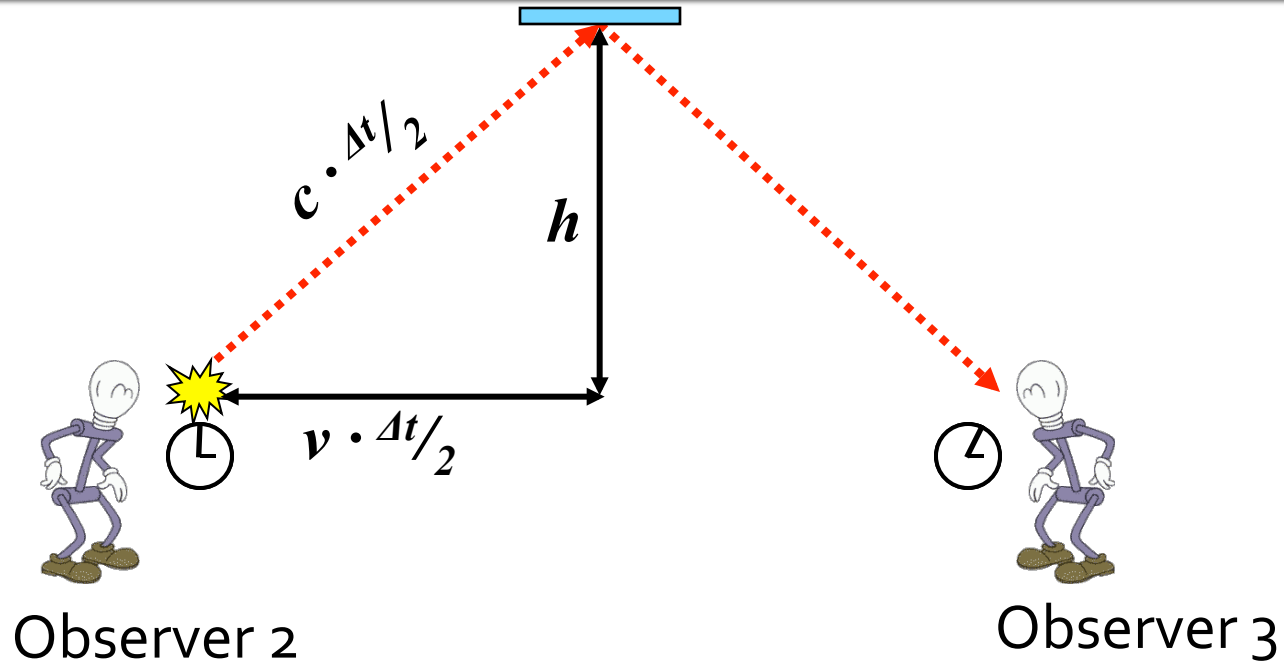
Observer 1 measures the time interval: $\Delta t' = \Delta t_0 = 2h/c$

Time Dilation



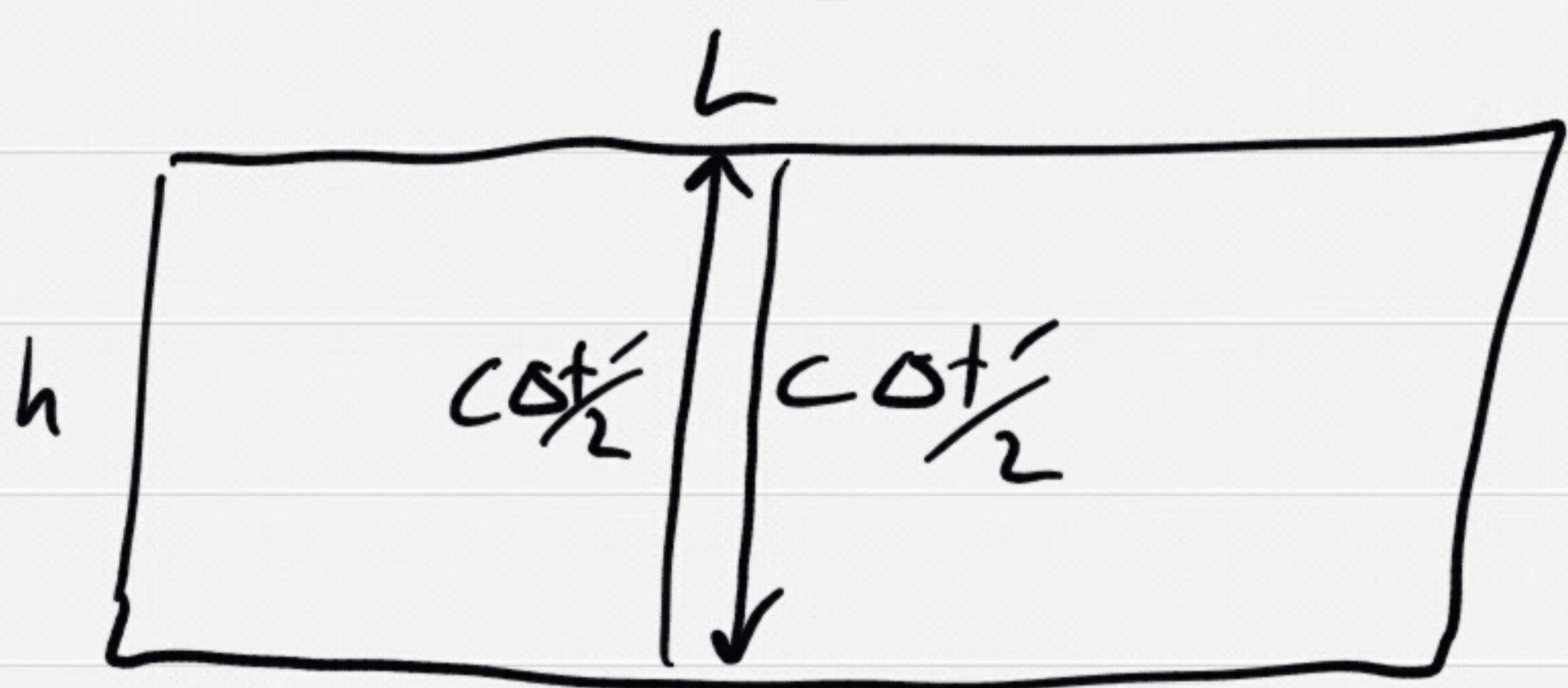
Note: This experiment requires two observers.

Time Dilation



12.1 | Special Relativity

Time dilation

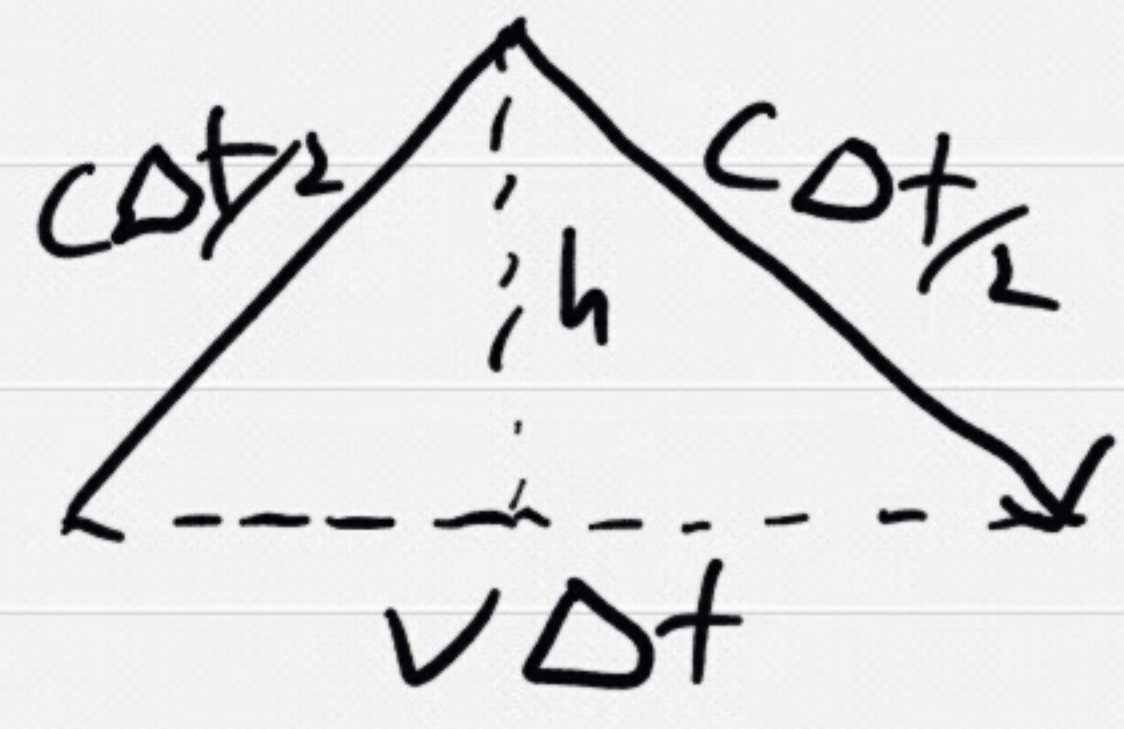


Frame S'

$$\begin{aligned} \Delta t' &= 2h/c \\ &= \Delta t_0 \\ &= \Delta \tau \\ &= \text{proper time interval} \end{aligned}$$



Frame S

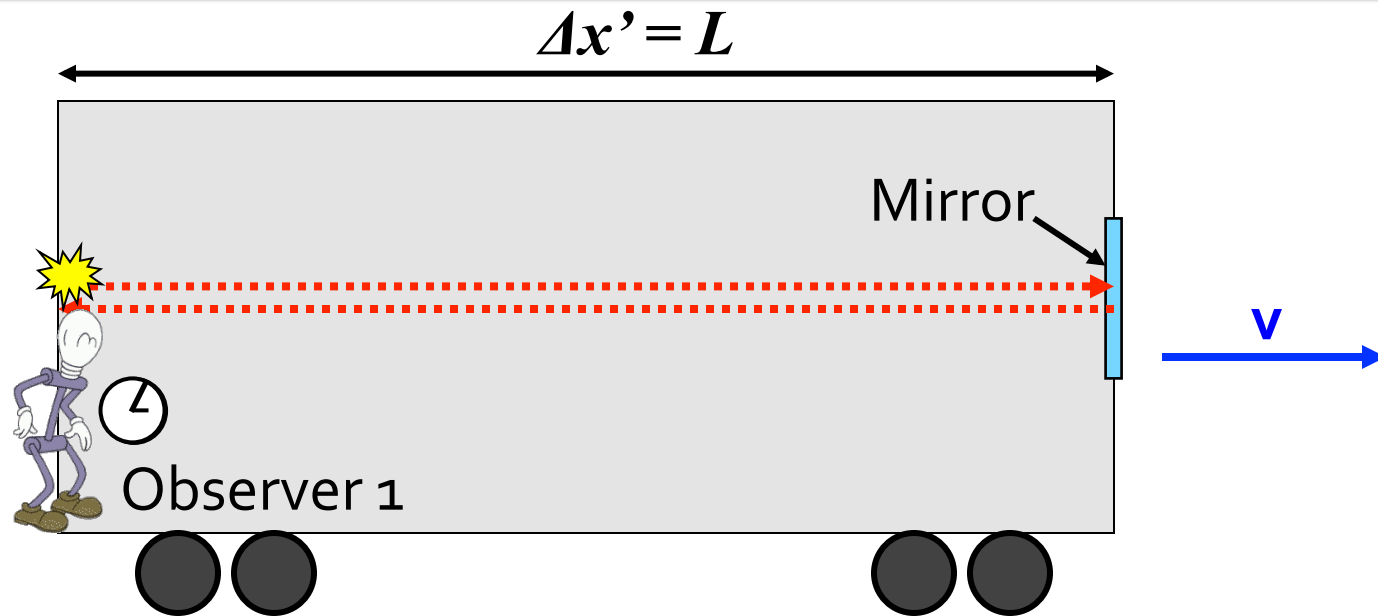


$$\begin{aligned} \left(\frac{c\Delta t}{2}\right)^2 &= \left(\frac{v\Delta t}{2}\right)^2 + h^2 \\ \Rightarrow \Delta t &= \frac{2h}{c} \cdot \frac{1}{\sqrt{1 - v^2/c^2}} \\ &= \gamma \Delta t' \end{aligned}$$

Proper Time

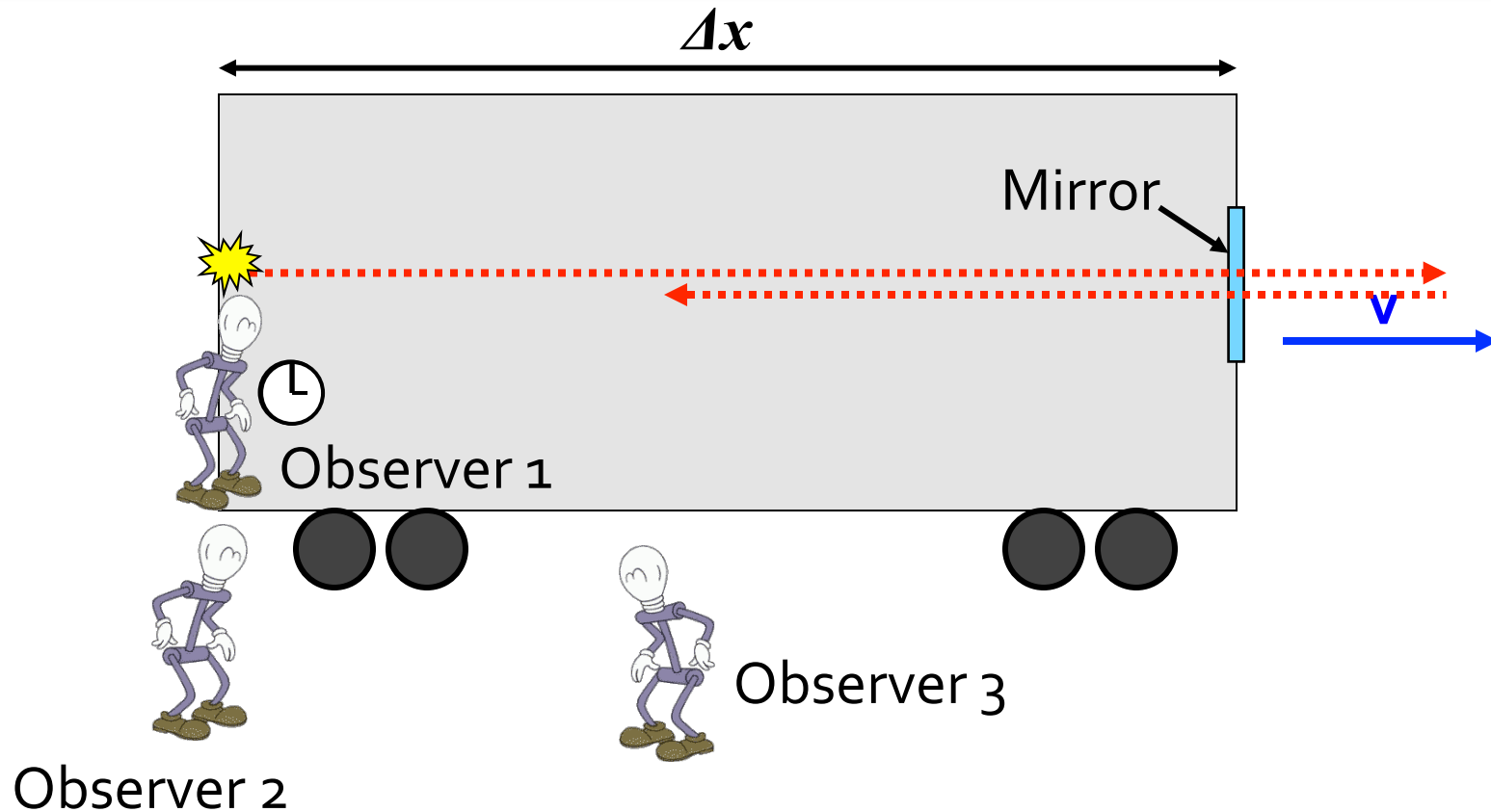
- “Proper Time” = $\Delta\tau = \Delta t_0$
 - The time interval measured in a frame where the two events occur at the same spatial coordinate – i.e. the frame moving with your clock
- The time interval Δt measured in **any** other frame moving with respect to this frame will be longer
 - $\Delta t = \gamma\Delta t_0$

Length Contraction



Observer 1 measures the time interval: $\Delta t' = \Delta t_0 = 2L/c$

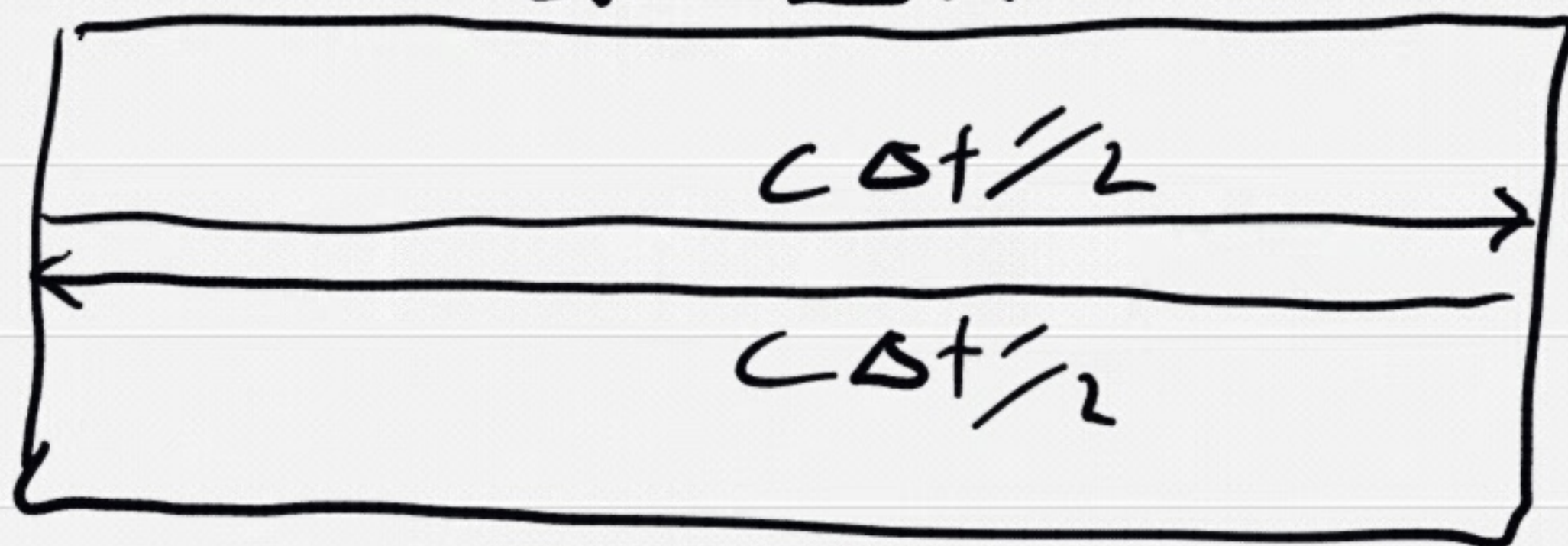
Length Contraction



Observers 2 and 3 measure the time interval:
 $\Delta t = \Delta x / (c - v) + \Delta x / (c + v) \Rightarrow \Delta x = \Delta x' / \gamma$

Length Contraction

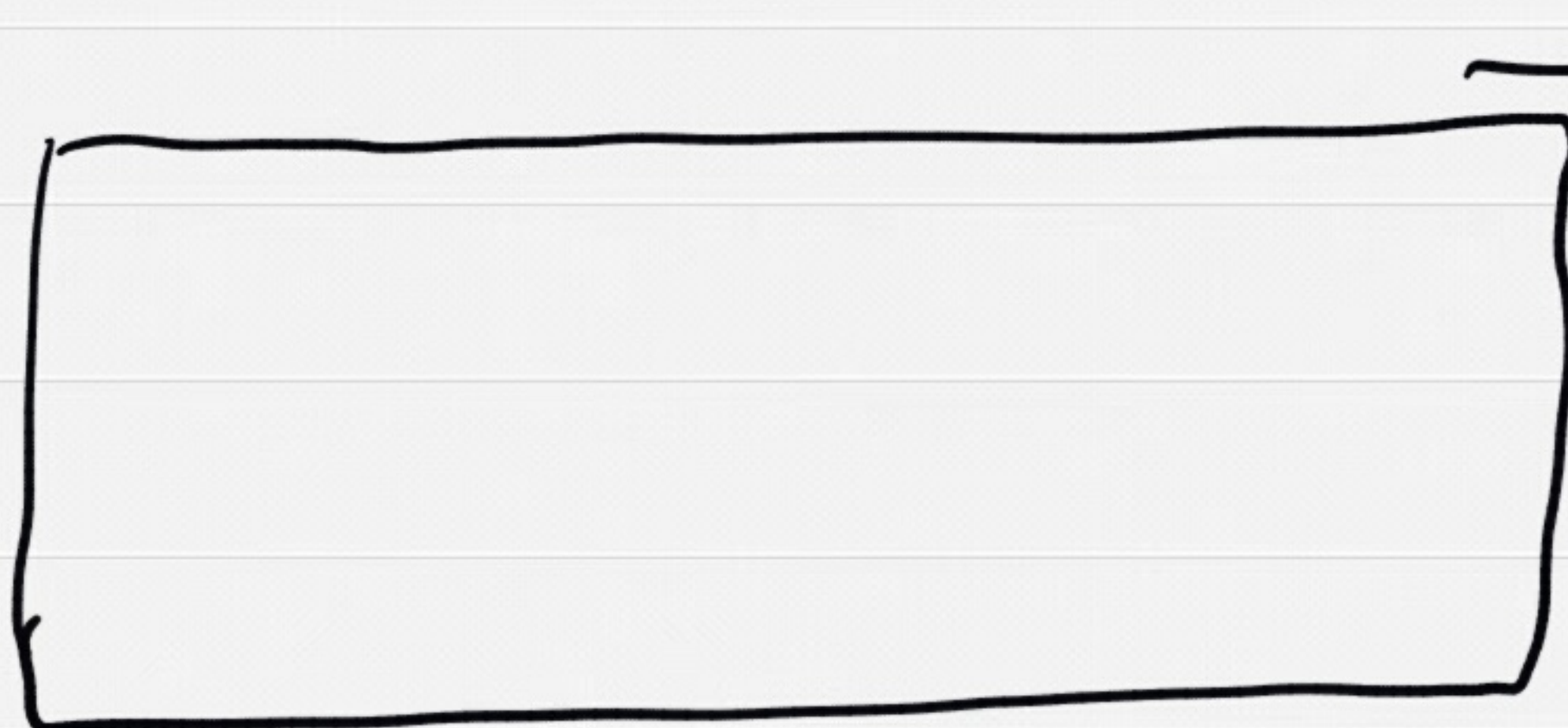
$$L_0 = \Delta x'$$



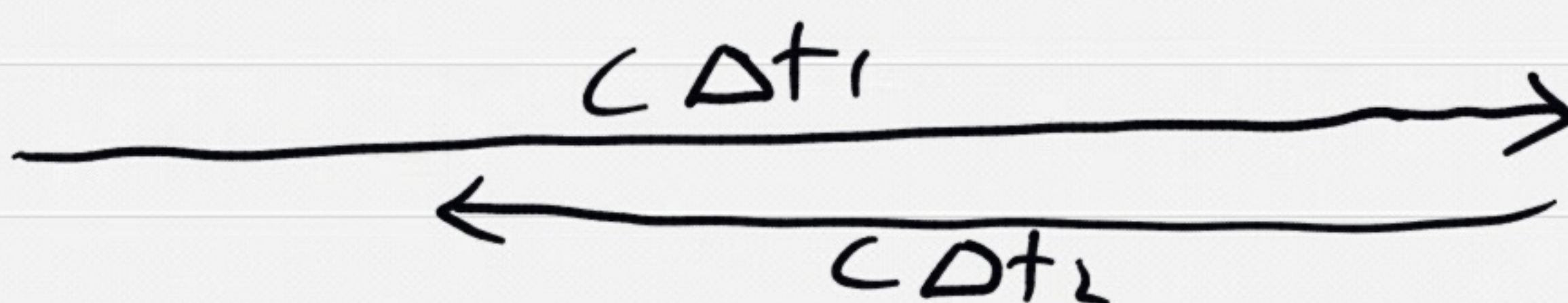
Frame S'

$$\Delta t' = 2L_0/c = 2\Delta x'/c$$

$$\Rightarrow \Delta x' = c\Delta t'/2 = L_0$$



Frame S



$$\Delta t_1 = (\Delta x + v\Delta t_1)/c$$

$$\Delta t_2 = (\Delta x - v\Delta t_2)/c$$

$$\Rightarrow \Delta t_1 = \frac{\Delta x}{c} \left(\frac{1}{1 - v/c} \right)$$

$$\Delta t_2 = \frac{\Delta x}{c} \left(\frac{1}{1 + v/c} \right)$$

$$\Delta t = \frac{\Delta x}{c} \frac{2}{1 - v^2/c^2}$$

$$\Delta x = \frac{c\Delta t}{2} \left(1 - v^2/c^2 \right)$$

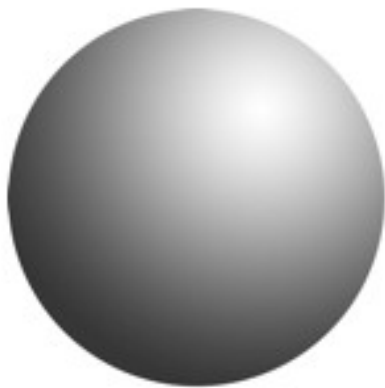
$$= \frac{c}{2} \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \left(1 - v^2/c^2 \right)$$

$$= \Delta x' \sqrt{1 - v^2/c^2} = \frac{\Delta x'}{\gamma} = \frac{L_0}{\gamma}$$

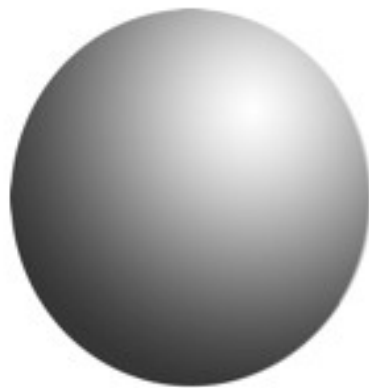
Proper Length

- “Proper Length” = L_0
 - The length of an object measured in a frame where it is at rest
- The length L measured in **any** other frame moving with a velocity with respect to this frame that has a component along the length will be shorter
 - $L = L_0/\gamma$

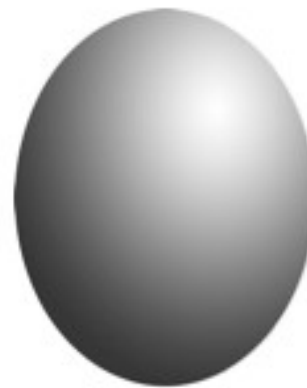
Length Contraction



$$V = 0$$



$$\rightarrow$$
$$V = 0.3C$$



$$\rightarrow$$
$$V = 0.6C$$



$$\rightarrow$$
$$V = 0.9C$$