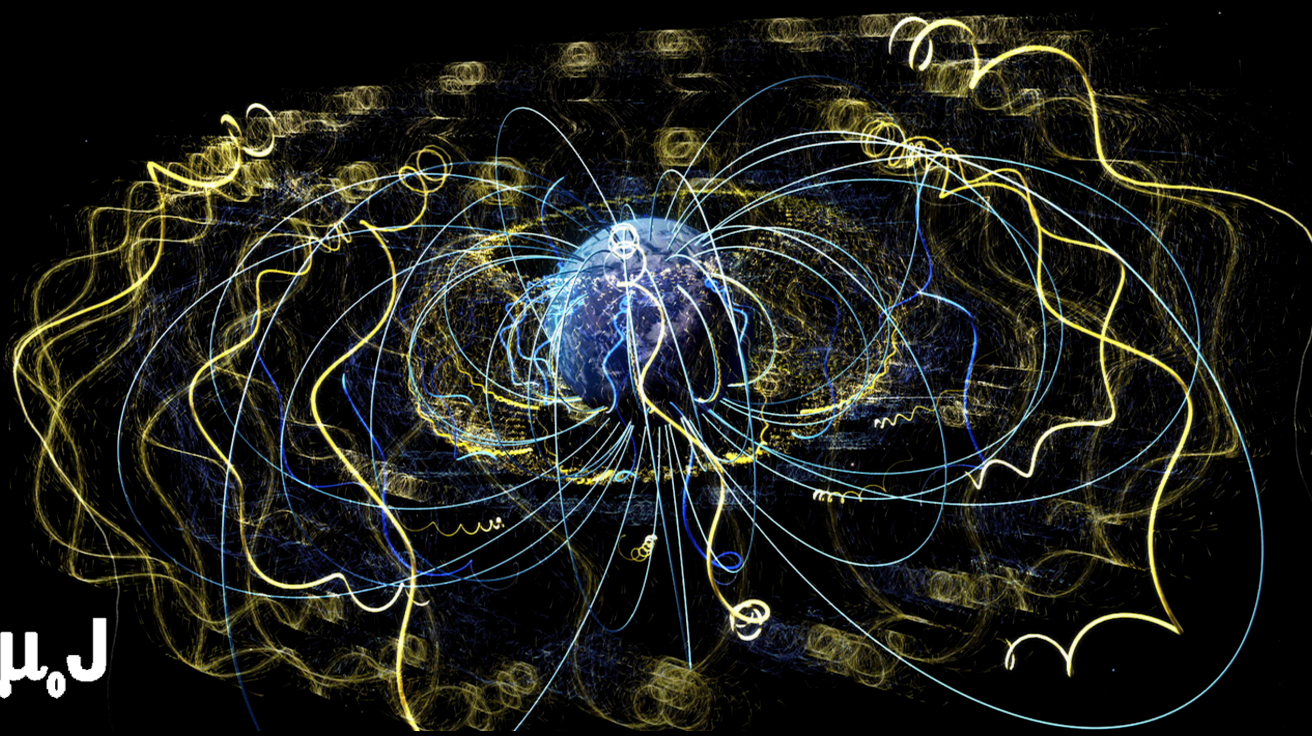
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

Virtual by Zoom!

MWF 9:30-10:20 Lecture

Announcements

- Final Exam
  - 3:00-5:00 pm (+1 hr grace) Monday 5/11 as scheduled
    - Take-home open-book (same format as Midterm 2)
  - Covers Chapters 7-12 in Griffiths
    - Final equation sheet posted
    - Last year's final & answers posted
  - Problem solving session Wednesday 5/6
  - Review Friday 5/8

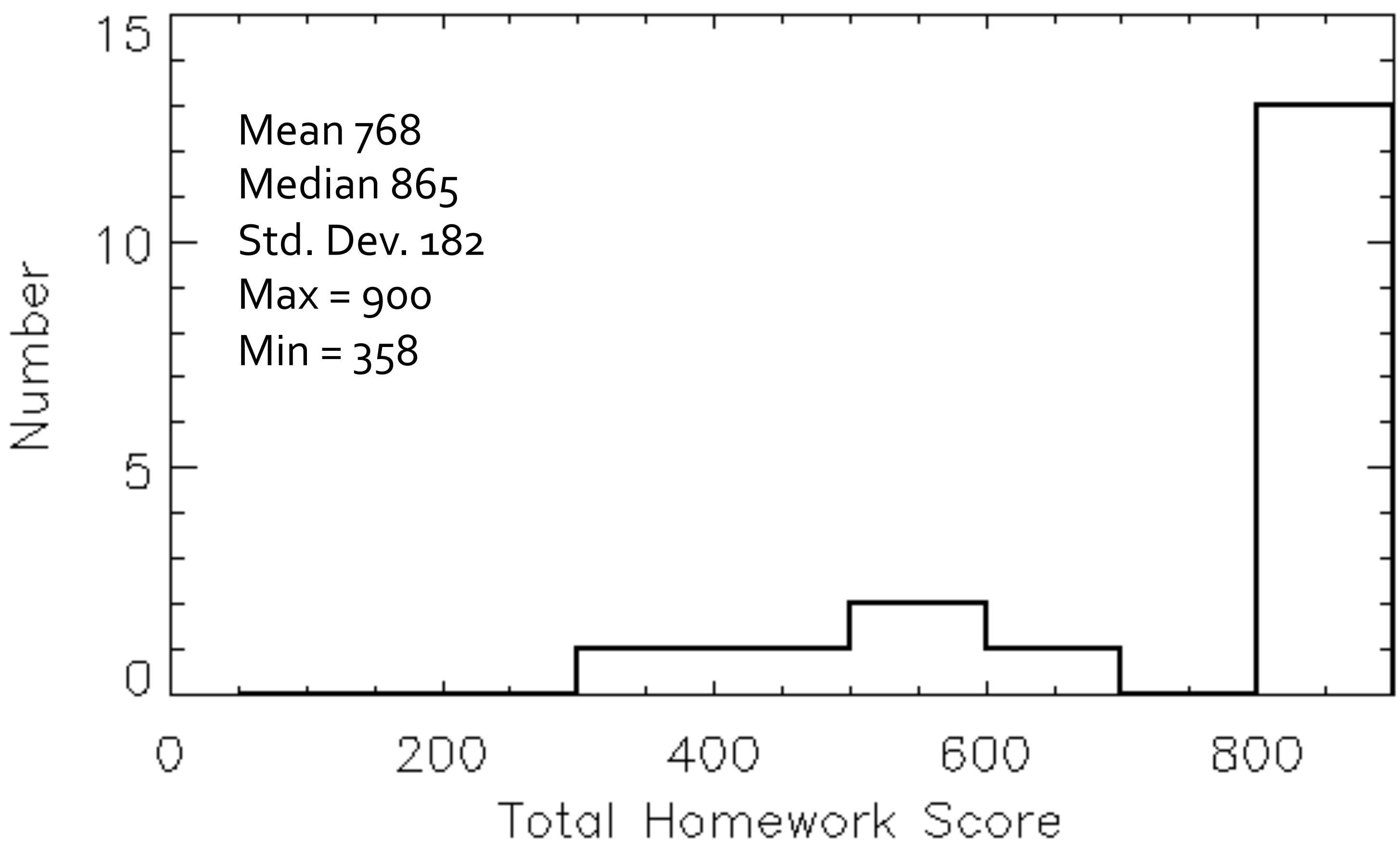
Announcements II

- Final Exam will cover Ch. 7-12, with the exception of the following sections:
  - 7.3.4 Magnetic charge
  - 8.2.4 Angular momentum
  - 8.3 Magnetic forces do no work
  - 9.4.3 Frequency dependence of permittivity
  - 9.5 Guided waves
  - 10.1.4 Lorentz force law in potential form
  - 10.2.2 Jefimenko's equations
  - 11.2.2-11.2.3 Radiation reaction
  - 12.2.3-12.2.4 Relativistic kinematics and dynamics
  - 12.3.3 The field tensor

Final Exam Instructions

- The exam will be posted on the course web page by 3:00pm. You must submit your answers to me by e-mail by 6:00pm. The exam is intended to take roughly two hours – the extra hour is grace period to check your work, scan it, and submit it.
- This exam is open book and open notes. However, it is not open internet, open solutions manual, or open classmate. Please do not utilize solutions or consult with anyone to solve the problems. I trust you all not to abuse this unique situation.
- Read all the questions carefully and answer every part of each question. Show your work on all problems. Partial credit may be granted for correct logic or intermediate steps, even if the final answer is incorrect. Make sure to clearly indicate (e.g. circle) your final answers.
- The solution for each of the five problems is ~½ page. If it looks like your solution is going to require substantially more work, you may be doing it the hard way!
- Unless otherwise instructed, express your answers in terms of fundamental constants like μ_o and ε_o, rather than calculating numerical values.
- Please ask if you have any questions, including clarification about the instructions, during the exam. A Zoom meeting (ID 992-770-55648) will be open during the exam.

Homework Scores



Field & Dual Tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

$$G^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z/c & E_y/c \\ -B_y & E_z/c & 0 & -E_x/c \\ -B_z & -E_y/c & E_x/c & 0 \end{pmatrix}$$

Field Tensor Transformation

$F^{\mu\nu\prime}$ =

$$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix} \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_x{}' = E_x,\ E_y{}' = \gamma(E_y - uB_z),\ \ E_z{}' = \gamma(E_z + uB_y)$$

$$B_x{}' = B_x,\ B_y{}' = \gamma\left(B_y + \frac{u}{c^2}E_z\right), B_z{}' = \gamma\left(B_z - \frac{u}{c^2}E_y\right)$$

Tensor Electrodynamics

$\rho = Q/V$

$\vec{J} = \rho \vec{v}$ if all charges flow together

Proper charge density

$\rho_0 = Q/V_0$ w/ V_0 = volume in rest frame

$V = V_0/\gamma$

$\Rightarrow \rho = \gamma \rho_0 = \rho_0 / \sqrt{1 - v^2/c^2}$

$\vec{J} = \rho \vec{v} = \rho_0 \vec{v} / \sqrt{1 - v^2/c^2}$

$J^\mu = \rho_0 \eta^\mu$

$= (c\rho, J_x, J_y, J_z)$

This is a 4-vector

Continuity Equation

$$\nabla \cdot \vec{J} = -\partial \rho / \partial t$$

or $\partial J_x/\partial x + \partial J_y/\partial y + \partial J_z/\partial z = -\partial \rho/\partial t$

write as $\partial J^1/\partial x^1 + \partial J^2/\partial x^2 + \partial J^3/\partial x^3 = -\partial J^0/\partial x^0$

$$\Rightarrow \boxed{\partial J^\mu / \partial x^\mu = 0}$$

4-d divergence of J^μ is 0

Maxwell's Equations

$$\boxed{\partial F^{\mu\nu}/\partial x^\nu = \mu_0 J^\mu \qquad \partial G^{\mu\nu}/\partial x^\nu = 0}$$

$$\partial F^{0\nu}/\partial x^\nu = 0 + \partial/\partial x (E_x/c) + \partial/\partial y \left(\frac{E_y}{c}\right) + \partial/\partial z \left(\frac{E_z}{c}\right)$$

$$= \mu_0 J^0 = \mu_0 c \rho$$

$$\Rightarrow \frac{1}{c} \nabla \cdot \bar{E} = \mu_0 c \rho$$

$$\Rightarrow \boxed{\nabla \cdot \bar{E} = \rho / \varepsilon_0}$$

$$\partial F^{1\nu}/\partial x^\nu = \partial/\partial(ct)\left(-E_x/c\right) + 0 + \partial B_z/\partial y - \partial B_y/\partial z$$
$$= -\frac{1}{c^2}\partial E_x/\partial t + \partial B_z/\partial y - \partial B_y/\partial z$$
$$= \mu_0 J^1$$
$$= \mu_0 J_x$$

similarly $-\frac{1}{c^2}\partial E_y/\partial t - \partial B_z/\partial x + \partial B_x/\partial z = \mu_0 J_y$

$$-\frac{1}{c^2}\partial E_z/\partial t + \partial B_y/\partial x - \partial B_x/\partial y = \mu_0 J_z$$

$$\Rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \partial \vec{E}/\partial t}$$

similarly:

$$\partial G^{0\nu}/\partial x^\nu = 0 \Rightarrow \boxed{\nabla \cdot \vec{B} = 0}$$

$$\partial G^{1\nu}/\partial x^\nu,\ \partial G^{2\nu}/\partial x^\nu,\ \partial G^{3\nu}/\partial x^\nu = 0$$

$$\Rightarrow \boxed{\partial \vec{B}/\partial t = -\nabla \times \vec{E}}$$

Force

$$K^\mu = q\,\eta_\nu F^{\mu\nu} = dp^\mu/d\tau$$

$$K^0 = q\left[-\gamma c \cdot 0 + \gamma v_x \cdot E_x/c + \gamma v_y E_y/c + \gamma v_z E_z/c\right]$$

$$= q\,\vec{v}\cdot\vec{E} \Big/ \sqrt{1 - v^2/c^2}$$

$$= d/d\tau \left(\frac{W}{c} \right)$$

$$= \frac{1}{c} \cdot \frac{1}{\sqrt{1 - v^2/c^2}}\, dW/dt$$

$$\Rightarrow \boxed{dW/dt = q\,\vec{v}\cdot\vec{E} = \vec{F}\cdot\vec{v}}$$

Energy Equation

$$K^1 = q\left[-\gamma c \cdot -E_x/c + 0 + \gamma v_y B_z - \gamma v_z B_y\right]$$

$$K^2 = q\left[-\gamma c \cdot -E_y/c - \gamma v_x B_z + 0 + \gamma v_z B_x\right]$$

$$K^2 = q\left[-\gamma c \cdot -E_z/c + \gamma v_x B_y - \gamma v_y B_x + 0\right]$$

$$\Rightarrow \vec{K} = \gamma q(\vec{E} + \vec{v}\times\vec{B})$$

$$= d\vec{p}/d\tau = \gamma\, d\vec{p}/dt$$

$$\Rightarrow \boxed{\vec{F} = d\vec{p}/dt = q(\vec{E} + \vec{v}\times\vec{B})}$$

Lorentz Force

Potentials

$$A^\mu = \left(\frac{V}{c}, A_x, A_y, A_z\right)$$

$$F^{\mu\nu} = \partial A^\nu / \partial x_\mu - \partial A^\mu / \partial x_\nu$$

$$\partial F^{\mu\nu} / \partial x^\nu = \mu_0 J^\mu$$

$$\Rightarrow \partial/\partial x^\nu \left(\partial A^\nu / \partial x_\mu\right) - \partial/\partial x^\nu \left(\partial A^\mu / \partial x_\nu\right)$$

$$= \partial/\partial x_\mu \left(\partial A^\nu / \partial x^\nu\right) - \partial/\partial x^\nu \left(\partial A^\mu / \partial x_\nu\right) = \mu_0 J^\mu$$

Pick gauge w/ $\partial A^\nu / \partial x^\nu = 0$

same as $\partial/\partial(ct)\, \frac{V}{c} + \partial/\partial x\, A_x + \partial/\partial y\, A_y + \partial/\partial z\, A_z = 0$

$$\Rightarrow \nabla \cdot \vec{A} = -\frac{1}{c^2} \partial V / \partial t$$

= Lorenz Gauge

$$\Rightarrow \partial/\partial x_\nu \left(\partial A^\mu / \partial x^\nu\right) = -\mu_0 J^\mu$$

or $\boxed{\Box^2 A^\mu = -\mu_0 J^\mu}$

$$\Box^2 = \partial/\partial x_\nu\, \partial/\partial x^\nu$$

$$= \nabla^2 - \frac{1}{c^2} \partial^2/\partial t^2$$

the d'Alembertian