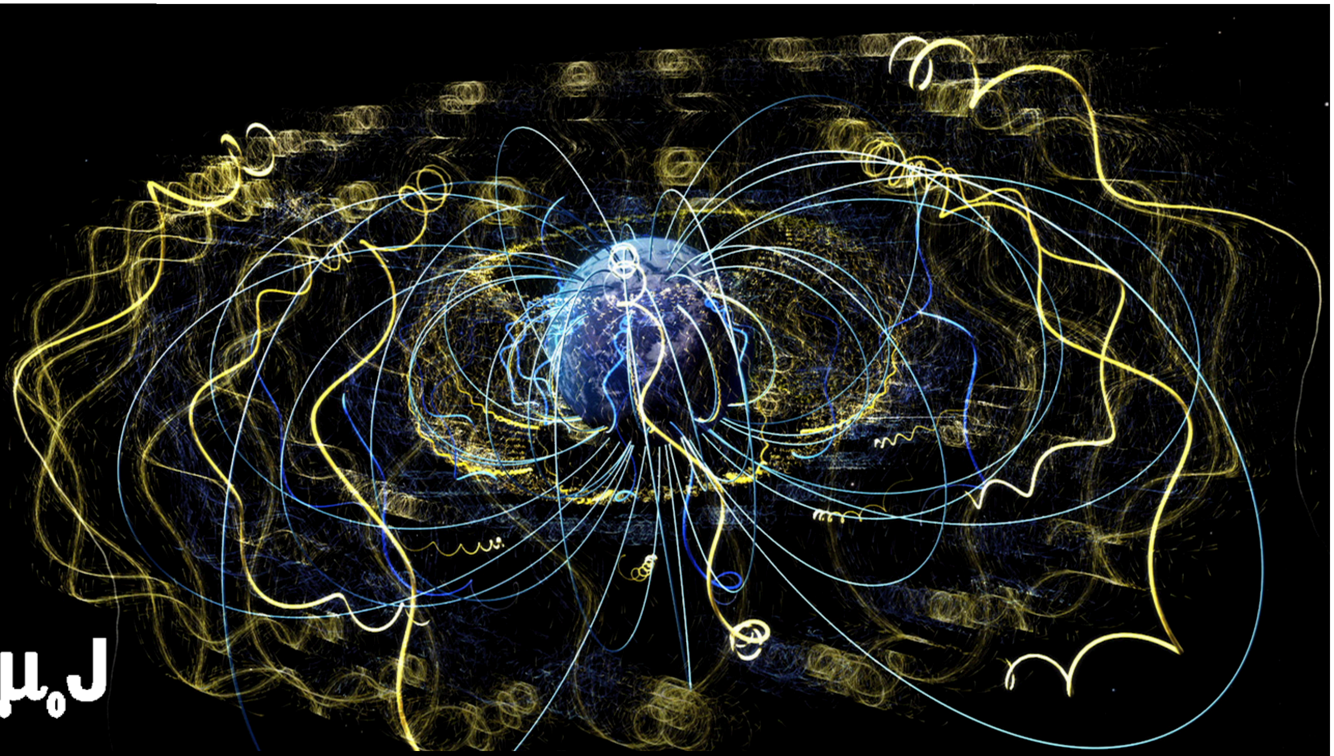
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

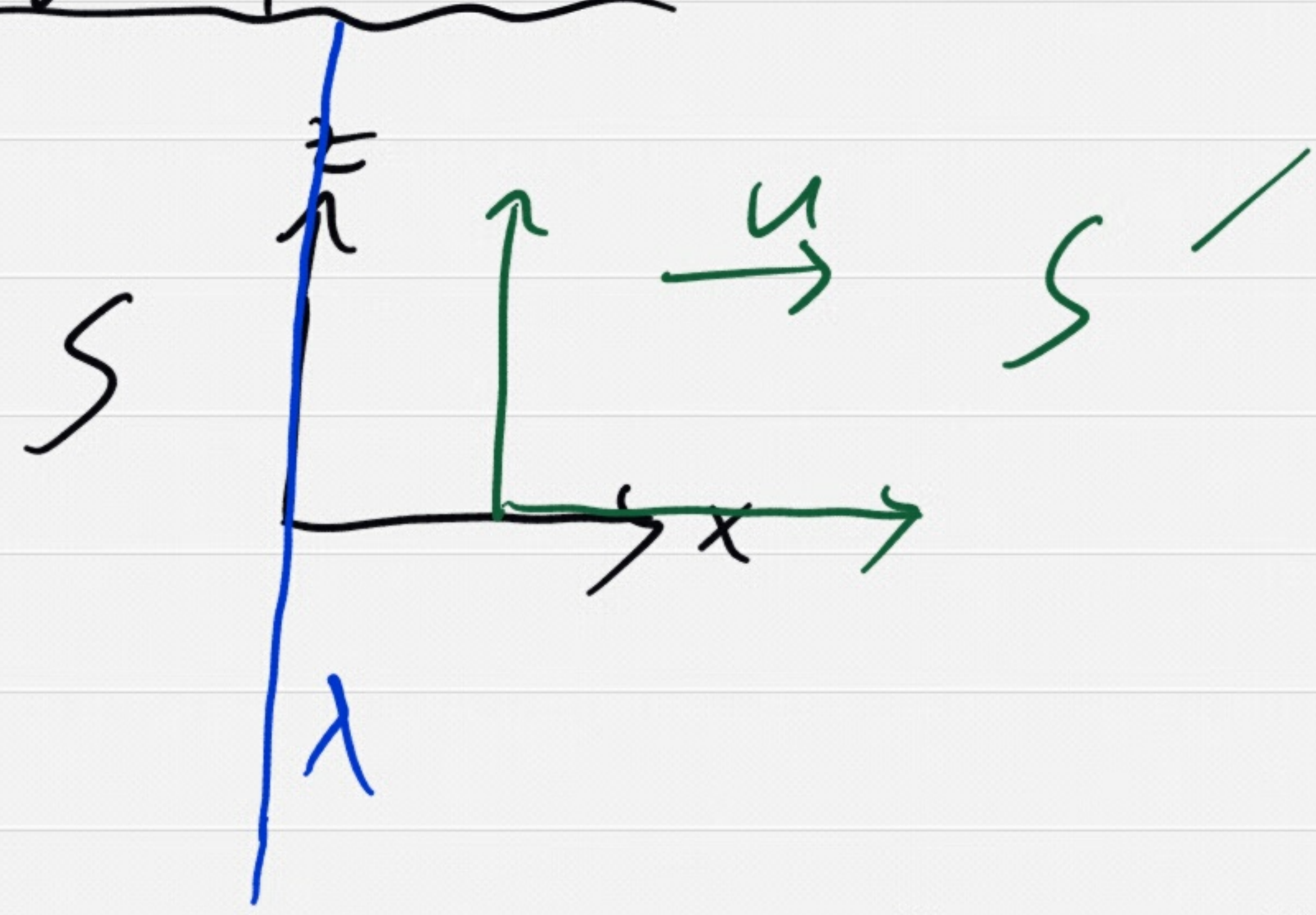
Virtual by Zoom!

MWF 9:30-10:20 Lecture

Announcements

- Final Exam
  - 3:00-5:00 pm (+1 hr grace) Monday 5/11 as scheduled
    - Take-home open-book (same format as Midterm 2)
  - Covers Chapters 7-12 in Griffiths (w/ exceptions noted)
    - Final equation sheet posted
    - Last year's final & answers posted
  - Problem solving session today
  - Review Friday 5/8
- Course evaluations are open through Sunday night
  - As always, feedback is much appreciated!
    - I have also added an extra question about how virtual instruction could be improved if it turns out to be necessary again in the future
  - Thanks very much to the 6 students who have responded so far

HW 10.3



$$\vec{E} = \frac{\lambda \hat{s}}{2\pi\varepsilon_0 s} = \frac{\lambda \vec{s}}{2\pi\varepsilon_0 s^2} = \frac{\lambda}{2\pi\varepsilon_0}\frac{x\hat{x}+y\hat{y}}{x^2+y^2} \quad \text{in } S$$

$$E_x' = E_x \quad , \quad E_y' = \gamma E_y$$

$$\Rightarrow \vec{E}' = \frac{\lambda}{2\pi\varepsilon_0}\frac{x\hat{x}+\gamma y\hat{y}}{x^2+y^2}$$

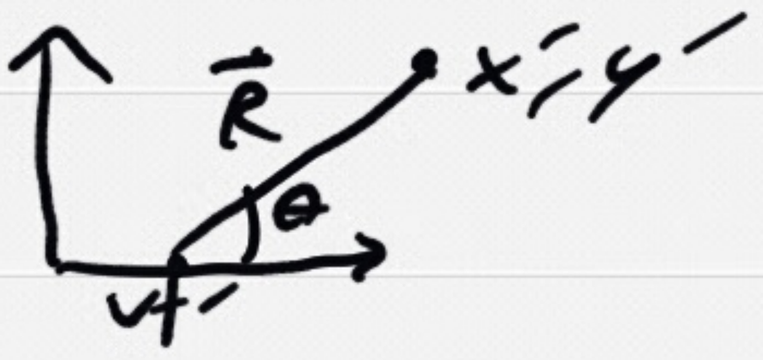
x & y do not go to x' & y' in this step!!

$$x = \gamma(x'+ut') = \gamma(x'-vt')$$

$$\Rightarrow \vec{E}' = \frac{\lambda}{2\pi\varepsilon_0}\frac{\gamma(x'-vt')\hat{x}+\gamma y'\hat{y}}{\gamma^2(x'-vt')^2+y'^2}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{\gamma}\frac{(x'-vt')\hat{x}+y'\hat{y}}{(x'-vt')^2+(1-v^2/c^2)y'^2}$$

Write in terms of



$$\vec{E}' = \frac{\lambda}{2\pi\varepsilon_0}\frac{\sqrt{1-v^2/c^2}\,\vec{R}}{R^2(1-v^2/c^2\sin^2\theta)}$$

Sample Problem I

- Consider an electromagnetic plane wave propagating in the x direction, polarized in the y direction, in frame S.

$$\mathbf{E}(x,y,z,t) = E_0 \cos(kx - \omega t)\,\hat{\mathbf{y}}, \quad \mathbf{B}(x,y,z,t) = \frac{E_0}{c}\cos(kx - \omega t)\,\hat{\mathbf{z}}$$

- Transform the fields to a reference frame S′ moving at speed u in the x direction with respect to S.

$$E_x{}' = E_x,\ E_y{}' = \gamma(E_y - uB_z),\ \ E_z{}' = \gamma(E_z + uB_y)$$
$$B_x{}' = B_x,\ B_y{}' = \gamma\left(B_y + \frac{u}{c^2}E_z\right), B_z{}' = \gamma\left(B_z - \frac{u}{c^2}E_y\right)$$

SPI

$$E_y' = \gamma(E_y - uB_z)$$
$$= \gamma\left(E_0\cos(kx-\omega t) - u\frac{E_0}{c}\cos(kx-\omega t)\right)$$
$$= \gamma\left(1-\frac{u}{c}\right)E_0\cos(kx-\omega t)$$
$$= \gamma\left(1-\frac{u}{c}\right)E_y$$

$$E_x' = E_z' = 0$$

$$B_z' = \gamma\left(B_z - \frac{u}{c^2}E_y\right)$$
$$= \gamma\left(\frac{E_0}{c}\cos(kx-\omega t) - \frac{u}{c^2}E_0\cos(kx-\omega t)\right)$$
$$= \gamma\left(1-\frac{u}{c}\right)\frac{E_0}{c}\cos(kx-\omega t)$$
$$= \gamma\left(1-\frac{u}{c}\right)B_z$$

$$B_x' = B_y' = 0$$

Note $|\vec{E}|/|\vec{B}| = c$ in both frames!

Sample Problem II

- Starting with the plane wave from the previous problem, rewrite the fields so they depend on the coordinates x′ and t′ rather than x and t, using the inverse Lorentz transformation

- Use $\omega/k = c$ to simplify

Lorentz transformation

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{u}{c^2}x)$$

S P II

Transform $kx - \omega t$

$$x = \gamma(x' + ut')$$
$$t = \gamma(t' + \frac{u}{c^2}x')$$

so $kx - \omega t \rightarrow k\gamma(x' + ut') - \omega\gamma(t' + u/c^2 x')$

$$= \gamma[(k - \omega u/c^2)x' - (\omega - ku)t']$$
$$= \gamma[k(1 - u/c)x' - \omega(1 - u/c)t']$$

so $k' = \gamma(1 - u/c)k$

$\omega' = \gamma(1 - u/c)\omega$

Note $\omega'/k' = \omega/k = c$

Sample Problem III

- Consider an infinitely extended neutral wire carrying a current *I* along the x-axis in frame S
- Transforming to a frame S′ moving at velocity u in the x direction with respect to S, find the resulting line charge density λ in S′

$$J^\mu = \rho_0 \eta^\mu = (c\rho_0, \rho_0 v_x, \rho_0 v_y, \rho_0 v_z)/\sqrt{1 - v^2/c^2} = (c\rho, J_x, J_y, J_z)$$

$$a^{\mu\prime} = (\gamma[a^0 - \beta a^1], \gamma[a^1 - \beta a^0], a^2, a^3)$$

Sp III

$$J^\mu = \left(0, \frac{I}{A}, 0, 0\right)$$

$$J^{\mu\prime} = \left(-\frac{\gamma u}{c}\frac{I}{A}, \gamma\frac{I}{A}, 0, 0\right)$$

$$\Rightarrow c\rho' = -\frac{\gamma u}{c}\frac{I}{A}$$

$$\lambda' = \rho' A = -\frac{\gamma u}{c^2} I$$

Note: $I = 2\lambda v$ composed of of λ w/ velocity v and $-\lambda$ w/ velocity $-v$ so that $\lambda_{total} = 0$

so $$\lambda' = -\frac{\gamma u}{c^2} I$$

$$= \frac{-2u\lambda v}{c^2\sqrt{1 - u^2/c^2}}$$

– Griffiths gets this result w/ considerably more effort by looking at how λ and $-\lambda$ transform