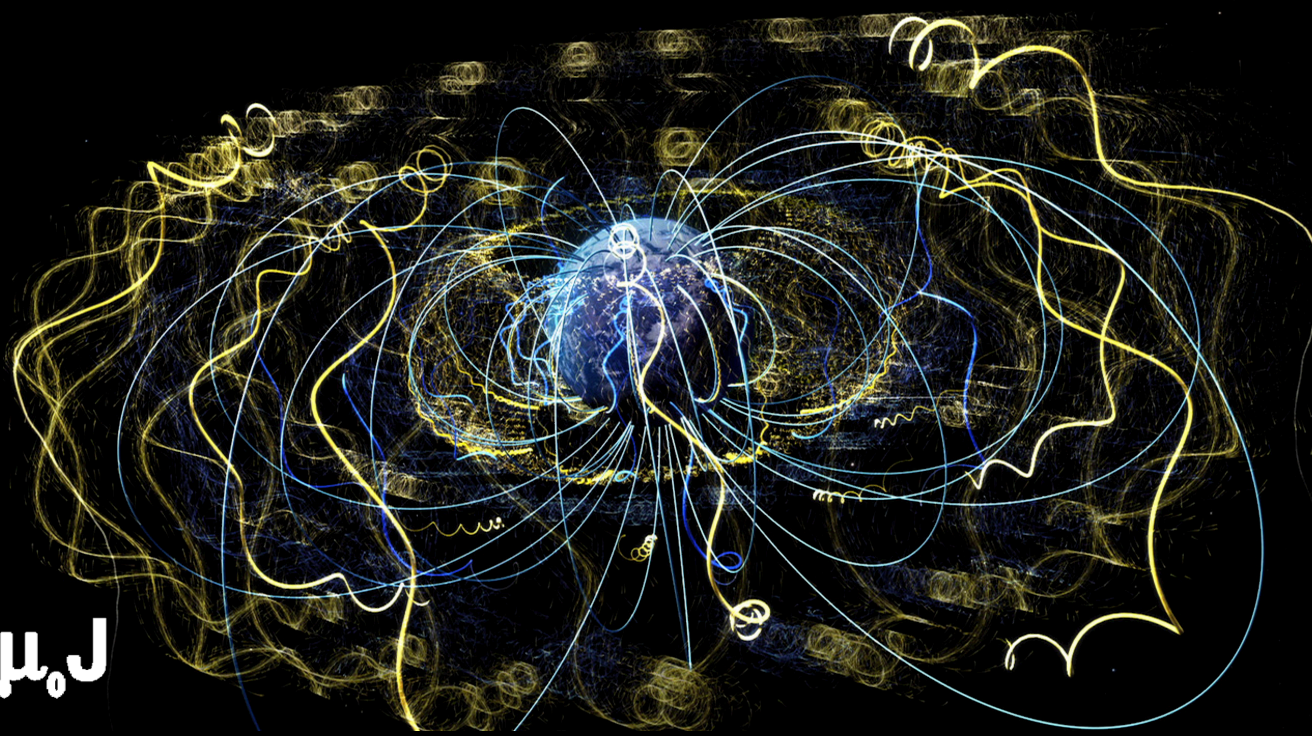
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



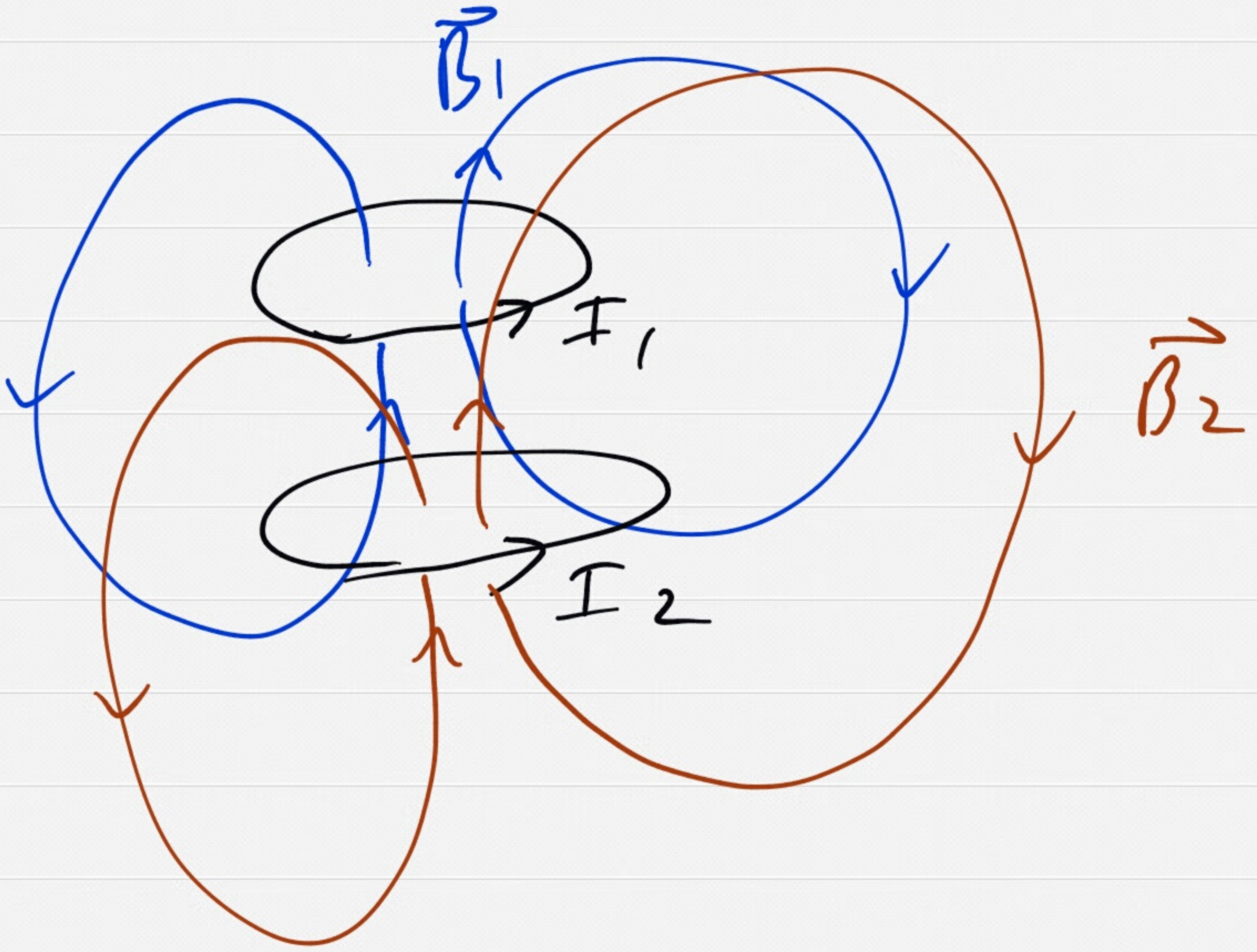
Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Reminder

- Homework #1 due in class (or before) this Friday Jan 31

Inductance



I_1 produces $\vec{B}_1$
I_2 produces $\vec{B}_2$

$\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2$ = magnetic flux of $\vec{B}_1$ through loop 2

$\Phi_1 = \int \vec{B}_2 \cdot d\vec{a}_1$ = magnetic flux of $\vec{B}_2$ through loop 1

$\Phi_2 = M_{21} I_1$ since $\vec{B}_1$ linearly dependent on I_1

M_{21} is function only of geometry

M_{21} = "Mutual Inductance"

$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2$$
$$= \int (\nabla \times \vec{A}_1) \cdot d\vec{a}_2$$
$$= \oint \vec{A}_1 \cdot d\vec{l}_2 \quad \text{by Stokes' Thm.}$$
w/ $d\vec{l}_2$ around loop 2

- For current loop
$$\vec{A}_1(\vec{r}) = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\vec{l}_1}{|\vec{r} - \vec{r}_1|}$$

w/ $d\vec{l}_1$ around loop 1

- Combine

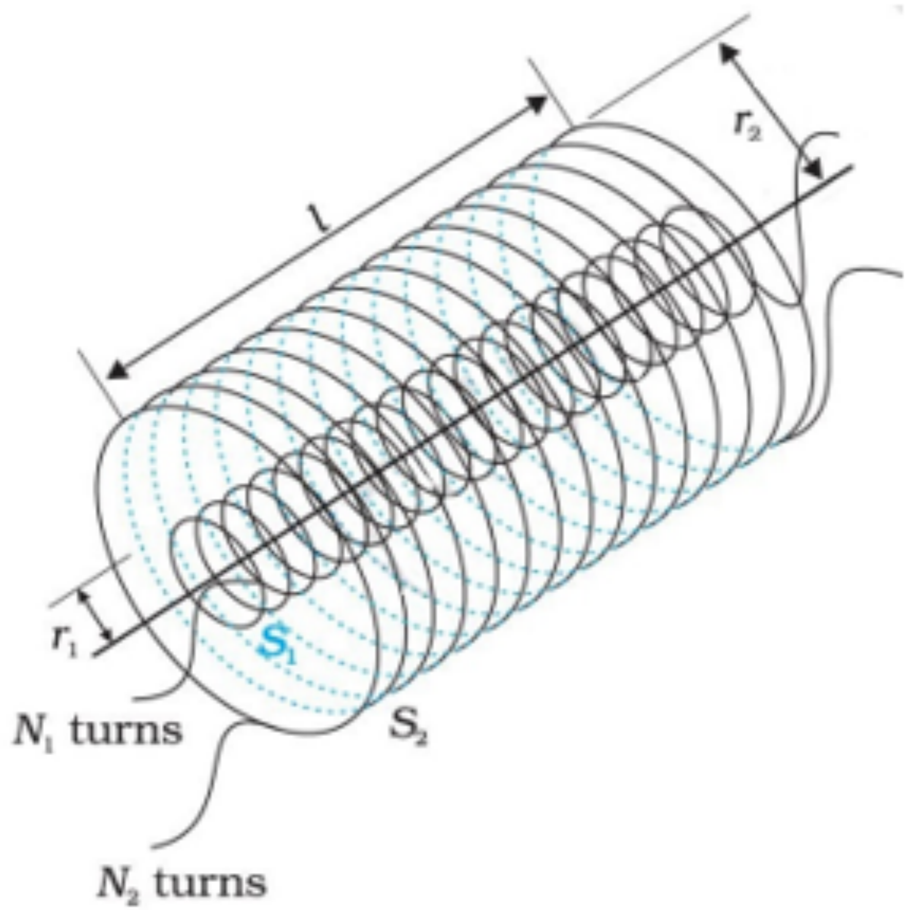
$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left[\oint \frac{d\vec{l}_1}{|\vec{r} - \vec{r}_1|} \right] \cdot d\vec{l}_2$$
$$= \frac{\mu_0 I_1}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|}$$

$$\Rightarrow \boxed{M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|}}$$

Neumann Formula

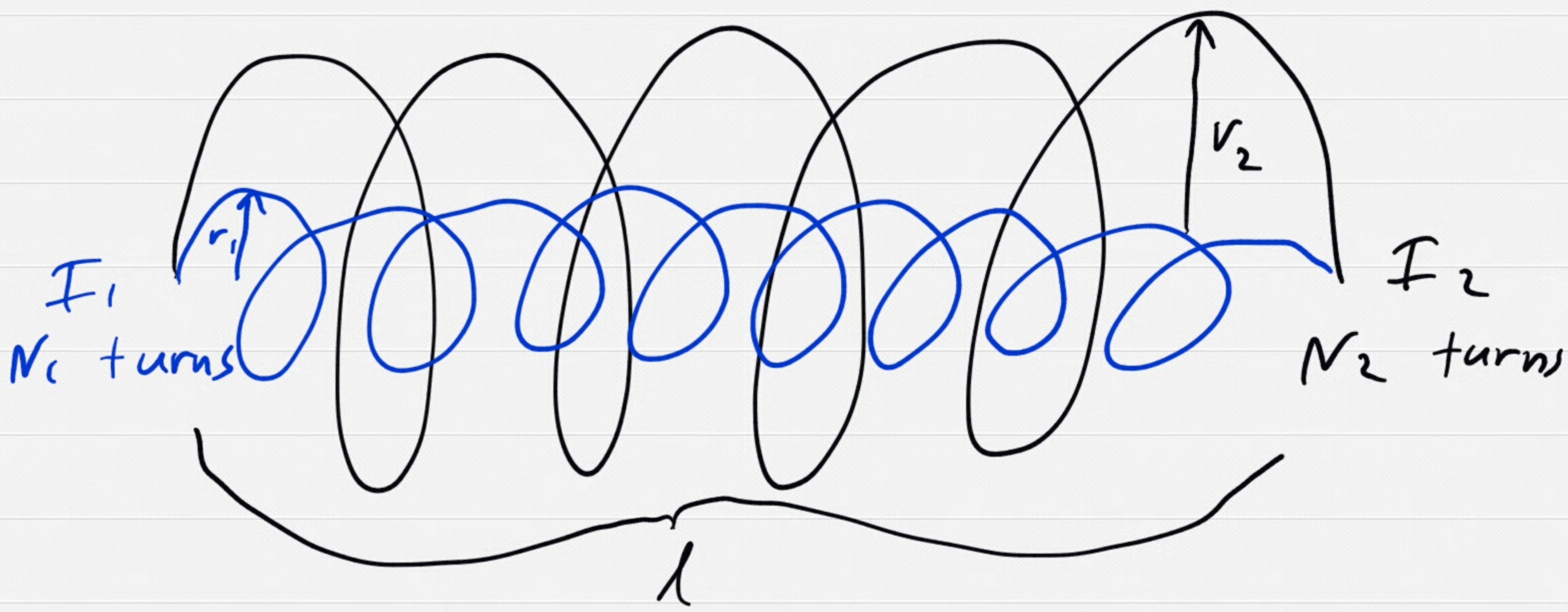
- Note: By inspection,
$M_{21} = M_{12} = M$

Mutual Inductance



Example

Nested solenoids



$$B_1 = \mu_0 n_1 I_1 \qquad (n_1 = N_1/l)$$
$$B_2 = \mu_0 n_2 I_2 \qquad (n_2 = N_2/l)$$

$$\begin{aligned}\Phi_2 &= B_1 \cdot \pi r_1^2 \cdot N_2 \\ &= B_1 \cdot \pi r_1^2 \cdot n_2 l \\ &= \mu_0 n_1 n_2 \cdot \pi r_1^2 l \cdot I_1\end{aligned}$$

$$\Rightarrow \boxed{M = \mu_0 n_1 n_2 \cdot \pi r_1^2 l}$$

$$\begin{aligned}\Phi_1 &= B_2 \cdot \pi r_1^2 \cdot N_1 \\ &= B_2 \cdot \pi r_1^2 \cdot n_1 l \\ &= \mu_0 n_1 n_2 \cdot \pi r_1^2 \cdot l \cdot I_2\end{aligned}$$

$$\Rightarrow \boxed{M = \mu_0 n_1 n_2 \cdot \pi r_1^2 \cdot l}$$

$\boxed{\text{Self-Inductance}}$

- Coil also has magnetic flux through itself

self-inductance $L \equiv \Phi_B / I$

For solenoid:

$$B = \mu_0 n I$$

$$\begin{aligned}\Phi_B &= B \cdot \pi r^2 \cdot N \\ &= B \cdot \pi r^2 \cdot n \cdot l \\ &= \mu_0 n^2 \cdot \pi r^2 \cdot l \cdot I\end{aligned}$$

$$\Rightarrow \boxed{L = \mu_0 n^2 \cdot \pi r^2 \cdot l}$$

$\boxed{\text{Induced EMF}}$

$$\mathcal{E} = -d\Phi_B/dt$$

Mutual Inductance:

$$\mathcal{E}_2 = -d/dt\,(M I_1)$$
$$= -M\, dI_1/dt$$

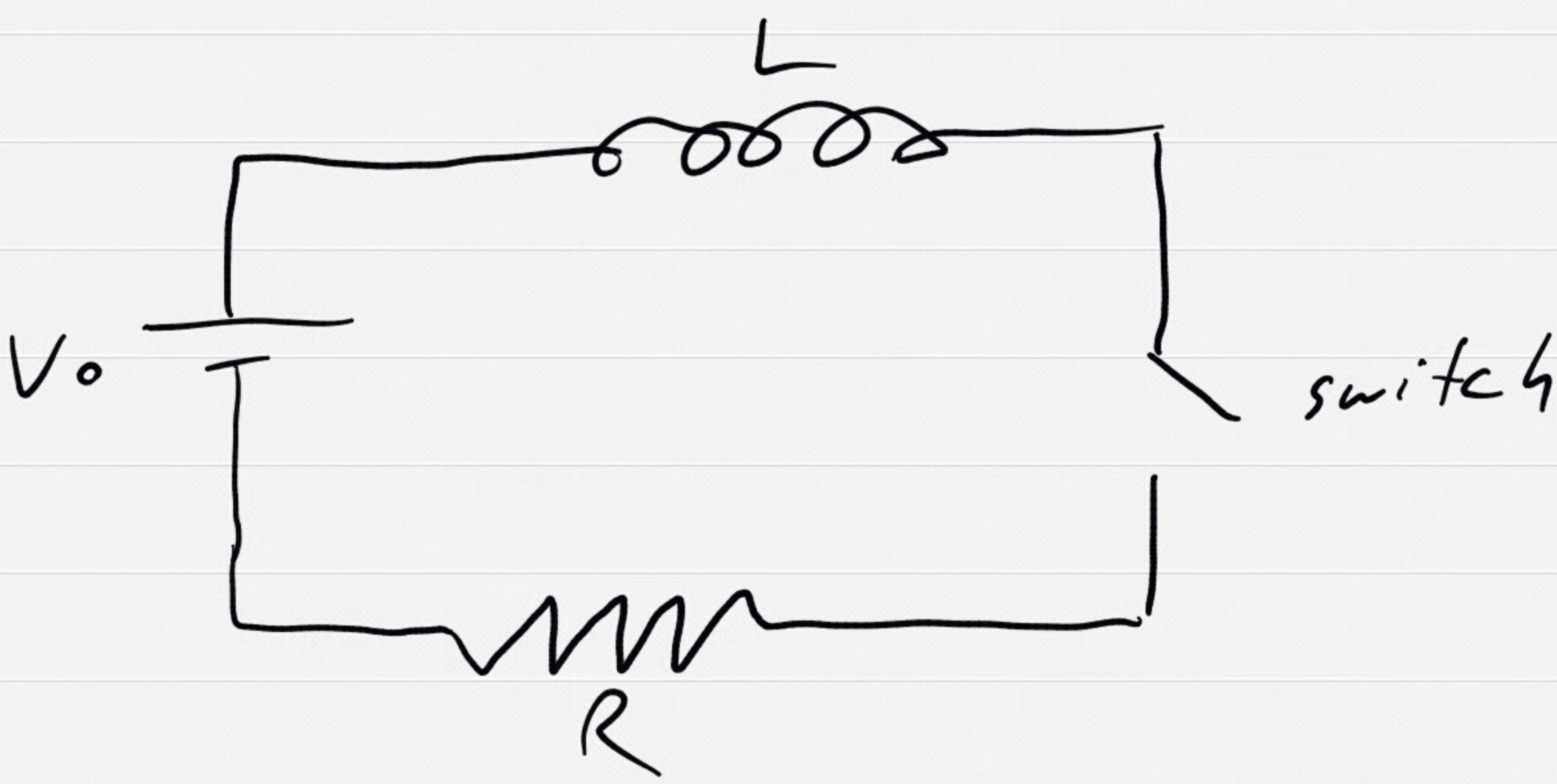
$$\mathcal{E}_1 = -d/dt\,(M I_2)$$
$$= -M\, dI_2/dt$$

Self-Inductance

$$\mathcal{E} = -d/dt\,(L I)$$
$$= -L\, dI/dt$$

- All circuits have some self-inductance - which leads to "back EMF"

Inductance in Circuits



$\mathcal{E}_0 = V_0$

$\mathcal{E}_{total} = V_0 - L\, dI/dt = IR$

$\Rightarrow L\, dI/dt = -(IR - V_0)$

$dI/dt = -\frac{R}{L}(I - V_0/R)$

$dI/(I - V_0/R) = -R/L\, dt$

$\ln(I - V_0/R)\Big|_{I_0}^{I} = -R/L\Big|_0^t$

$\ln\left(\frac{I - V_0/R}{I_0 - V_0/R}\right) = -R/L\, t$

$\frac{I - V_0/R}{I_0 - V_0/R} = e^{-\frac{R}{L}t}$

$\Rightarrow I(t) = \frac{V_0}{R} + (I_0 - V_0/R\, e^{-\frac{R}{L}t})$

$I_0 = 0 \Rightarrow \boxed{I(t) = V_0/R\left[1 - e^{-\frac{R}{L}t}\right]}$