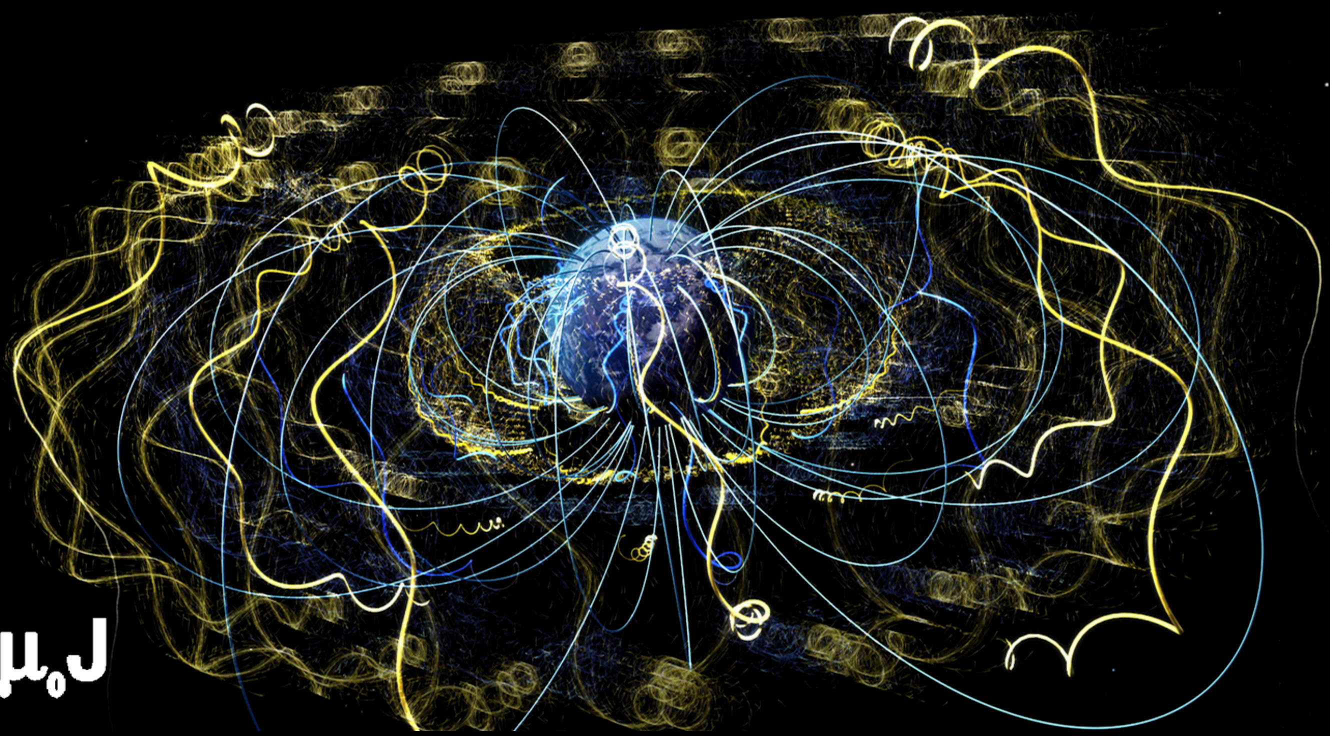
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



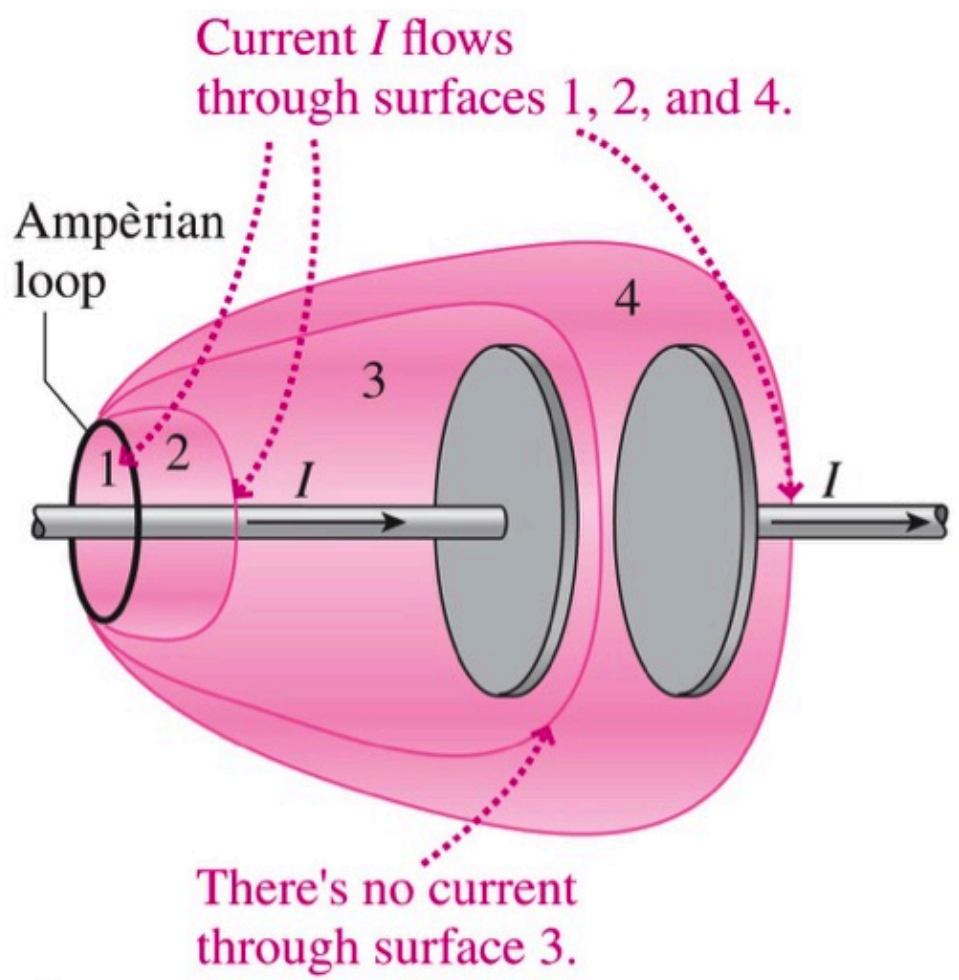
Electricity and Magnetism II: 3812

Professor Jasper Halekas

Van Allen 70

MWF 9:30-10:20 Lecture

Displacement Current



Consistency of E&M, Integral Viewpoint

Faraday's Law

$$\mathcal{E} = -d\Phi_B/dt$$

$$\oint \vec{E} \cdot d\vec{l} = -d/dt \left(\int \vec{B} \cdot d\vec{a} \right)$$

$\int \vec{B} \cdot d\vec{a}$ same over all surfaces enclosed by loop since $\nabla \cdot \vec{B} = 0 \Rightarrow \oint \vec{B} \cdot d\vec{a} = 0$

Amperes Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a}$$
$$= \mu_0 I_{enc}$$

- But for charging capacitor, if I deform surface to go between plates then $I_{enc} = 0$

- This is because $\int \vec{J} \cdot d\vec{a}$ depends on surface, because $\oint \vec{J} \cdot d\vec{a} \neq 0$

$$\boxed{\text{Displacement Current}}$$

Fix w/ help of continuity

$$\nabla \cdot \vec{J} = -\partial\rho/\partial t$$

converging current $\Rightarrow$ charge buildup

$$\rho = \varepsilon_0 \nabla \cdot \vec{E} \quad \text{(Gauss's Law)}$$

so

$$\nabla \cdot \vec{J} = -\partial/\partial t\,(\varepsilon_0 \nabla \cdot \vec{E})$$
$$= -\nabla \cdot (\varepsilon_0\, \partial \vec{E}/\partial t)$$

or

$$\nabla \cdot (\vec{J} + \varepsilon_0\, \partial \vec{E}/\partial t) = 0$$

If we take

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \varepsilon_0\, \partial \vec{E}/\partial t)$$

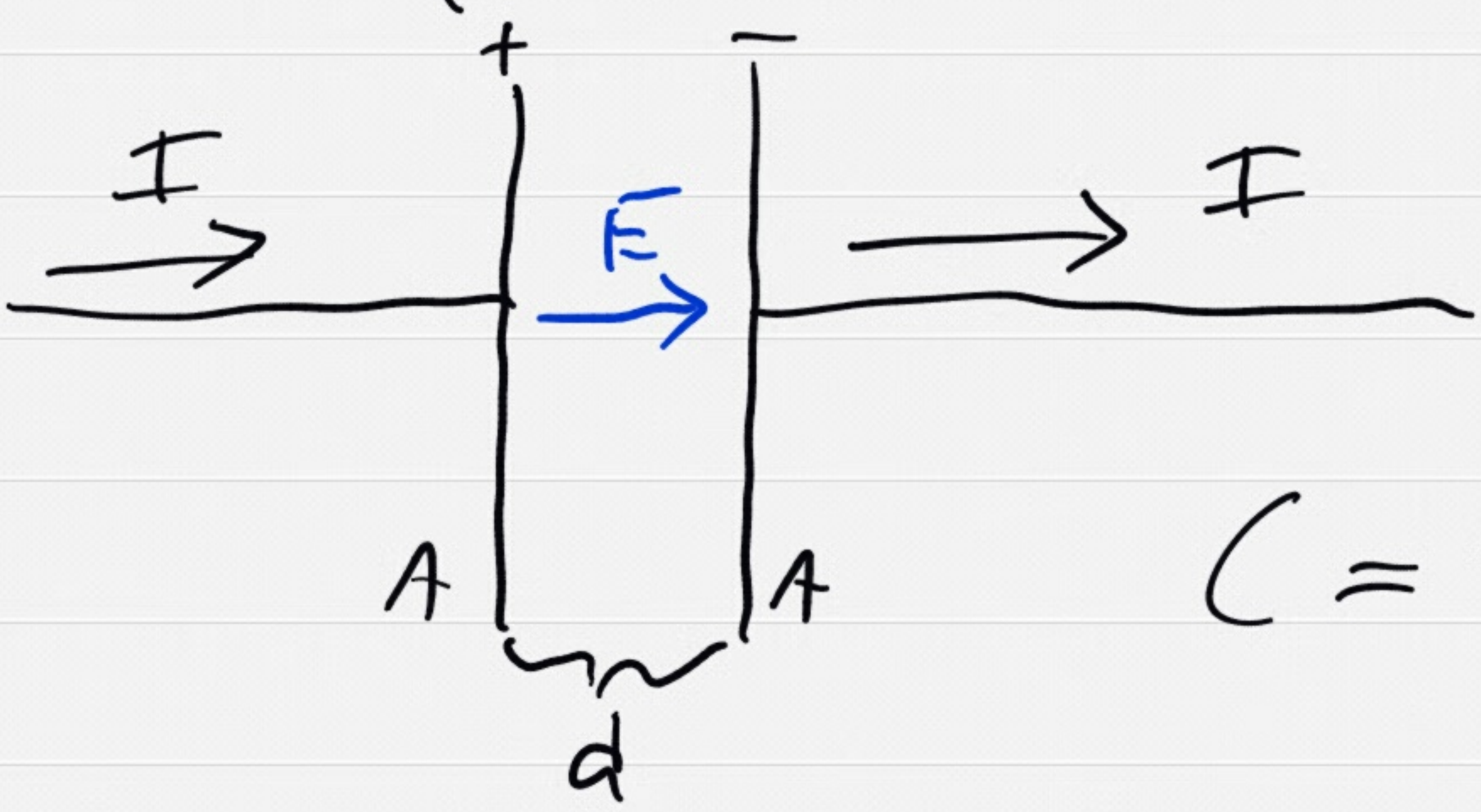
$$\nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J} + \mu_0 \varepsilon_0\, \partial \vec{E}/\partial t)$$
$$0 = 0 \,//$$

Displacement current $\vec{J}_d = \varepsilon_0\, \partial \vec{E}/\partial t$

Full Ampere's Law

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0\, \partial \vec{E}/\partial t}$$

Capacitor Paradox Resolved



$C = \varepsilon_0 A / d$

$E = \Delta V / d$ w/ $\Delta V = Q / C$

$\Rightarrow \quad E = Q / Cd = Q / \varepsilon_0 A$

$$
\begin{aligned}
I_d &= J_d \cdot A \\
&= \varepsilon_0 \, \partial E / \partial t \cdot A \\
&= \varepsilon_0 \cdot \partial / \partial t \left(Q / \varepsilon_0 A \right) \cdot A \\
&= \partial Q / \partial t \\
&= I
\end{aligned}
$$

– So the displacement current I_d "continues" the charging current I in the gap

Electrostatics/Magnetostatics => Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

Maxwell's Equations → Light

$$\left.\begin{aligned}\nabla \times \vec{E} &= -\partial \vec{B}/\partial t \\ \nabla \cdot \vec{E} &= 0 \\ \nabla \times \vec{B} &= \mu_0 \varepsilon_0 \partial \vec{E}/\partial t \\ \nabla \cdot \vec{B} &= 0\end{aligned}\right\} \rho = 0, \vec{J} = 0$$

Take $\nabla \times (\nabla \times \vec{E})$

$$= \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$= -\nabla^2 \vec{E}$$

$$= \nabla \times (-\partial \vec{B}/\partial t)$$

$$= -\partial/\partial t\,(\nabla \times \vec{B})$$

$$= -\partial/\partial t\,(\mu_0 \varepsilon_0 \partial \vec{E}/\partial t)$$

$$\Rightarrow \quad \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \partial^2 \vec{E}/\partial t^2$$

wave equation
w/ phase velocity

$$v = 1/\sqrt{\mu_0 \varepsilon_0}$$

$$= 1/\sqrt{4\pi \times 10^{-7} \cdot 8.85 \times 10^{-12}}$$

$$= 3 \times 10^8 \text{ m/s}$$

Look familiar??

$\boxed{\text{E\&M in Vacuum}}$

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0$$
$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \partial \vec{E}/\partial t$$

$+ \quad \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

- Describes everything!
- Charges generate fields
- Fields move charges

Note: $\nabla \cdot \vec{E} = \rho/\varepsilon_0$, so why not
$\nabla \cdot \vec{B} = \mu_0 \rho_m$?

$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \partial \vec{E}/\partial t$
so why not
$\nabla \times \vec{E} = -\partial \vec{B}/\partial t - \mu_0 \vec{J}_m$?

No magnetic monopoles!

(At least never found)