Physics IV [2704] Practice Midterm II

Directions: This exam is closed book. You are allowed one 8.5"x11" sheet with equations etc., which should be turned in with your test. Read all the questions carefully and answer every part of each question. Please show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Please use a calculator only to check arithmetic – all steps of calculations should be explicitly shown. Unless otherwise instructed, you can express your answers in terms of fundamental constants like $k, h, \hbar, c, \varepsilon_0$ rather than calculating numerical values. If the question asks for an explanation, please write at least a full sentence explaining your reasoning. Please ask if you have any questions, including clarification about any of the instructions, during the exam.

Good luck!

A few useful numbers:

Speed of light $c = 3x10^8$ m/s
Planck's constant $h = 6.6x10^{-34}$ J s = $4x10^{-15}$ eV s
Compton wavelength $h/(m_e c) = 2.4x10^{-12}$ m
Photon energy = $h\nu = hc/\lambda \sim 1240$ eV-nm /(λ in nm)
Bohr energies $E_n = -me^4/(32\pi^2\varepsilon_0^2\hbar^2) * Z^2/n^2 = -13.6eV * Z^2/n^2$
Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/(me^2) = 0.0529$ nm

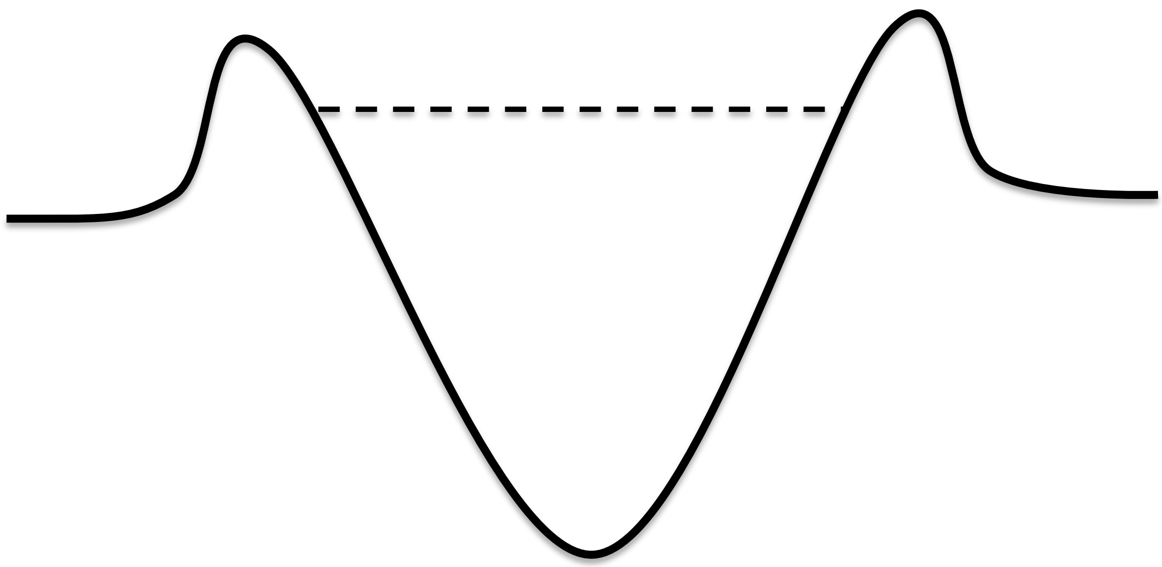
---

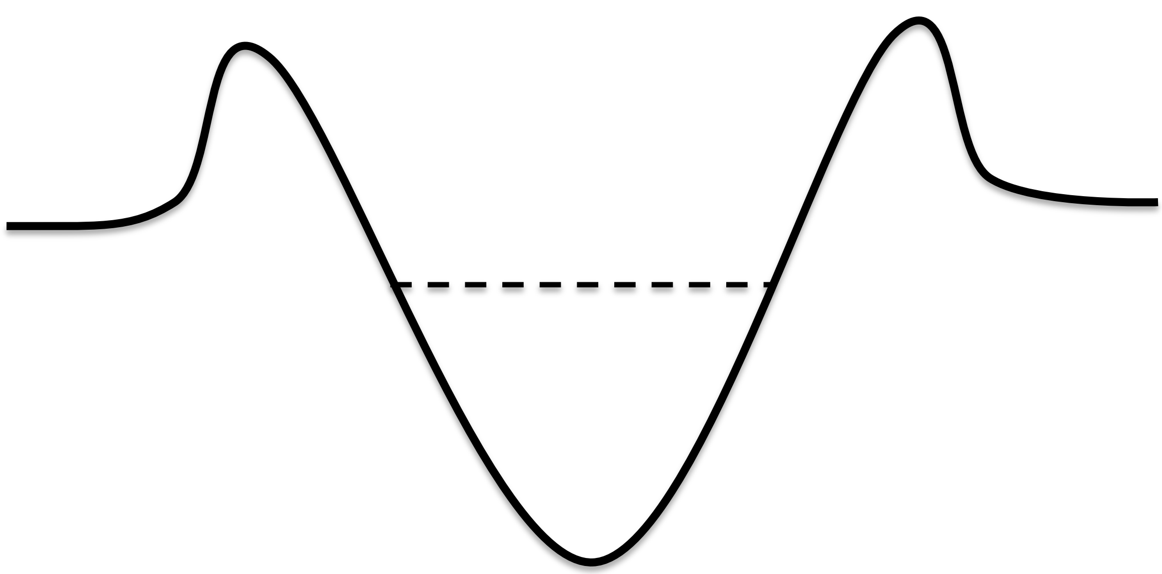
Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name_____________________________________

Print Your Name_____________________________________

Question 1 (20 points): Draw the real component of two wave functions that might occur in a symmetric potential like that shown below, for electrons at the energy levels shown. Make sure your wave functions include at least three full wavelengths, and are qualitatively accurate in how both their wavelength and amplitude vary across the drawing.





Question 2 (30 points): In some situations, the behavior of an electron can be approximated as if the electron were bound to an equilibrium position by a spring force $F(x) = -kx$ corresponding to a potential energy $U(x) = kx^2/2$.

2a (20 points): Show that the time-independent wave function $\psi(x) = Axe^{-ax^2}$ with $a = \sqrt{km}/(2\hbar)$ is a solution to the time-independent Schrödinger equation for this potential, and determine the total energy E that corresponds to this wave function.

2b (10 points): Write down an equation (do not try to solve it) that would allow you to determine the value of the normalization constant A.

Question 3 (15 points): An atom is almost completely ionized, with only one electron remaining around the nucleus (so it is "hydrogenic"). We observe that the lowest energy photon absorbed by the atom when the electron is in its ground state (i.e. the n = 1 to n = 2 transition) has an energy of 40.8 eV. What is the atomic number Z of the atom?

Question 4 (20 points): The 2*p* ($n = 2$, $l = 1$) radial wave function of an electron in atomic hydrogen is $R(r) = A\frac{r}{a_o}e^{-r/(2a_o)}$ where *A* is a constant.

4a (10 points): Find the most probable value of *r* (i.e. the most likely distance between the electron and the nucleus) from the radial probability density.

4b (10 points): List the possible sets of quantum numbers (*n, l, m_l, m_s*) that could describe an electron in the *2p (n=2, l = 1*) orbital. How many possible states are there?

Question 5 (15 points): Describe a physical rationale to explain the originally arbitrary Bohr quantization condition for orbital angular momentum $L = n\hbar$, and discuss why this conceptual model provides some accurate predictions, but fails to completely describe the hydrogen atom.