Understanding the Luminosity and Surface Temperature of Stars

1. The Hertsprung-Russell diagram

Astronomers like to plot the evolution of stars on what is known as the Hertsprung-Russell diagram. Modern versions of this diagram plot the luminosity of the star against its surface temperature, so to understand how a star moves on this diagram as it ages, we need to understand the physics that sets the luminosity and surface temperature of stars. Surprisingly, these can be understood at an approximate level using only basic physics, over most of the evolution of both low-mass and high-mass stars.

2. The role of heat loss

Stellar evolution is fundamentally the story of what happens to a star by virtue of it losing an enormous amount of heat. By the virial theorem, the total kinetic energy in a star equals half its total gravitational potential energy (in absolute value), as long as the gas in the star remains nonrelativistic. This implies that when a gravitationally bound object loses heat by shining starlight, it always contracts to a state of higher internal kinetic energy. For a nonrelativistic gas, the pressure P is related to the kinetic energy density KE/V by

$$P = \frac{2}{3}\frac{KE}{V} , \tag{1}$$

we can apply the nonrelativistic virial theorem

$$\frac{KE}{N} = \frac{1}{2}\frac{GMm_i}{R} \tag{2}$$

where m_i is the average ion mass (about the mass of a proton, usually) to assert that the characteristic pressure over the interior (i.e., the core pressure P_c) approximates

$$P_c \sim \frac{GM\rho}{R} \propto \frac{{}^2}{R^4} , \tag{3}$$

where $\rho \propto M/R^3$ is the mass density. Thus we see that whenever the star suffers a net loss of heat and contracts to smaller R, its internal gas pressure P_c rises. We note that the use of the nonrelativistic virial theorem implies that radiation is not contributing significantly to the pressure, and the electrons are not relativistic, both of which are true over the evolution to the main sequence of all but the highest and lowest mass stars.

Also notice that all of these statements continue to hold when fusion initiates, the role of fusion is simply to delay the net heat loss by self-regulating to replace the escaping energy while the fuel lasts. The heat loss drives the inexorable evolution of the star, and the evolving luminosity and surface temperature can be described using basic physics, through the basic stages of a fully convective interior (Hayashi track), the rise of radiative diffusion (Henyey track), the onset of core fusion (zero-age main sequence), hydrogen to helium conversion (evolution within the main sequence), and the development of a degenerate core (red giant phase) for low-mass stars. For high-mass stars, the gas remains ideal and the Henyey track is never really left until very late phases, and whether or not the star is undergoing core fusion determines whether or not the radius remains small or puffs out into a supergiant. Let us now go through each phase and derive an estimate of the luminosity and surface temperature, such that the evolutionary path on the H-R diagram can be understood. We can even characterize the evolutionary timescales in each phase, though all the quantifications will be only order unity in accuracy.

3. The fully convective phase and the "Hayashi track"

When a star first reaches a force balance, such that we can call it a star, it is generally very luminous and fully convective. The reason for this is that the surface temperature it would require to carry away the light that is radiatively diffusing through it would be less than the surface temperature for a fully convective star. That implies that the temperature gradient it would need, to be radiative, is steeper than what convection would produce, and that is the situation that leads to convection in the first place– a steep temperature gradient is convectively unstable, the hotter gas becomes buoyant if it starts to rise, so the rise continues in an unstable way. Of course there cannot be a net upward displacement of material, as that would violate the force balance, so cooler gas falls to take the place of the rising hotter gas. Upward motion of hot gas and downward motion of cool gas implies a net outward flux of heat, which carries the luminosity of the star up to the surface. Convection is generally very efficient, so can carry any luminosity required; the necessary luminosity is set by the surface temperature that results from convection, and the radius, via the Stefan-Boltzmann law

$$L \propto T_s^4 R^2 . \tag{4}$$

To determine T_s, we use the fact that a fully convective star has the same temperature structure that would result if the same parcel of gas were moved adiabatically all over the star, responding to the pressure stratification required by force balance. Thus we can use the well-known result that for a monatomic adiabatic gas, the ratio of the density at any two places in the interior equals the pressure ratio to the 2/5 power. Since it is actually the temperature T that we care about, we use the ideal gas law

$$T \propto \frac{P}{\rho} \qquad (5)$$

to eliminate the density (ρ) ratio, resulting in

$$T \propto P^{3/5} , \qquad (6)$$

which implies

$$\frac{T_s}{T_c} = \left(\frac{P_s}{P_c}\right)^{3/5} . \qquad (7)$$

Now we use the fact that the surface pressure is approximated by the weight per unit area of an optical depth unity of gas to estimate the surface pressure as

$$P_s \sim \frac{g}{\kappa} \propto \frac{M}{\kappa R^3} , \qquad (8)$$

whereas the core pressure obeys eq. (3). Thus the T ratio obeys

$$\frac{T_s}{T_c} \propto \left(\frac{P_s}{P_c}\right)^{2/5} \propto \left(\frac{R}{\kappa M}\right)^{2/5} . \qquad (9)$$

With the virial theorem $T_c \propto M/R$, this implies

$$T_s \propto \frac{M^{3/5}}{R^{3/5}\kappa^{2/5}} . \qquad (10)$$

To find T_s, we must treat the opacity κ. Pre-main-sequence stars will be found to have low T_s, which implies an important role for H minus opacity. That type of opacity roughly obeys

$$\kappa_{H-} \propto P_s T_s^3 \propto \frac{M T_s^3}{\kappa_{H-} R^2}, \qquad (11)$$

which physically is related to the fact that at the T_s of interest here, hydrgen is largely neutral so the necessary free electrons to make the H^- ion come from ionizing the metals, which occurs more frequently at higher T. Also, for given T, higher P causes more of those free electrons to bind to the H atoms and make the H^- ion. Using this opacity law then yields

$$\kappa_{H-} \propto \frac{M^{1/2} T_s^{3/2}}{R} . \qquad (12)$$

Thus we obtain by substituting for κ_{H-}

$$T_s \;\propto\; \frac{M^{2/5}}{R^{1/5}T_s^{3/5}} \;, \tag{13}$$

and solving for T_s yields

$$T_s \;\propto\; \frac{M^{1/4}}{R^{1/8}} \;. \tag{14}$$

The result in eq. (14) exhibits only a weak dependence on M, and an especially weak dependence on R, which is the reason that fully convective stars (on the "Hayashi track") always have a fairly consistent T_s, typically in the range 3000 – 4000 K. To place this on the H-R diagram, we need only replace R in favor of T_s and L, using $L \propto T_s^4 R^2$, giving

$$L \;\propto\; \frac{T_s^{20}}{M^4} \;. \tag{15}$$

The high power of T_s implies that the Hayashi track is nearly vertical. We can also express this as

$$L \;\propto\; R^{5/2} M \;, \tag{16}$$

which shows that L falls significantly as heat is lost and R drops, and that higher M stars will have proportionately higher L when they reach the same given R.

4. The onset of radiative diffusion and the "Henyey track"

The star continues to be fully convective until its luminosity L has dropped sufficiently that the rate of radiative diffusion through its interior can carry a greater luminosity than convection is supporting. At this point, the T_s needed to carry off the radiatively diffusing luminosity is actually higher than the T_s that convection would produce, so the radiative T gradient is softer than the adiabatic gradient. This ushers in convective stability, and two things happen: convective heat transport throughout most of the interior is replaced by radiative diffusion, and the T_s begins to rise. We shall see in a moment that this implies the star makes a transition from nearly constant T_s with dramatically falling L, to a rising T_s and a nearly constant L. This is the transition from the "Hayashi track" to the "Henyey track," and corresponds to a sharp turn in the evolution on the H-R diagram, ushering in the approach to the main sequence.

To determine L and T_s on the Henyey track, we assume the internal heat transport is predominantly by radiative diffusion, so the L is given by the ratio of the total radiative energy content, proportional to $T_c^4 R^3$, to the diffusion time t. The diffusion time can be estimated by approximating the effect of diffusion as reducing the average speed at which the radiation makes "headway" in its escape process from c to c/τ, where τ is the number of mean-free-paths in a straight line from core to surface, called the "optical depth" of the star. Since $\tau \propto \rho\kappa R$, we have

$$L \propto \frac{T_c^4 R^3}{R\tau} \propto \frac{T_c^4 R^4}{M\kappa} . \tag{17}$$

Since by the virial theorem, $T_c^4 R^4 \propto M^4$, we obtain the remarkable result that if κ is regarded as fixed, the L does not depend on either R or T_c, only on M. This is the origin of the nearly horizontal (i.e., constant L) quality of the Henyey tracks. At some point the T_c rises to around 15 million K, at which point fusion of H initiates, but since the fusion rate is self-regulated to merely replace the heat that is radiatively diffusing outward, the onset of the main sequence implies only minor adjustments of the internal structure of the star, and little change in L. Hence, we can understand the main-sequence L in terms of the pre-main-sequence radiative diffusion, and thereby derive the celebrated "mass-luminosity" relationship without specific reference to the details of fusion physics.

Historically, the main-sequence mass-luminosity relationship was understood in this way, before it was even known that fusion existed. Thus it is clear that explanations of the mass-luminosity relationship that make specific reference to fusion are incorrect. This problem is exacerbated by the common inclusion of a claim that higher-mass stars have higher fusion rates because they have higher P_c, when an elementary application of eq. (3) above, given a known T_c for fusion to initiate, demonstrates easily that higher-mass stars have significantly *lower* P_c. Hence we should not say that faster fusion in higher-mass stars causes their higher L, we should say that a faster rate of radiative escape, from a "bigger bucket leaking light" if you will, is what causes the higher L, and that in turn causes the fusion to self-regulate to whatever T_c will suffice to replace that L.

The steep dependence of L on M implied by eq. (17) is steepened even a little more when it is included that higher-M stars have somewhat higher interior T_c (to replace their higher L), and they also have lower P_c, so this gives them a somewhat lower κ due to the higher state of ionization. Hence the observed relation is more characteristically $L \propto M^{3.5}$ on average, rather than the simple $L \propto M^3$ that results from assuming a constant κ.

5. Evolution within the main sequence

Once a star begins fusing H, the core H will be turned more and more into He. This leads to a generally rising L, though the reasons for it are often poorly explained. It is certainly true that when the core has a different composition than the rest of the star, many of our simplifying assumptions are taxed, but we can still get a rough understanding of why and how much L rises simply by noticing that fusion of a gas with 75 % H and 25 % He by weight, into one with 100 % He, implies turning 14 free electrons into 8. So the Thomson (free electron) opacity falls by the factor 8/14, and we can thus expect the radiative diffusion to increase by roughly the inverse of that, or almost a factor of 2. Indeed, this does characterize the increase in L for stars during their main-sequence lifetime. We can also use the rough $\propto M^{3.5}$ result, along with $M_c \propto M$, to assert the expectation that main-sequence lifetime should scale with M like $M^{-2.5}$, which is indeed borne out in more detailed simulations.

6. Low-mass post-main-sequence evolution

Once core hydrogen runs out, what happens next depends on the mass of the star. Lower mass stars needed to lose more heat to get their cores to contract enough to start fusion, so once core H runs out and the core continues to lose heat, these stars rapidly approach the fascinating regime of all systems that have lost sufficient heat: the realm of quantum physics. Here, the de Broglie wavelength of the electrons becomes of the same order as the interparticle spacing, and the system can no longer lose heat because it can no longer contract. This is a natural consequence of two key principles of quantum physics: wave/particle duality (expressed via the de Broglie wavelength) and the Pauli exclusion principle for electrons (which technically arises because electrons are indistinguishable fermions). The first principle says that the electron cannot be confined to a box of scale smaller than its de Broglie wavelength ($\lambda = h/p$), which is the scale over which its "wave function" (the wave that in some sense tells the electron what to do) can set up destructive interference outside that box. The second principle says that the electron must be confined to such a box, because it is not allowed to share the same state as another electron (photons, for example, do not have this property– they can crowd into the same state without consequence). Putting these two principles together implies that the electron de Broglie wavelength must not exceed the interparticle spacing $n_e^{-1/3}$, where n_e is the electron number density. The reason the "size of the box" corresponds to the tiny interparticle spacing, and not the huge stellar radius, is that the particles are indistinguishable, so interchanging electrons over the scale of the interparticle spacing does not represent a physical change to the system– the possible electron states are limited to the states that fit within their own box of scale $n_e^{-1/3}$, and that cannot be made smaller than the de Broglie wavelength λ because of the wave nature of the electron behavior.

When a self-gravitating system loses heat and contracts, the kinetic energy of the electrons rises, which means their momentum p rises too, so their de Broglie wavelength $\lambda = h/p$

gets smaller. So you might think this was not approaching the limit where λ gets larger than $n_e^{-1/3}$, but actually, λ does not fall as fast with R_c as $n_e^{-1/3}$ does, so λ will ultimately catch up with $n_e^{-1/3}$, even as both drop. We can see this because its actual KE/N drops in proportion to M_c/R_c by the virial theorem, but by the quantum mechanical limitations, we have $\lambda \lesssim n_e^{-1/3}$. Thus we have

$$\frac{KE}{N} \gtrsim \frac{p^2}{2m_e} \propto n_e^{2/3} \propto \frac{M^{2/3}}{R^2} . \tag{18}$$

So as the core loses heat which is not replaced, and its R_c drops, its minimum kinetic energy rises like R_c^{-2}, but its acdtual KE/N is only rising like R^{-1}, so at some point the minimum necessary KE attempts to rise higher than the actually available KE. At this point, we can say that the core electrons have reached their "quantum mechanical ground state," and no further heat loss is possible. We can also say the core is "degenerate," for technical reasons that are largely jargon. What matters is that electrons obey the Pauli exclusion principle, which means you cannot have less than one state to go around per electron, that is, no more than one electron in each state.

The importance of this situation for the core is that it implies a core mass-radius relation, because equating the virialized KE to the minimum quantum mechanically allowed KE, for given core mass M_c, implies

$$\frac{M_c^2}{R_c} \propto M_c \frac{KE}{N} \propto \frac{M_c^{5/3}}{R_c^2} . \tag{19}$$

This in turn implies $R_c \propto M_c^{-1/3}$, which is the celebrated mass-radius relationship for degenerate cores. If you keep the appropriate physical constants to obtain an order-unity estimate of this result, you will find (try it) that a solar mass of gas produces a core that is about the size of a terrestrial planet, so that is quite small for such a huge mass– we are talking about a very dense core, and this is why it is only relevant to lower mass stars, which already achieved higher densities in order to reach the the main sequence. Still, the core density must rise considerably for most stars, so the initiation of a degenerate core, and the subsequent creation of a red giant star (as we shall see), takes some time after the end of the main sequence.

In fact, if we include the constants in the above expression, we find that the radius of a degenerate stellar core that contains about a solar mass is about the size of the Earth. As we shall see, the envelope puffs out to a radius of up to 100 times solar, so the core is some 10^4 times smaller than the rest of the star. The properties of the inert core have a huge influence on the fusing shell, and the stellar luminosity, so that tiny core is of extreme importance.

Knowing the mass-radius relationship for the core allows us to determine the shell temperature, T_s, by the condition that the gas is virialized to the core gravity. Hence, we have

$$T_{sh} \propto \frac{M_c}{R_c} \propto M_c^{4/3} . \qquad (20)$$

What this means is, the T_{sh} is determined by the core mass, and rises as helium ash builds up in the core. Even more importantly, it means T_{sh} is not self-regulated by the shell fusion like it was for core fusion, and this tends to produce very high rates of fusion. Ultimately the shell must self-regulate the mass in the shell, M_{sh}, to turn down the spectacular fusion rate at the high T_{sh}, and the way it accomplishes that is by dumping that excess heat into the envelope. This causes the envelope to expand mightily, which in turn reduces its weight because it increases the radius and thereby weakens the force of gravity. So this is the cause of the red giant phenomenon– the shell fuses too rapidly at first, dumping heat into the envelope in more or less the opposite way that heat originally leaked out of the star and caused its contraction. The result of the heat dumping in the envelope is to turn down the shell fusion rate, by reducing the weight of the envelope, reducing the balancing shell pressure P_{sh}. Since the size of the shell is pegged to the size of the core, reducing P_{sh} requires having gas expand out of the shell and into the envelope. This is what regulates M_{sh} and allows for an equilibrium between the fusion rate and the radiative diffusion rate out of the shell. Thus, to estimate this luminosity, we must find both the fusion rate, and the diffusion rate, and equate them.

Let us begin with the radiative diffusion rate. As for the Henyey track, we say

$$L \propto \frac{T_{sh}^4 R_{sh}^3}{t_{sh}} \propto \frac{T_{sh}^4 R_c^4}{M_{sh}\kappa} , \qquad (21)$$

where here t_{sh} is only the time to diffuse out through the shell, not the core, and we take $R_{sh} \sim R_c$ so that no distinction need be made between the two (in our homologous treatment). Taking κ to be constant, perhaps close to the free-electron Thomson opacity, and using the virialized $T_{sh} \propto M_c/R_c$, we obtain for the radiatively diffusing luminosity

$$L \propto \frac{M_c^4}{M_{sh}} , \qquad (22)$$

which looks a lot like the previous Henyey luminosity except M is replaced by just M_c, and there is an additional factor of M_c/M_{sh}. These may not seem like important factors for order unity work, but actually the dependence on M_c will be found to be very steep, such that the accumulated increases in M_c are quite important to the red giant evolution, and the ratio M_c/M_{sh} ultimately becomes quite large, as the shell pressure is reduced by the unweighting of the expanded envelope. It is this large factor M_c/M_{sh} that is ultimately responsible for the huge, perhaps factor thousand, increase in the L of a red giant, and we can see that it is due to the simple fact that the light does not need to diffuse through nearly as much gas as it did on the Henyey track. In effect the radiation gets a "leg up" from starting in the shell rather than the core, and much of the shell mass being expelled into the envelope, along with the very low density of that expanded envelope, allows for much easier and more rapid escape of that light.

So far we only have a reason to understand why L rises, we still have not quantified it because we do not know how M_{sh} self-regulates to reach equilibrium. To understand that, we need to model the fusion rate. Since we are dealing with high T_{sh}, the H fusion occurs via the CNO cycle, which is highly T sensitive. Indeed, it is common to model the H fusion in the shell by the expression

$$L \ \propto \ M_{sh}\rho_{sh}T_{sh}^{18} \ \propto \ \frac{M_{sh}^2 T_{sh}^{18}}{R_c^3} \ , \tag{23}$$

where the steep T_{sh}^{18} dependence plays an important role since it means that rises in T_{sh} due to the increasing M_c creates an overproduction of fusion that is ultimately responsible for the expansion of the envelope and the self-regulation of M_{sh}. Applying $T_{sh} \propto M_c/R_c$ and $R_c \propto M_c^{-1/3}$ from above leads to

$$L \ \propto \ M_c^{25} M_{sh}^2 \ \propto \ \frac{M_c^4}{M_{sh}} \ , \tag{24}$$

where the first expression is for the fusion rate and the second is for the radiative diffusion rate. Equating them results in $M_{sh} \propto M_c^{-7}$, which shows the extreme sensitivity of M_{sh} to M_c, and it also shows how M_{sh} must drop as M_c accumulates. Substituting this result back into either L expression then yields $L \propto M_c^{11}$, showing that even a factor of nearly 2 increase in M_c via the accumulation of He ash can produce a factor of a thousand increase in L.

To place this result in the H-R diagram, we also need the evolution of the surface T_s. This is accomplished through $L \propto T_s^4 R^2$, where T_s is determined by the fact that the puffed out envelope is again fully convective, so once again we have a Hayashi track situation. This implies T_s will always be in the vicinity of 3000 K, which is why they are called "red" giants. Since we can also estimate L using the above expressions, but leaving the constants in them, we find that L rises to about 10^3 times solar, and R rises to about 100 solar. This is true regardless of the mass of the star (as long as it is a low-mass star), because it is all determined by the mass of the core, not the mass of the whole star. So as the core builds up mass, all these low-mass stars follow a similar path up the red giant branch.

6.1. The helium flash

If the core continued to pile up helium ash until the entire mass of the star was in the core, then the ultimate L of any red giant would depend on its total M. However, this is not the case. Instead, the same peak L is reached over a wide range of M from about 0.6 solar to about 8 solar. This is because the T_c rises with M_c until, at the same M_c of about 0.6 solar masses, it gets hot enough for the He to begin fusing, and the structure of the star reverts to a more dwarflike status because the luminosity must once again diffuse out from near the center. That returns the star closer to its old Henyey luminosity, but not quite that low because there is still a shell fusing H, and the light from that shell does not need to diffuse all the way out from the core. Stars fusing He in their cores are called "horizontal branch" stars, because they lie in the H-R diagram along a horizontal branch, not too far from the main sequence. Before they get there, however, they undergo an interesting event, called the helium flash.

The helium flash is the result of having a star initiate fusion in a degenerate core. Degenerate gas has some unusual properties, the most striking being that essentially all the kinetic energy that is responsible for the extreme pressure holding up the core against its self-gravity is in the electrons, not the ions. But the fusion rate is regulated by the kinetic energy in the ions (here, helium), so when fusion initiates and heat is released, it does not experience the thermostatic stabilization of fusion in an ideal gas. The excess heat adds to the kinetic energy of the core and produces the same expansion that it would in an ideal gas (in contrast to descriptions you will generally find elsewere), but the work done by this expansion does not reduce the temperature, keeping the fusion self-regulated. Instead, the work done comes from the kinetic energy of the electrons, but this is highly decoupled from the kinetic energy in the ions. That decoupling is a standard feature of degenerate electrons, the same thing happens in a metal spoon– if you hold one to your cheek, it will feel cool, because the ions in the metal are responsible for the kinetic energy associated with its temperature, even though the electrons in the metal hold vastly more kinetic energy than do the ions.

As can be seen from a more detailed investigation, almost all the heat released by fusion goes into the helium ions, just as almost all the heat from putting a spoon over a flame would go into the metal ions, whereas the PdV work from the expanding stellar core comes out of the electron kinetic energy. This makes the fusion unstable– the more fusion, the more heat goes into the ions, the more their temperature rises, even though expansion is reducing the kinetic energy of the electrons. The fusion runs away until degeneracy is lifted, and ions and electrons ultimately share the kinetic energy more equally, as in ideal gases. I have never seen this correctly explained anywhere else.

Once the core returns to being an ideal gas, the thermostatic self-regulation sets in once again, but not before an explosive release of the fusion energy of some of the helium is released. That is the "flash," which lasts less than a second, and is not observable at the surface of the star because the diffusion timescales are much longer. The energy release only helps give the star enough of a jolt to begin restructuring itself into the more dwarf-like status of when core fusion is occurring. It is perhaps counterintuitive that a star that is undergoing fusion in its core and in a shell, instead of just in a shell, would drop down to a much smaller radius, but the basic reason is that the helium flash expands the core, reducing its gravitational potential energy, which in turn reduces the temperature in the shell. This greatly reduces its fusion rate, so the shell is no longer under any requirement to lift weight off itself to reduce its pressure. The envelope, stripped of heat input from the copious shell fusion rate, leaks excess heat into space and shrinks back down again.

6.2. The asymptotic giant branch

It is of course inevitable that the core will eventually run out of helium to burn, just as it did hydrogen. Helium fusion results in the creation of carbon (which is 3 helium nuclei fused together), and some oxygen (which is 4). There is no stable element that involves 2 helium nuclei fused together, so the fusion must actually be a 3-body process, where 3 nuclei come together all at once. That this only happens in combination with high temperature and high *density* is the reason that carbon and the heavier elements are only created in stellar cores. But once the core is all carbon and oxygen, fusion ceases once again, and a very similar evolution repeats– the star becomes a red giant for the second time, following a path that is "asymptotically" similar to the one it followed previously. The star can ultimately reach even higher luminosities than before because there is no "carbon flash"– almost all of the mass of the star can be put into its core without ever getting hot enough to fuse carbon, so M_c is allowed to rise even higher the second time around. Also, there is then a shell of helium fusion along with the shell of hydrogen fusion, which adds to the luminosity. To support its higher luminosity, the "asymptotic giant" envelope puffs out even more than before, and our own Sun will puff out to a radius of about 1 AU at this point. (The Earth will not be swallowed though– the Sun also loses some mass to a strong wind during this phase, and a lower-mass Sun requires that the Earth move outward in its orbit.)

6.3. The creation of a planetary nebula and a white dwarf

Since the carbon/oxygen core never gets hot enough to fuse, the shell fusion continues to add energy to the envelope, ultimately driving a strong wind that completely strips the envelope of the star, exposing the naked core. The envelope can create a beautiful nebula, called a planetary nebula (because it looks like a planet in an old-fashioned telescope). The core is still a degenerate carbon/oxygen gas that is now exposed, and is about the size of the Earth, so gets called a "white dwarf" (so it is the kind of dwarf that is not a main-sequence star, and is way smaller than a red or yellow or blue dwarf). Hence, whenever you see a planetary nebula, look for the little white spot near the center– that's the core of the star that remains after shedding its envelope. The white dwarf is close to its quantum mechanical ground state, but is not completely there, so it still shines from its white hot (non-degenerate) surface. But over time, it will simply cool down, as it has no heat source and cannot contract further unless more mass gets added to it somehow. When mass is added by a binary companion, we get a very different result, but that is a topic for later on when we talk about binaries.

7. High-mass stars

For stellar masses above about 8 times the Sun, the post-main-sequence evolution is completely different. The reason is, the core does not go degenerate before it starts fusing its helium. Even so, the ideal-gas core, prior to helium fusion, must still contract as it loses heat to the envelope, so the envelope still puffs out. Also, hydrogen fusion continues in a shell, just as for lower mass stars. So the high-mass star also becomes giant in size, and since it was already larger than lower-mass stars when it was a dwarf, when it puffs out it gets called a "supergiant" (and the highest mass stars even get called hypergiants). The main difference is, the L of the supergiant does not rise significantly, because the gravitational contraction of the core never ceases, and so the core is still an important source of the luminosity of the star. This means the luminosity must still (mostly) diffuse out through the entire star, and the Henyey luminosity continues to be a reasonable benchmark, though the shell fusion adds a bit more luminosity to the supergiant. As such, these stars tend to evolve horizontally to the right in the H-R diagram.

When the core gets hot enough to fuse helium, which is about 100 million K, the behavior is similar to the low-mass stars, in the sense that the star shrinks back down. What is essentially happening here is that since high-mass stars have ideal-gas cores, rather than degenerate cores, they find there are two rather different states that they can be in, one supergianlike and one dwarflike, and the star is a bit "schizophrenic" about which state to be in. Relatively minor internal adjustments, like whether the central heat source is fusion or gravitational contraction, end up having a major influence in determining which of the two possible solutions actually occur. When the star is core fusing, it is dwarflike, and when it runs out of a fuel in the core, it becomes supergiantlike, all while maintaining close to the Henyey luminosity, so it moves horizontally back and forth from left to right as it sequentially fuses hydrogen into helium, then helium into carbon, then oxygen, then silicon, and all the way up to iron. The rest of the star continues to fuse in shells, creating a kind of "onion layers" composition in the interior.

Once the core reaches iron, there is no more heat that can be released by fusion, because iron has a very low energy per nucleon. Indeed, this is why nuclei lighter than iron release heat when they fuse, but nuclei heavier than iron release heat when they fission. Since the core has no more nuclear fuel, it continues to lose heat until it also goes degenerate. However, the cores of massive stars do not become white dwarfs, because of the way relativity changes the electron behavior.

7.1. Relativistic electrons and core-collapse supernovae

The degenerate mass-radius relationship from low-mass stars tells us that as the iron core of a massive star builds up iron ash and increases M_c, it shrinks and raises the kinetic energy of the electrons. However, since there is more M to begin with, and nothing left to fuse, the M_c rises even higher, and so does the kinetic energy of the electrons. The rest mass of an electron, in energy units, is 455 eV, so when the kinetic energy of the electrons begins to exceed this level, the electrons behave more and more relativistically. This changes their connection between kinetic energy and momentum ($KE = pc$ rather than $KE = p^2/2m$), which alters the character of the virial theorem. This has extremely important ramifications for the star.

The nonrelativistic virial theorem asserts that the kinetic energy is (minus) half the potential energy, which implies that to increase the KE by one unit, you need to extract one unit of heat from the system. However, the highly relativistic virial theorem changes this to saying that the kinetic energy equals (minus) the potential energy. This seemingly innocent factor of 2 difference means that if you contract a relativistic self-gravitating gas, it attains no more kinetic energy than it needs to remain in force balance at the smaller radius– there is no need to extract any heat! So a relativistic gas does not need to wait to lose heat, it is already extremely easy to contract. This leads to runaway collapse any time there is any mechanism that can extract heat from the system, and many such mechanisms do indeed exist at the high temperatures that core contraction leads to. So the cores of massive stars are given to catastrophic collapse as soon as the electrons go relativistic. This does not happen to the electrons in low-mass stars, which go degenerate (so reach their quantum mechanical ground state) prior to going relativistic.

The difference is extreme, because when the core of a star collapses catastropically, it releases such a huge burst of gravitational energy that it can explode the rest of the star. This is called a "core-collapse supernova," so we see the reason why stars with masses above about 8 solar masses will die in dramatic explosions, rather than create white dwarfs. The remnant left behind from the core collapse is either a neutron star, or a black hole, depending on the mass of the core that collapses. The resulting supernova remnant surrounding the remnant looks more violent and choppy than do planetary nebulae, and the gas is moving at much higher speeds, a few percept of the speed of light.

8. Timescales for each stage

The above scaling laws can all be turned into quantitative estimates simply by keeping the fundamental physical constants in the expressions. This allows us to quantify the L at the order-unity level, and that in turn lets us estimate the timescales for each of these phases of evolution. We can look at these timescales in more detail later if we get the opportunity, but suffice it to say that the higher the M of a star, the faster it moves through these various phases. Also, most stars spend the majority of their lives, say up to about 90 %, in the main-sequence phase. This does not count the time they might spend as white dwarfs or neutron stars, as that timescale is potentially essentially infinite.