Cosmology

- Assumptions in cosmology
- Olber’s paradox
- Cosmology à la Newton
- Cosmology à la Einstein
- Cosmological constant
- Evolution of the Universe

Assumptions in Cosmology

Copernican principle:

- We do not occupy a special place.
- There are no special places.
- The universe is homogeneous if viewed at sufficently large scales.
- The laws of physics are the same everywhere.

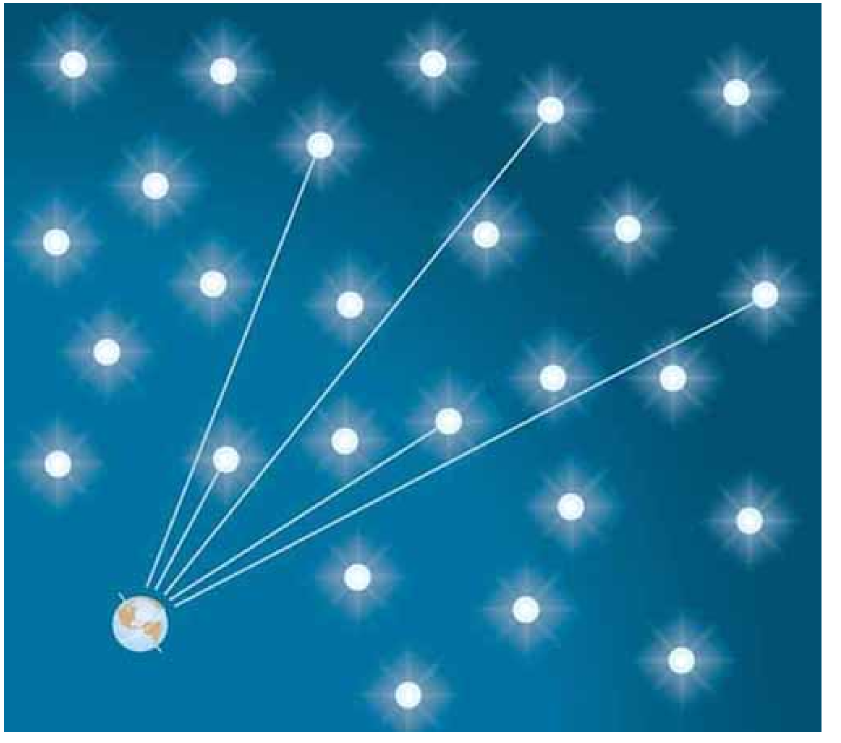
Implications of the Copernican Principle

- The average density of matter and energy is the same throughout the Universe.
- The same Hubble expansion law is seen for all observers anywhere in the Universe.
- The curvature of the Universe is the same everywhere.

How can we test the Copernican principle?

- Does the Universe look the same in all directions? (Isotropy)
- Are the spectral lines from atoms the same in distant galaxies?
- Do the same laws of gravity apply in other galaxies?

Why is the night sky dark? (Olber's Paradox 1826)



- Or what is the temperature of the sky?
- Assume universe is static, infinite, and full of stars like the Sun.
- Then every ray extending out from the Earth will eventually intersect a star.
- So, the brightness of the sky at that point will be determined by the surface brightness of that star.
- But surface brightness is independent of distance, so the whole sky should be as bright as the Sun.

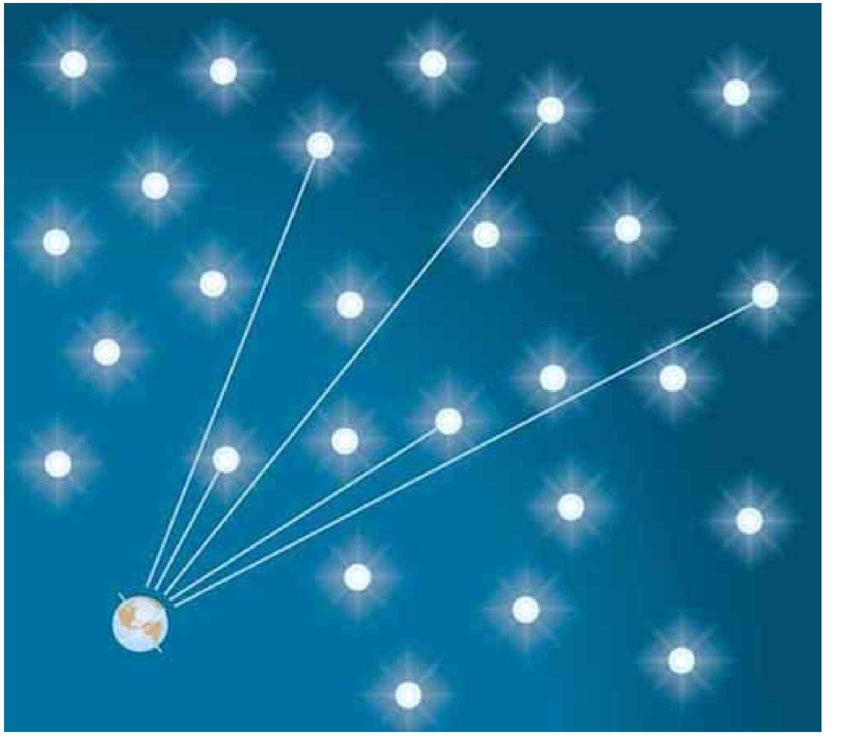
$$F \propto d^{-2} \qquad \Omega \propto d^{-2} \qquad \Sigma = \frac{\mathrm{F}}{\Omega} \propto d^{0}$$

Why is the night sky dark? (Olber's Paradox 1826)

• One of the assumptions (static, infinite, and full of stars like the Sun) must be incorrect.

• Thus, to have a dark night sky, the Universe must be some combination of

  • dynamic
  • finite in time
  • finite in extent



Scale Factor

- Assume expansion of Universe is homogeneous and isotropic
- Then expansion can be described by a scale factor $a(t)$, such that

$$r(t) = a(t)\, x$$

where $x = r(t_0 = \text{now})$ is "comoving coordinate"

a is "cosmic scale factor", $a(t_0) = 1$

Hubble Parameter

- Scale factor $a(t)$, such that $r(t) = a(t)\,x$
- Hubble law $v = H(t)r$
- Becomes

$$v = \frac{d}{dt} r = \frac{da}{dt} x = \dot{a}\, x = \frac{\dot{a}}{a} r = H(t) r$$

- Thus

$$H(t) = \frac{\dot{a}}{a}$$

Cosmology à la Newton

- Model universe as homogeneous sphere with mass M and radius r, consider test mass m at surface. Then energy is:

$$\frac{1}{2} m v^2 - \frac{GMm}{r} = -\frac{1}{2} Kmc^2 x^2$$

- Rewrite in terms of scale factor and density ρ

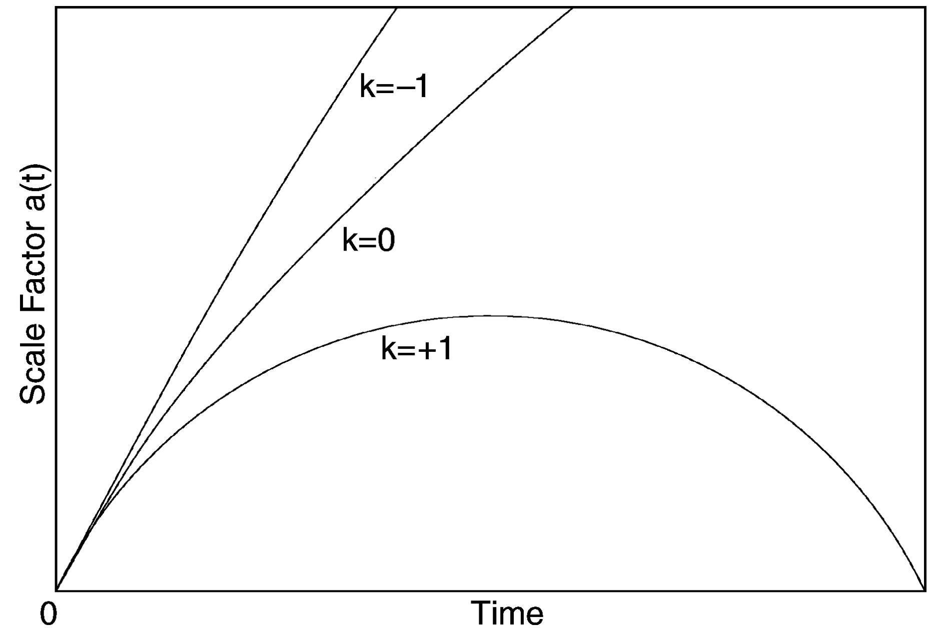
$$r = ax \qquad v = \dot{a} x \qquad M = \frac{4}{3} \pi r^3 \rho$$

$$\frac{1}{2} \dot{a}^2 x^2 - \frac{4 \pi G \rho a^3 x^3}{3 a x} = -\frac{1}{2} K c^2 x^2$$

$$\dot{a}^2 = \frac{8 \pi G}{3} \rho a^2 - K c^2$$

$$\frac{\dot{a}^2}{a^2} = \frac{8 \pi G}{3} \rho - \frac{K c^2}{a^2}$$

Cosmology à la Newton



$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - K c^2$$

- K ~ total energy of a comoving particle
- $K < 0$, RHS > 0, since da/dt > 0 today, universe will expand forever.
- $K = 0$, RHS > 0, universe will expand forever but da/d$t \rightarrow 0$ as $t \rightarrow \infty$.
- $K > 0$, RHS = 0 when $a = 8\pi G\rho_0/3Kc^2$, at this point expansion stops and universe starts to contract.
- Critical density is defined for $K = 0$

$$\rho_{cr} = \frac{3H_0^2}{8\pi G}$$

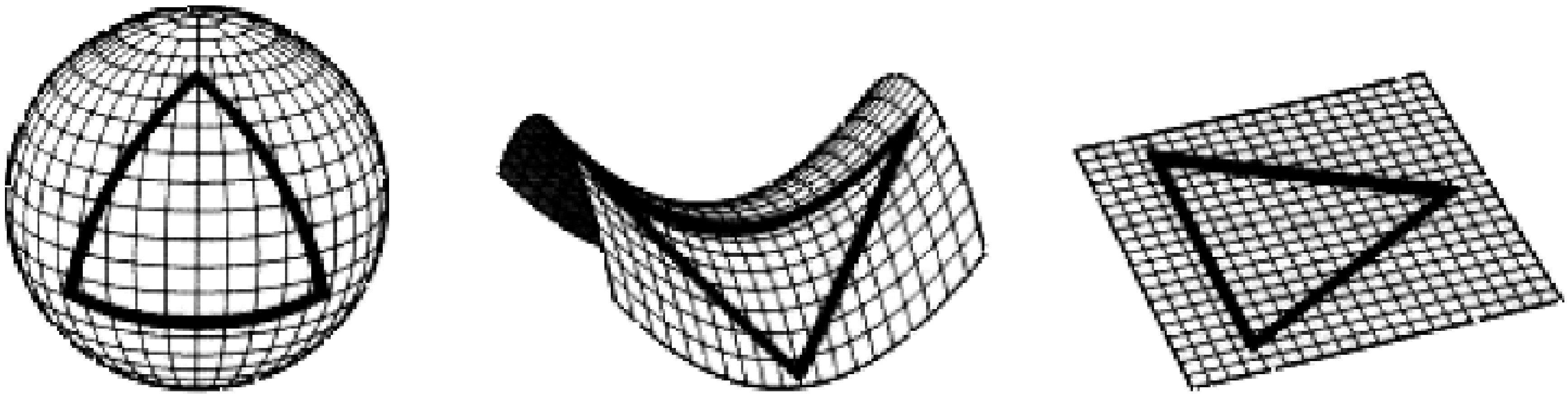
Cosmology à la Einstein

- Add energy density due to radiation
- Add "cosmological constant" or vacuum energy, Λ
- Change previous equation to:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$$

- Friedman-Lemaitre expansion equation

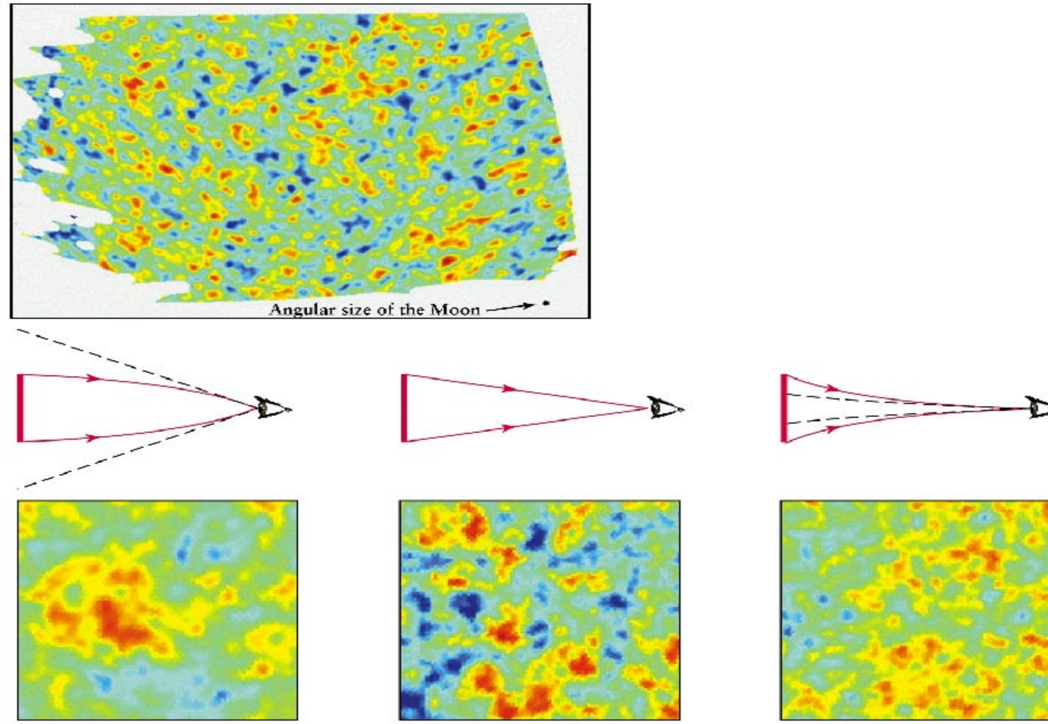
Cosmology à la Einstein



Closed Open Flat

- Interpret K in terms of curvature of spacetime:
  - $K > 0$ positive curvature (sphere)
  - $K < 0$ negative curvature (saddle)
  - $K = 0$ flat.

Fluctuations in CMB



a If universe is closed, "hot spots" appear larger than actual size

b If universe is flat, "hot spots" appear actual size

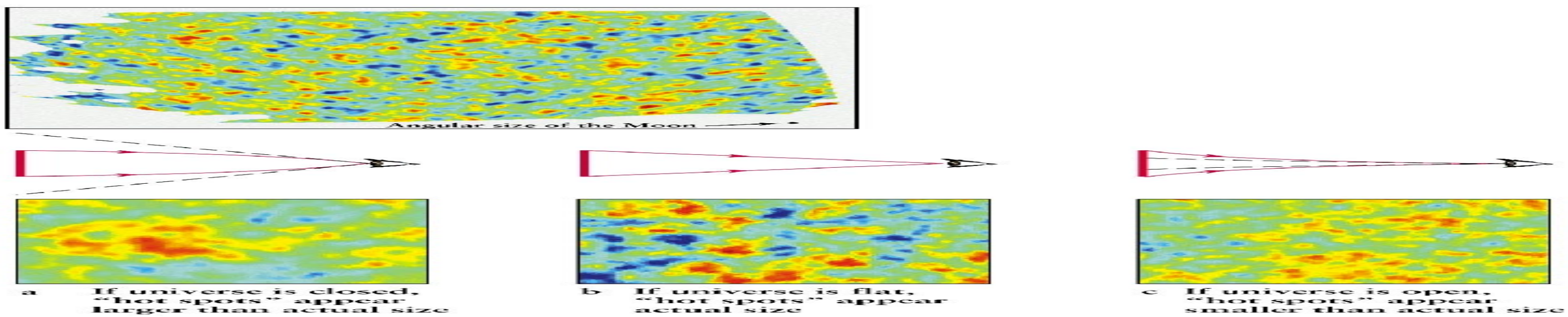
c If universe is open, "hot spots" appear smaller than actual size

- Properties of the CMB are set by physics of ionized plasma at ~4000 K – well known physics.

- Temperature variations in the Cosmic Microwave Background (CMB) are observed to be about 0.0003 K, agrees with calculations of sound waves propagating through plasma.

- The expected physical size of the hot/cold regions can be calculated. What can we do with this?

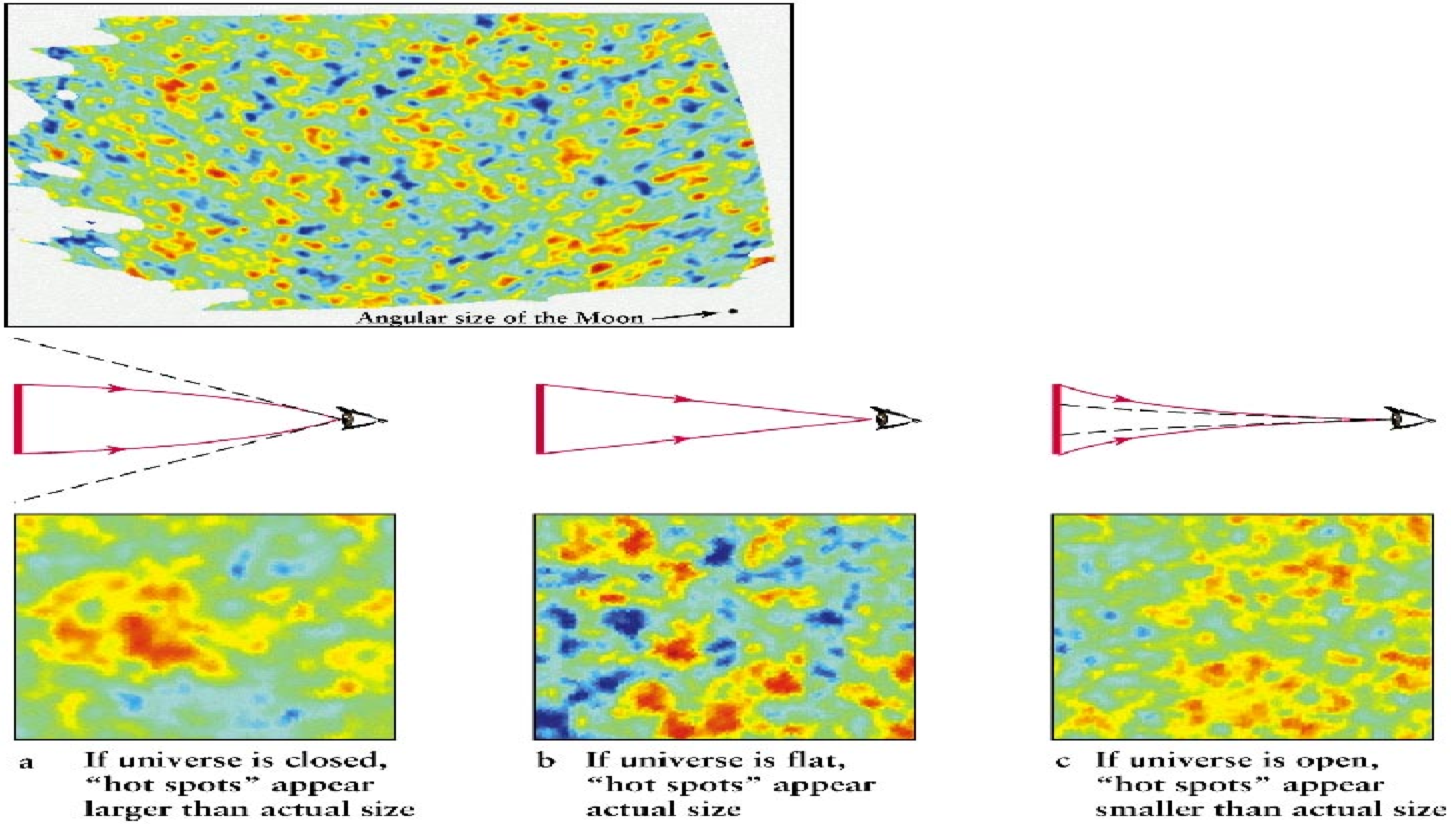
Angular size in curved space



$$\alpha = \frac{D}{R \cdot \sin(d/R)} \geq \frac{D}{d} \qquad \alpha = \frac{D}{d} \qquad \alpha = \frac{D}{R \cdot \sinh(d/R)} \leq \frac{D}{d}$$

- We can draw triangles!
- Angles in triangles in curve space do not equal 180°.
- This affects the angular size

Angular size of CMB fluctuations



- In positively curved Universe, fluctuations appear larger than calculated
- In negatively curved Universe, fluctuations appear smaller
- In flat Universe, fluctuations appear at expected size

Curvature of the Universe

The curvature of the Universe is determined by the density parameter Ω_0

$$\Omega_0 = \frac{\rho}{\rho_C}$$

$\Omega_0 < 1 \Rightarrow$ negative curvature

$\Omega_0 > 1 \Rightarrow$ positive curvature

Measurement of CMB fluctuations gives

$$\Omega_0 = 1.02 \pm 0.02$$

Evolution of energy densities

- Energy density of Λ is constant in time.
- Energy density of matter (normal or dark)
  - Assume non-relativistic particles, then energy is dominated by rest mass
  - Rest mass is not red-shifted, so energy density varies like number density of particles, decreases as volume of universe increases

$$\rho_m(t) = n(t)\varepsilon = n(t)mc^2 = mc^2 N/V \propto a(t)^{-3}$$

Evolution of energy densities

- Energy density of radiation
  - Number density of photons as volume of universe increases

    $n(t) = N/V \propto a(t)^{-3}$
  - Wavelength of photons increases as size of universe increases

    $\lambda(\mathrm{t}) \propto a(t)$ so $\varepsilon(\mathrm{t}) = \mathrm{hc}/\lambda(\mathrm{t}) \propto a(t)^{-1}$
  - Combine both factors

    $\rho_{\mathrm{r}}(t) = n(t)\varepsilon \propto a(t)^{-3} a(t)^{-1} \propto a(t)^{-4}$

Expansion equation

- Write densities as $\Omega_i = \rho_i/\rho_{cr}$ and use appropriate scaling for each, rewrite expansion equation as:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[a^{-4}\Omega_r + a^{-3}\Omega_m + a^{-2}\left(1-\Omega_m-\Omega_\Lambda\right)+\Omega_\Lambda\right]$$

- To find age of the universe, note $dt = da(da/dt)^{-1} = da/(aH)$, and integrate:

$$t(a) = \frac{1}{H_0}\int_0^a da\left[a^{-2}\Omega_r + a^{-1}\Omega_m + \left(1-\Omega_m-\Omega_\Lambda\right)+a^2\Omega_\Lambda\right]^{-1/2}$$

- Redshift $1+z = \lambda_{obs}/\lambda_a$ have $\lambda_{obs} = \lambda_a/a$ so $1+z = 1/a$

Friedmann/Lemaitre Equation

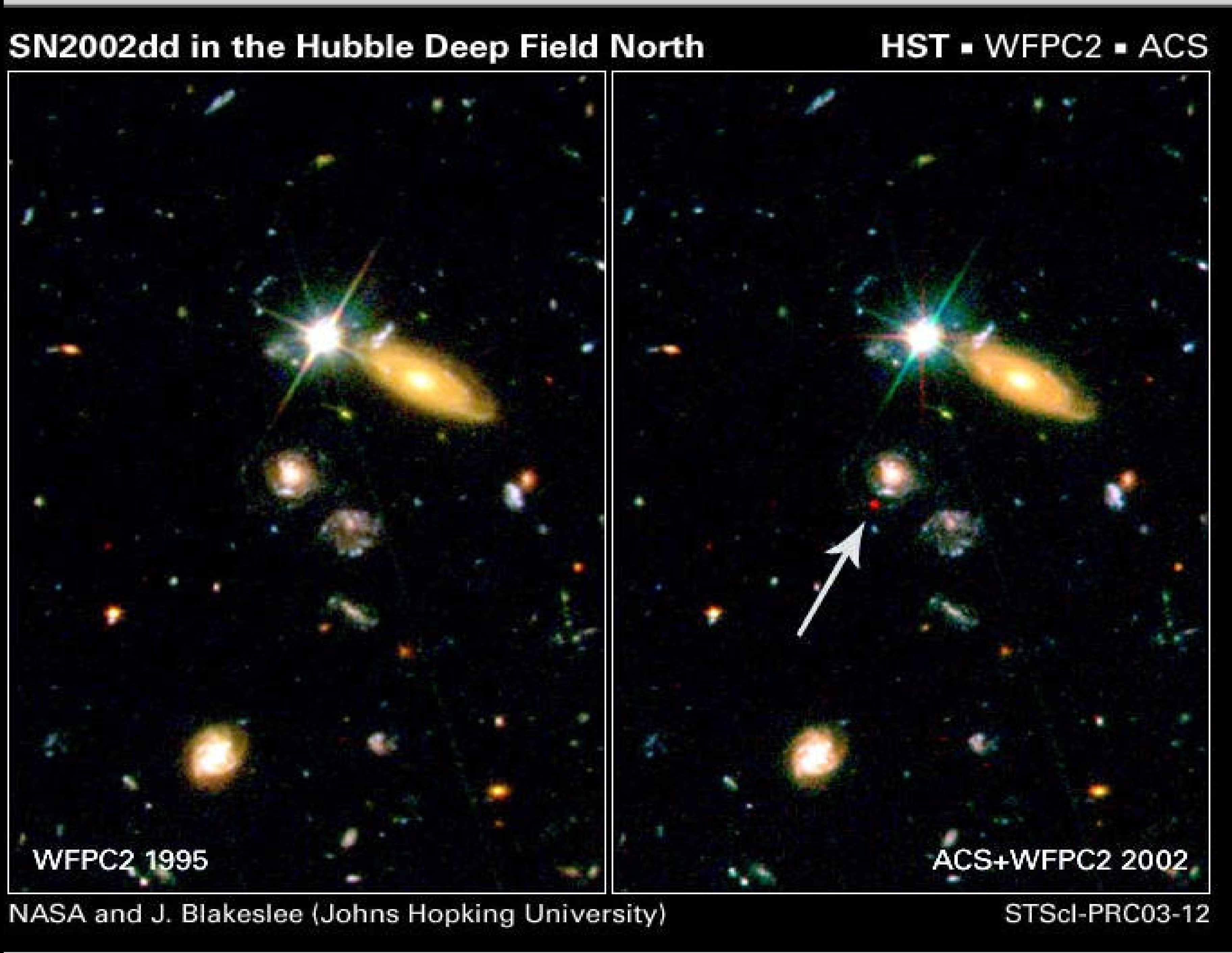
$$\dot{a} = H_0 \left[\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 \right]^{1/2}$$

$$\ddot{a} = H_0^2 \left[-\frac{\Omega_{r,0}}{a^3} - \frac{\Omega_{m,0}}{2a^2} + \Omega_{\Lambda,0} a \right]$$

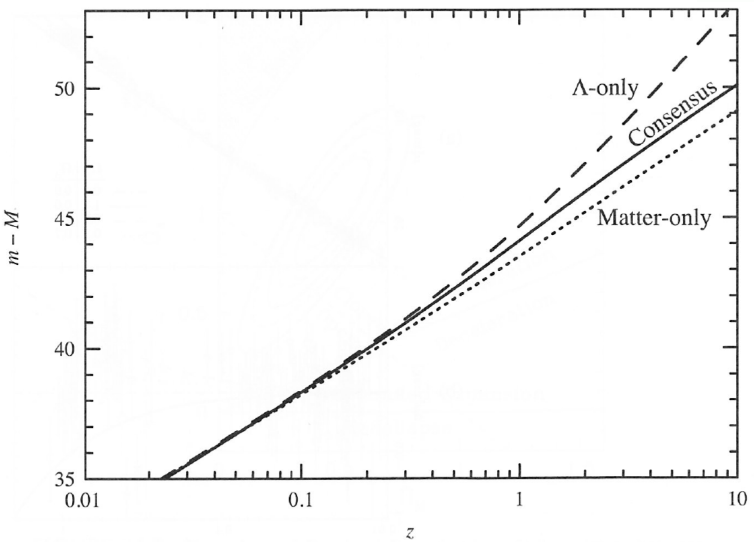
- Radiation and matter slow down expansion
- CC speeds up expansion
- Impossible to get static universe without CC

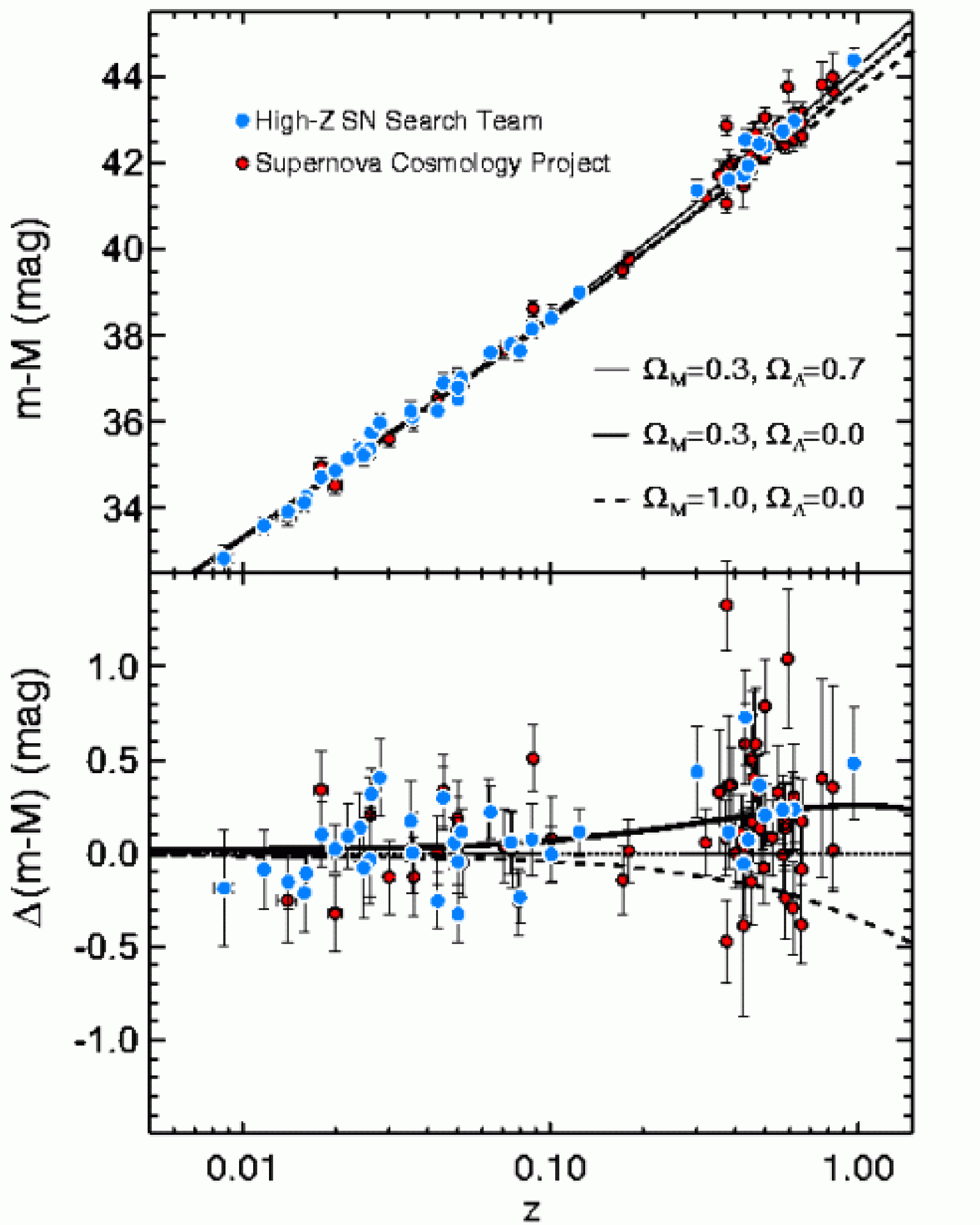
Einstein and Cosmology

- After Einstein wrote down the equations for General Relativity, he made a model of the Universe and found that the Universe had to be either expanding or contracting.
- He introduced a new term, the cosmological constant or Λ, in his equations representing a energy field which could create antigravity to allow a static model.
- After Hubble found the expansion of the Universe, Einstein called Λ his greatest blunder.
- Quantum physics predicts some energy fields that act like Λ.


SN2002dd in the Hubble Deep Field North
HST ▪ WFPC2 ▪ ACS
WFPC2 1995
ACS+WFPC2 2002
NASA and J. Blakeslee (Johns Hopking University)
STScI-PRC03-12

Accelerating Universe

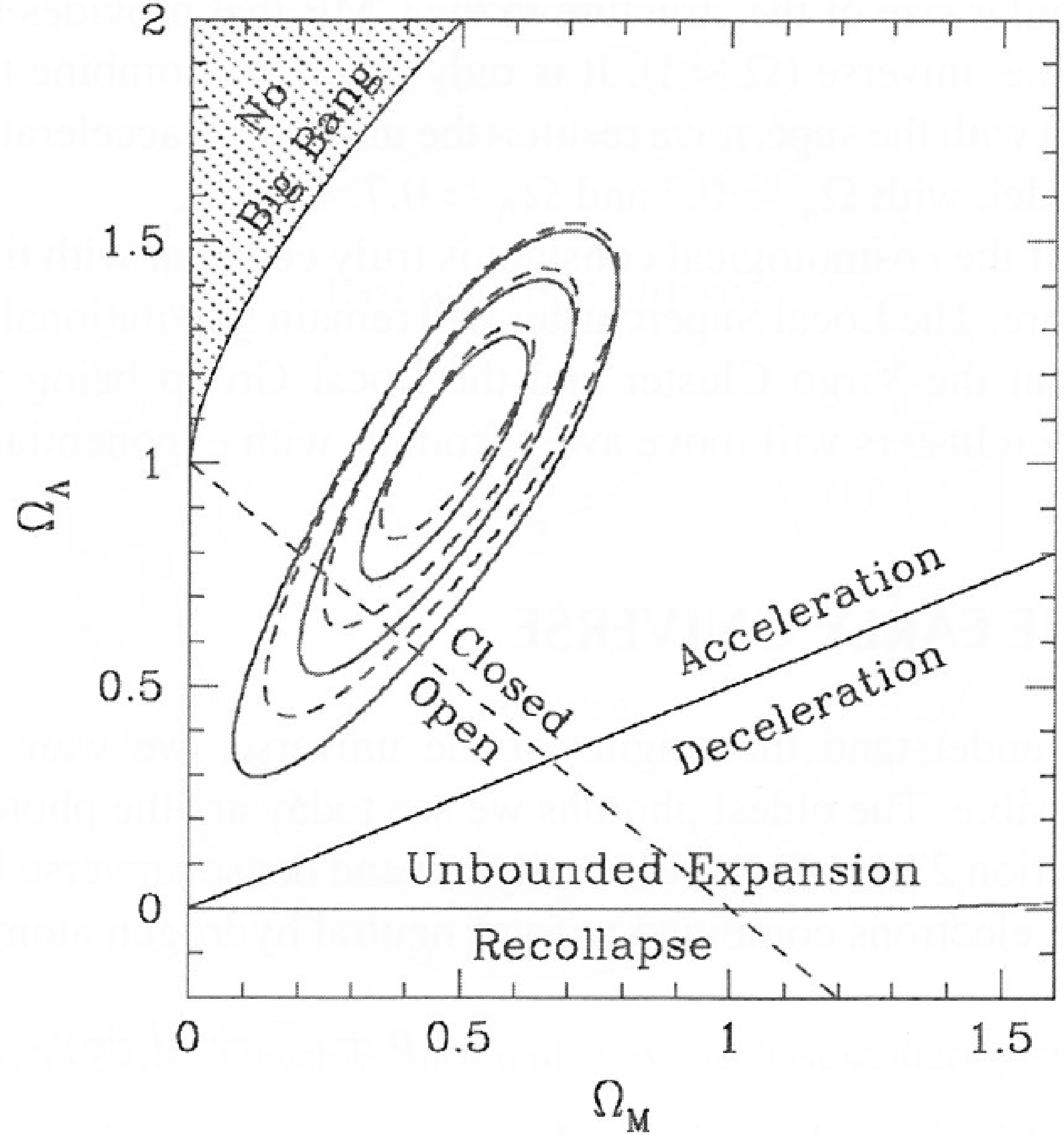




Accelerating Universe

- Hubble expansion appears to be accelerating
- Normal matter cannot cause acceleration, only deceleration of expansion
- Dark energy is required
  - may be cosmological constant
  - may be something else
  - major current problem in astronomy

Supernova constraints on Ωs



- Dashed vs solid are different SN samples
- Use curvature constraint Ω=1.02±0.02 to narrow range

Energy densities

Critical density

$$u_c = \rho_c c^2 = \frac{3H_0^2 c^2}{8\pi G} = 5200\ \mathrm{MeV\ m^{-3}}$$

Express densities in terms of density parameters:

$$\Omega_m = \frac{u_m}{u_c} \quad , \ldots$$

From CMB curvature measurement:

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1.02 \pm 0.02$$

Radiation Energy Density

Main component is CMB, star light is < 10%

$u_{\mathrm{CMB}} = 0.260$ MeV m^{-3}

$$\Omega_{CMB} = \frac{u_{CMB}}{u_c} = \frac{0.260\ \mathrm{MeV\ m^{-3}}}{5200\ \mathrm{MeV\ m^{-3}}} = 5.0 \times 10^{-5}$$

There are also likely neutrinos left over from the big bang, produced when nucleons froze out

$u_{\mathrm{nu}} = 0.177$ MeV m^{-3}

$$\Omega_{CMB} = \frac{u_{CMB}}{u_c} = \frac{0.177\ \mathrm{MeV\ m^{-3}}}{5200\ \mathrm{MeV\ m^{-3}}} = 3.4 \times 10^{-5}$$

Total for radiation: $\Omega_r = 8.4 \times 10^{-5}$

Matter Energy Density

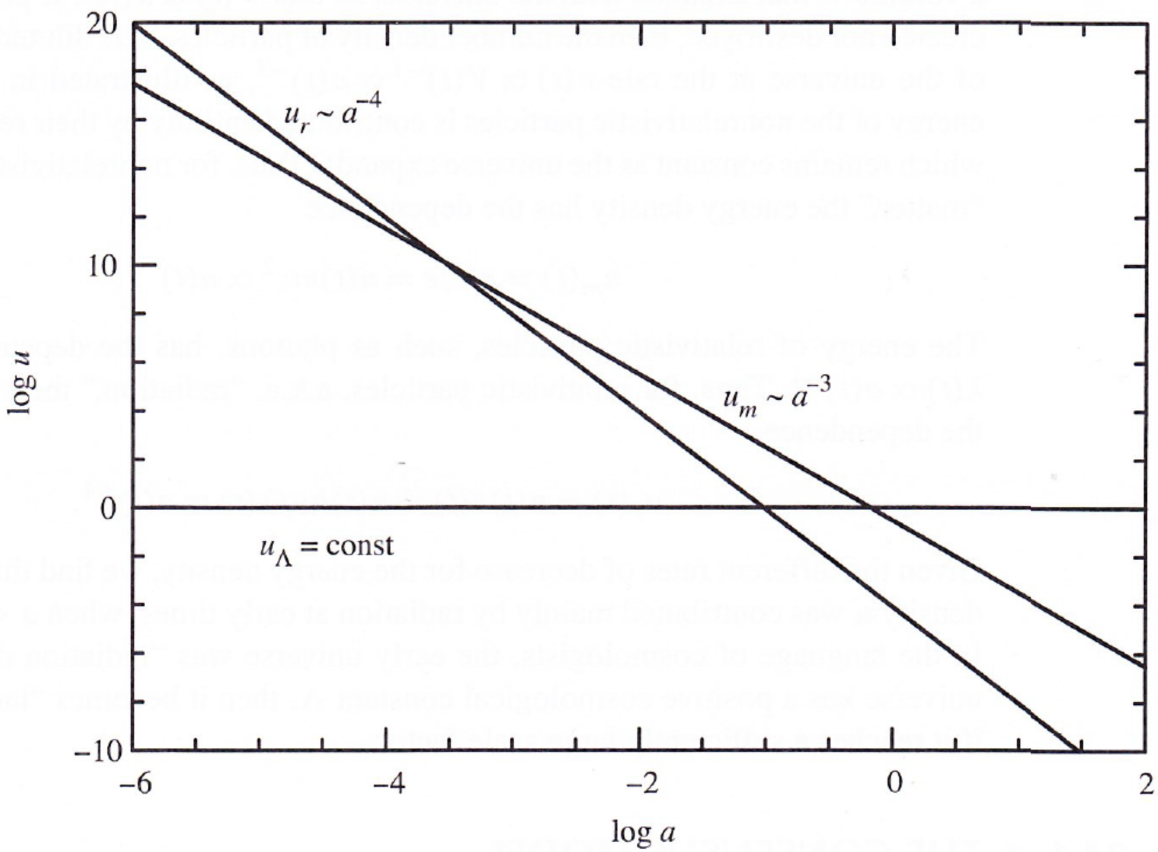
- Matter in baryons (protons, neutrons, electrons): $\Omega_{bary} = 0.04$
- Matter in clusters (part dark): $\Omega_{cluster} = 0.2$
- Best estimate of all matter (baryons+dark): $\Omega_{m} = 0.3$
- Ratio of photons to baryons $\sim 2 \times 10^{9}$

Consensus Model

Component	Ω
Photons	5.0×10^{-5}
Neutrinos	5.0×10^{-5}
Total radiation	5.0×10^{-5}
Baryons	0.04
Dark matter	0.26
Total matter	0.30
Cosmological constant	~0.7
Curvature	1.02 ± 0.02

- Hubble constant = 70 ± 5 km s^{-1} Mpc^{-1}

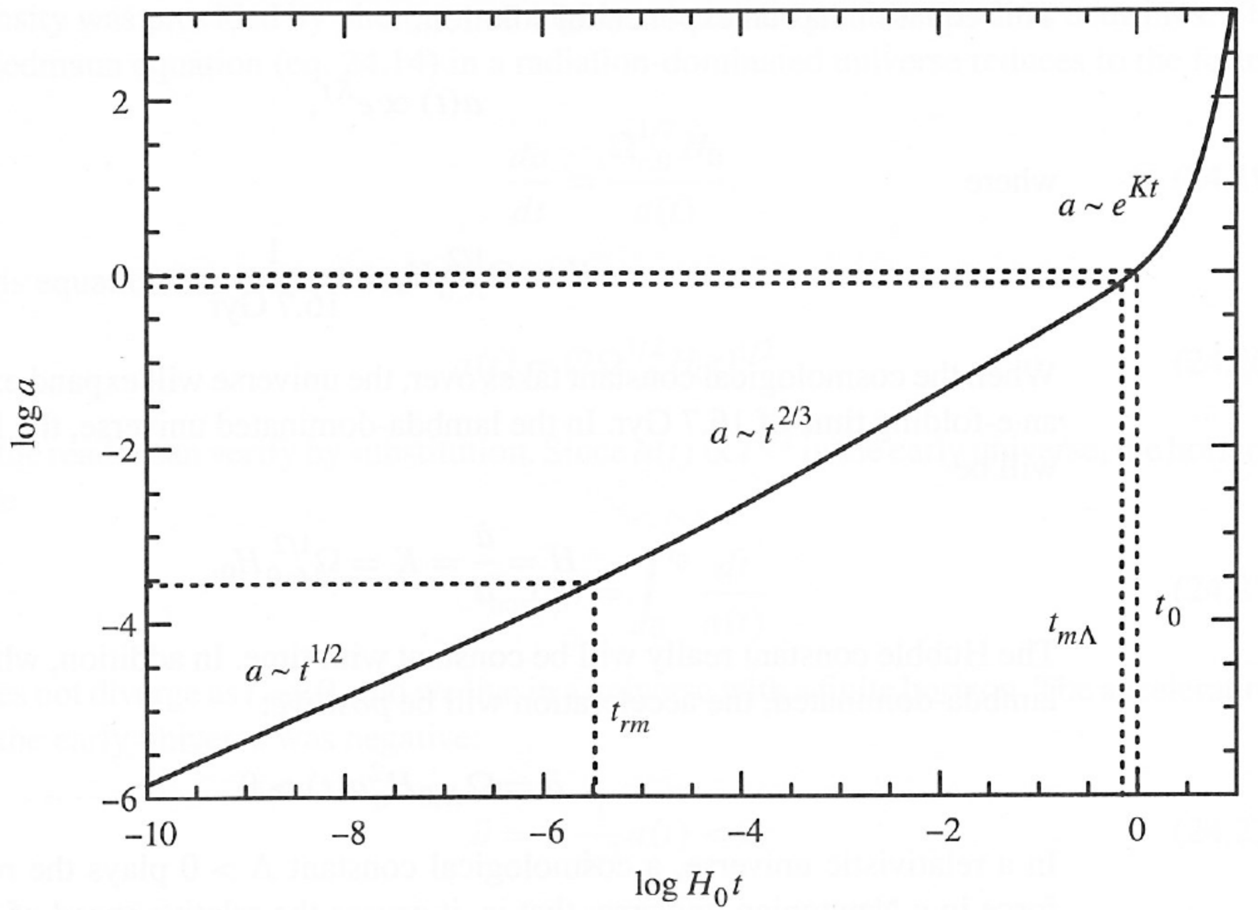
Energy density versus scale factor



$z = 1/a - 1$

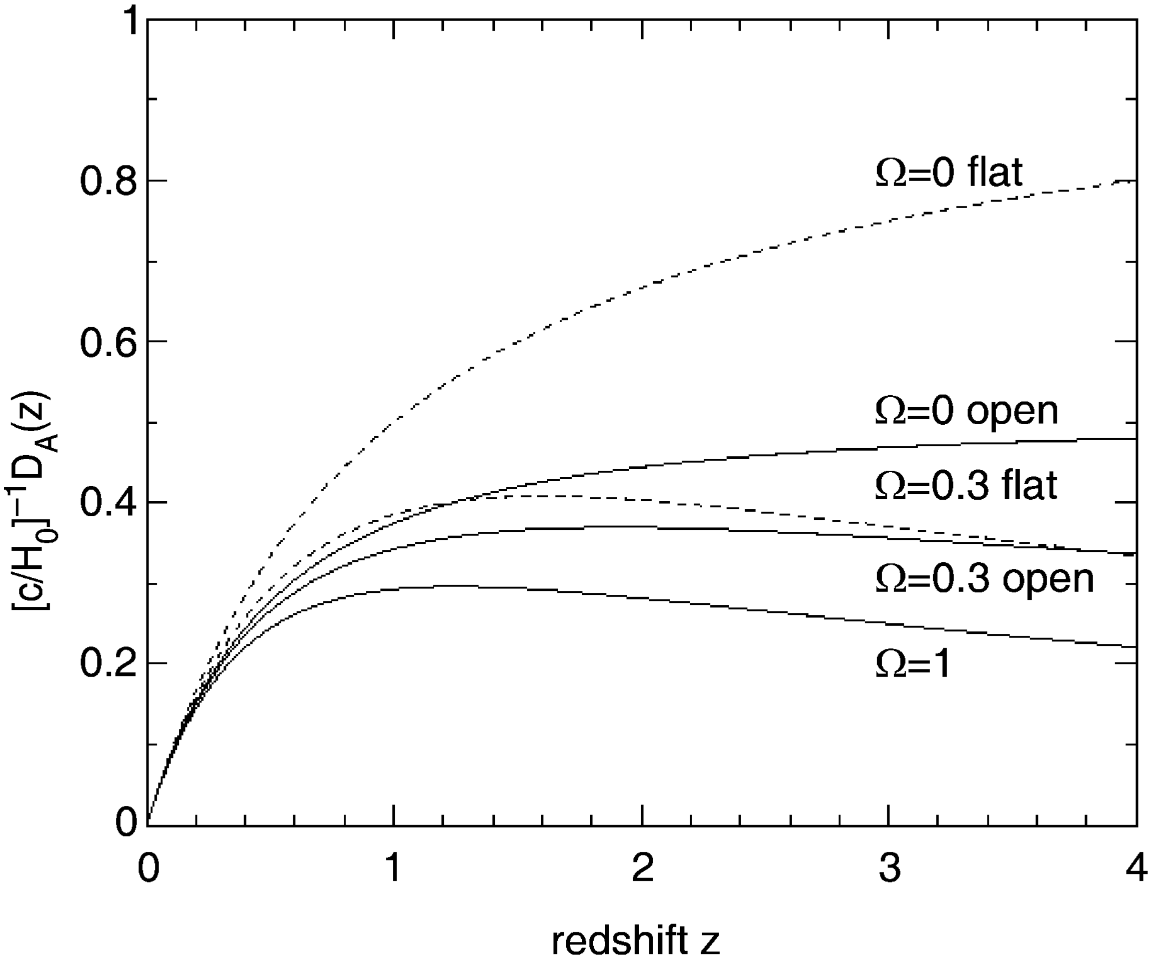
- Early times, z > 3600 or age < 47 kyr, were radiation dominated
- Matter dominated until 9.8 Gyr
- Current age 13.5 Gyr

Scale factor versus time



- Different slopes of expansion in radiation vs matter dominated epochs
- Exponential expansion in Λ dominated epoch (if cosmological constant)

Distances versus redshift



- Different distances are needed for different measurements: angular size versus luminosity

For next class

- Read WISE mission description.
- Download python to your laptop (see me if you don't have a laptop) and install numpy, matplotlib, and ipython.
- On Wednesday, 9/12, we will start looking at WISE data using a simple python program.