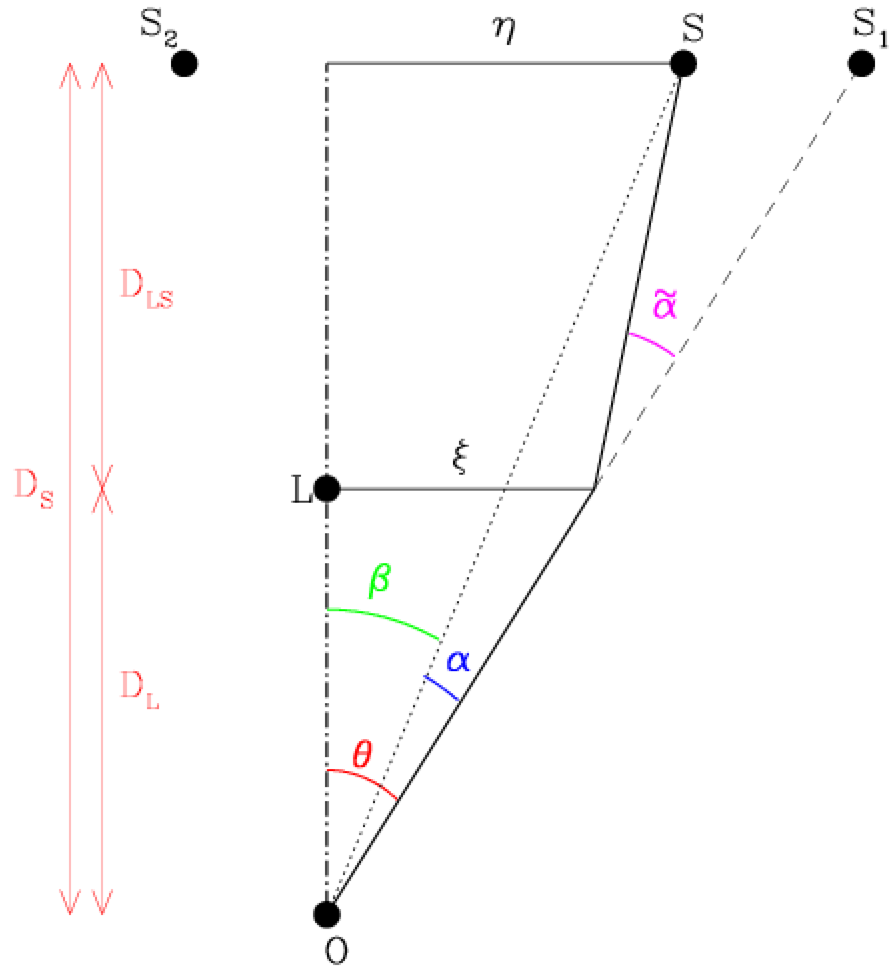
Gravitational Lensing

- Einstein deflection angle
- Lens equation, magnification
- Galaxy lenses
- Cluster lenses
- Weak lensing
- Time delays in lenses

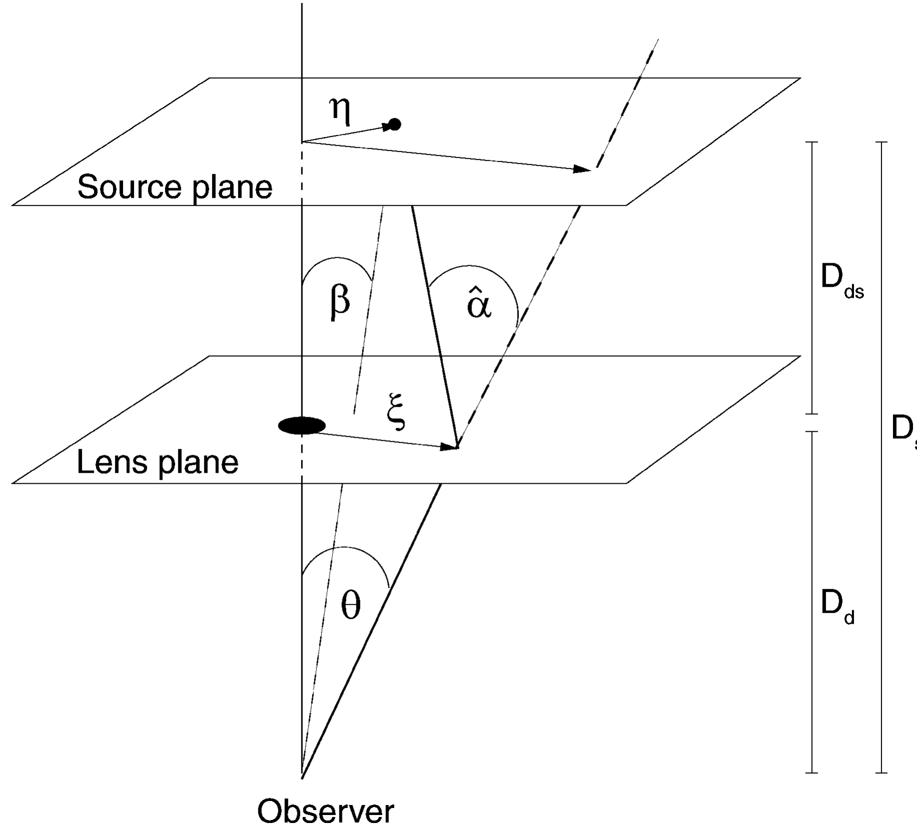
Einstein's deflection angle

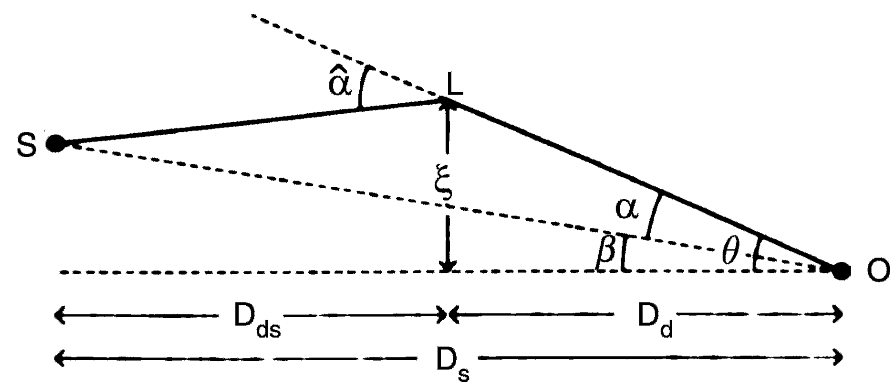
- In General Relativity, light is deflected in a gravitational field
- Deflection angle $\tilde{\alpha} = 4GM/c^2\xi$, where M = mass, ξ = distance at closest approach
- For the Sun, deflection angle = 1.74 arcseconds
- This was measured in 1919 during a solar eclipse and established General Relativity



Gravitational lens

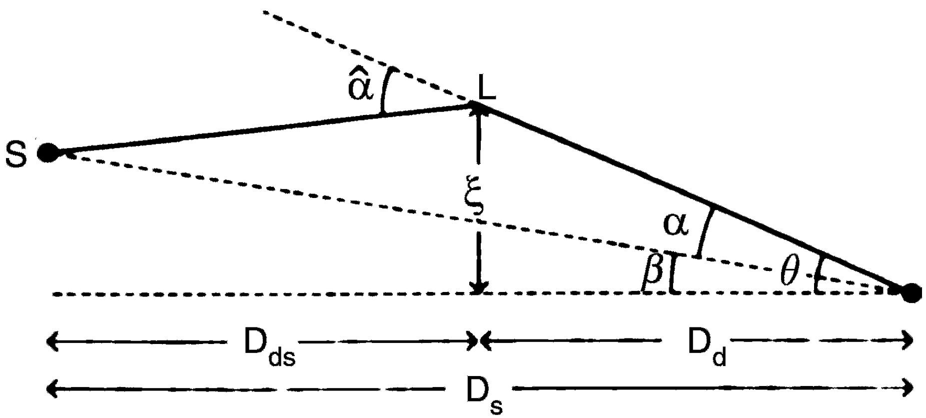
- Lens plane at distance D_d
- Source plane at distance D_s
- Distance lens to source plane D_{ds}
- True position of source = $\boldsymbol{\eta}$
- Position in lens plane = $\boldsymbol{\xi}$
- Source angular position (no lens) = $\boldsymbol{\beta} = \boldsymbol{\eta}/D_s$
- Observed source angular position = $\boldsymbol{\theta} = \boldsymbol{\xi}/D_d$
- Deflection angle = $\hat{\boldsymbol{\alpha}}$
- Lens equation $\boldsymbol{\eta} = \boldsymbol{\xi} D_s/D_d - \hat{\boldsymbol{\alpha}} D_{ds}$
- Lens equation $\boldsymbol{\beta} = \boldsymbol{\xi}/D_d - \boldsymbol{\alpha} D_{ds}/D_s = \boldsymbol{\theta} - \hat{\boldsymbol{\alpha}} D_{ds}/D_s$
- Lens equation $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}$

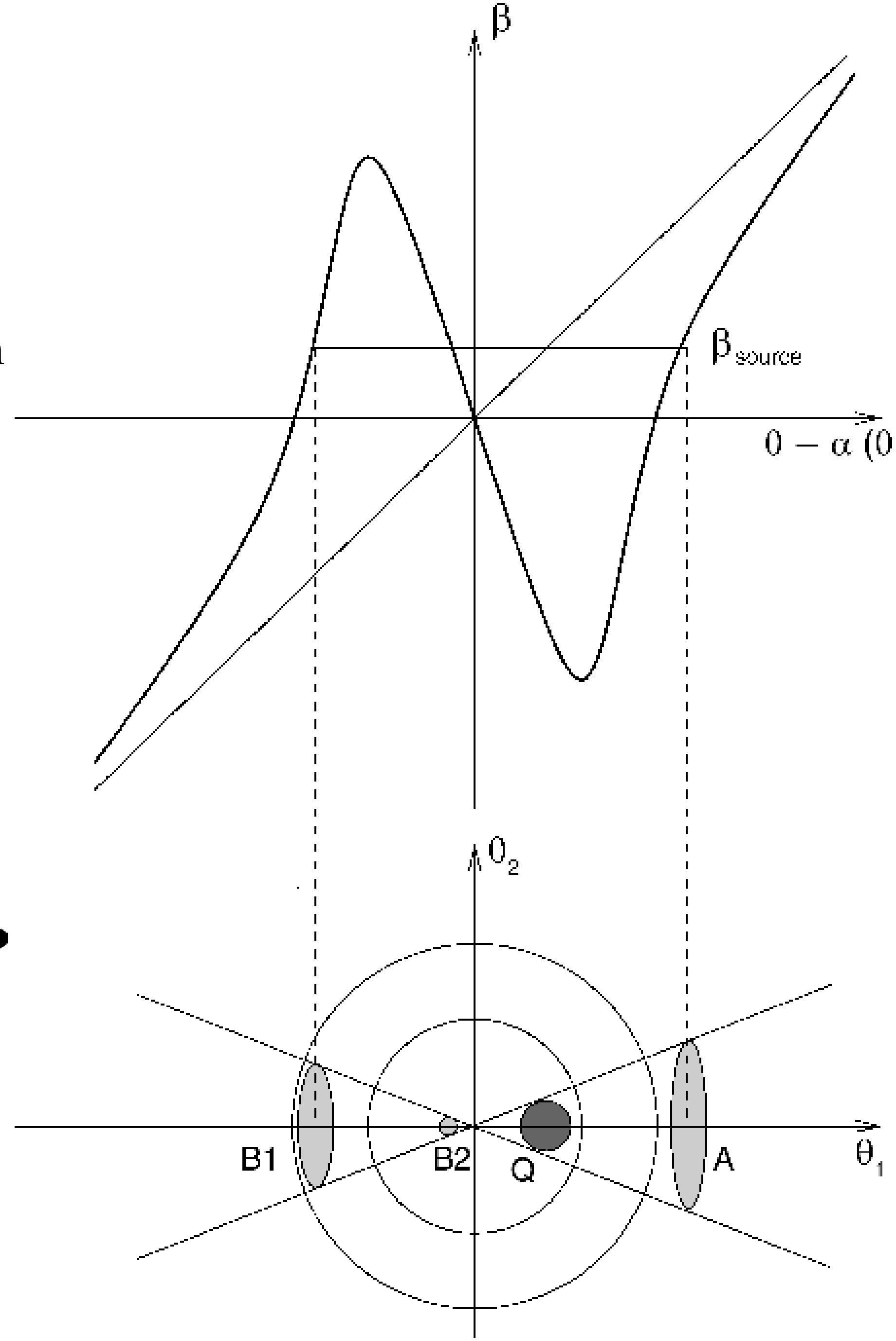




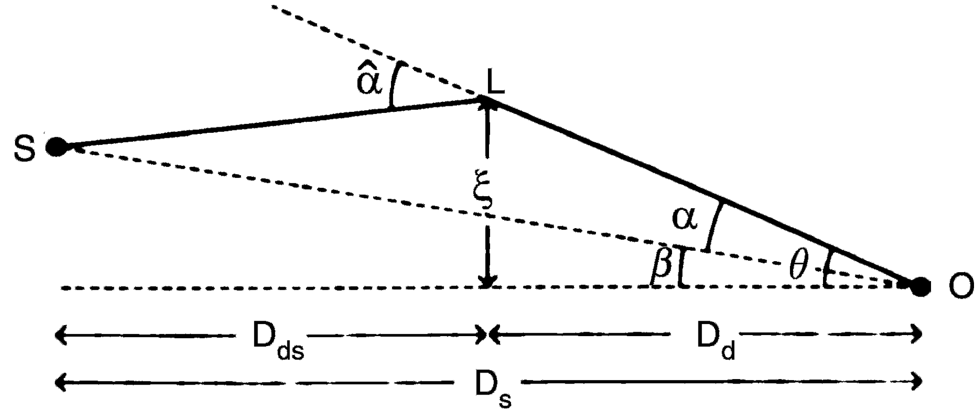
Lens equation

One sees a lensed image when the lens equation is satisfied. Lens equation $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$
If $\beta = 0$ one gets an Einstein ring.





Extended gravitational lens



Superposition of deflection angles: $\hat{\alpha} = \sum_i \hat{\alpha}_i = \sum_i \frac{4 G m_i}{c^2} \frac{\boldsymbol{\xi} - \boldsymbol{\xi}_i}{|\boldsymbol{\xi} - \boldsymbol{\xi}_i|^2}$

where $\boldsymbol{\xi}$ is position of light ray in the lens plane and $\boldsymbol{\xi}_i$ are mass elements positions.

Rewrite as: $\hat{\alpha} = \frac{4G}{c^2} \int d^2 \xi' \Sigma(\boldsymbol{\xi}') \frac{\boldsymbol{\xi} - \boldsymbol{\xi}'}{|\boldsymbol{\xi} - \boldsymbol{\xi}'|^2}$ where Σ is mass surface density.

Define scaled deflection angle: $\boldsymbol{\alpha} = \hat{\boldsymbol{\alpha}} D_{ds} / D_s$ then $\boldsymbol{\alpha} = \frac{1}{\pi} \int d^2 \theta' \kappa(\boldsymbol{\theta}') \frac{\boldsymbol{\theta} - \boldsymbol{\theta}'}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2}$

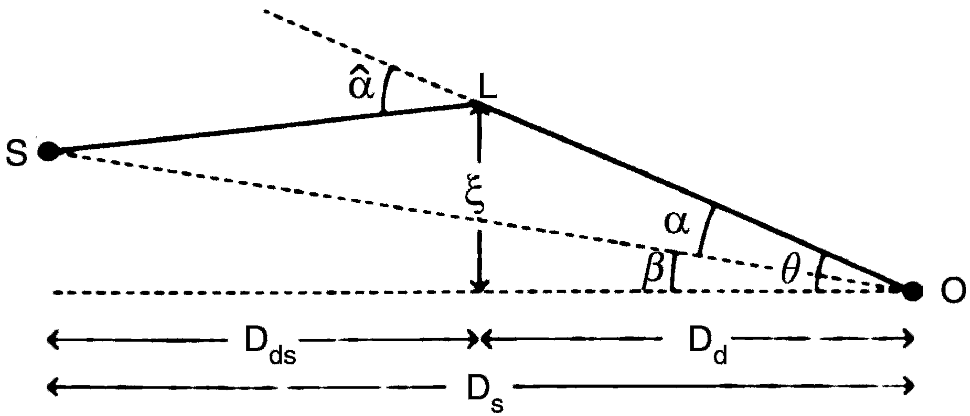
where $\kappa = \Sigma / \Sigma_{cr}$ and $\Sigma_{cr} = \frac{c^2 D_s}{4 \pi G D_d D_{ds}}$

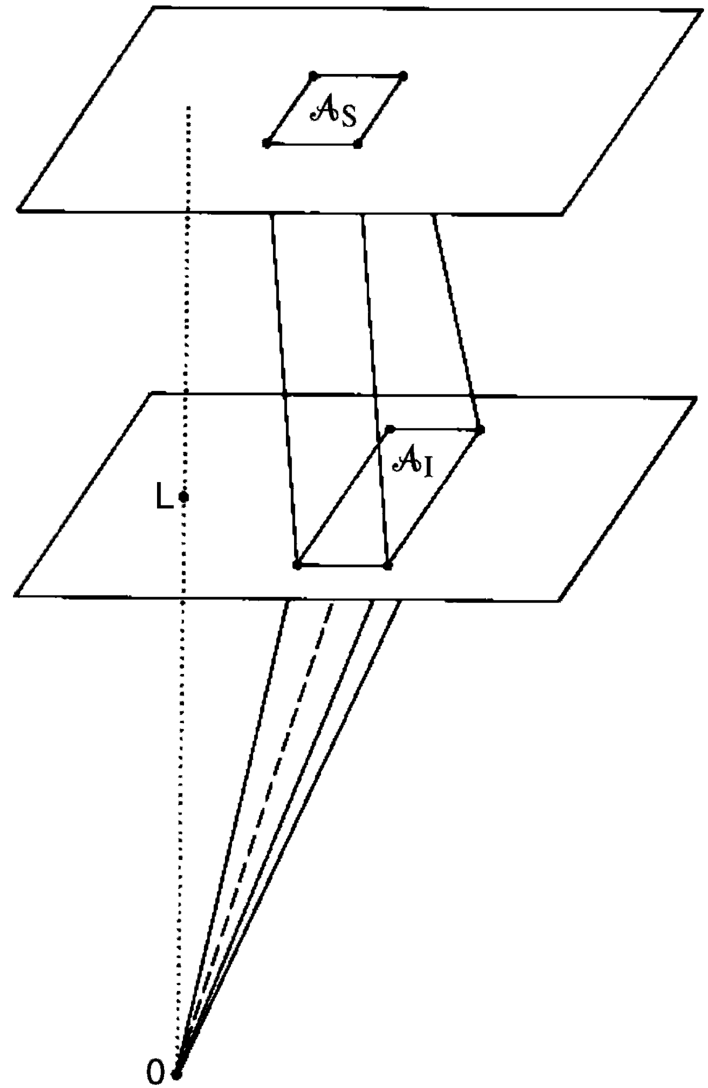
Σ_{cr} is called the critical surface mass density because if $\Sigma \geq \Sigma_{cr}$ at any point in the lens, then there are multiple images.

Magnification

Lensing can magnify source flux.

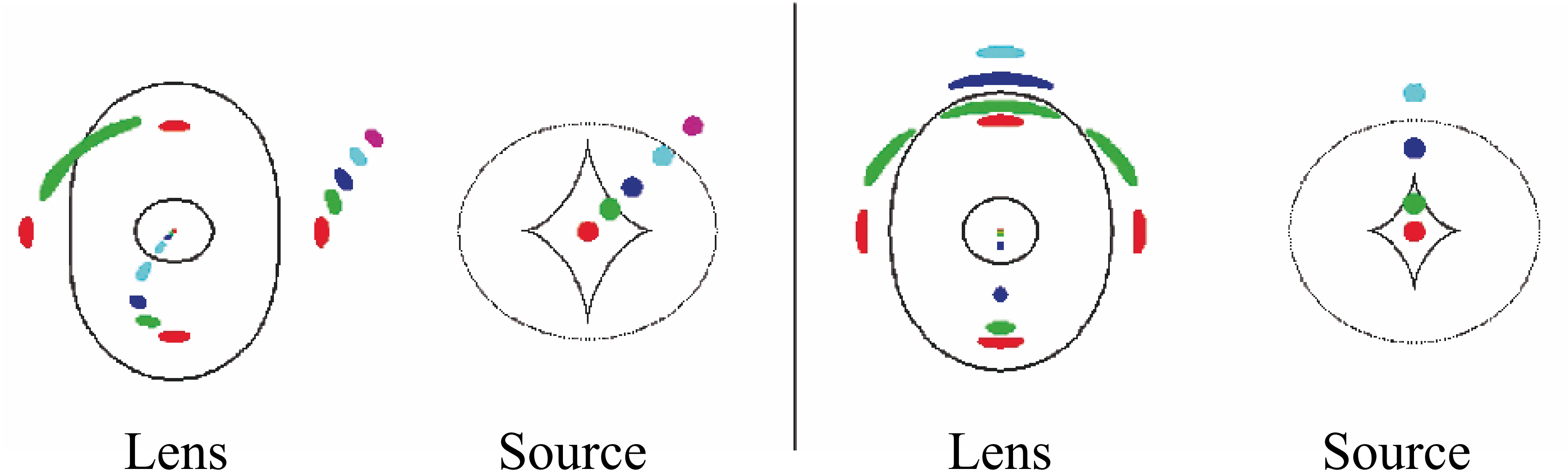
$$\text{Magnification} = \mu = \frac{S}{S_0} = \left| det\left(\frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}}\right)\right|^{-1}$$





Can have $\mu \rightarrow \infty$. Curves where $\mu \rightarrow \infty$ are called critical curves in the lens plane and caustics in the source plane.

Elliptical lens

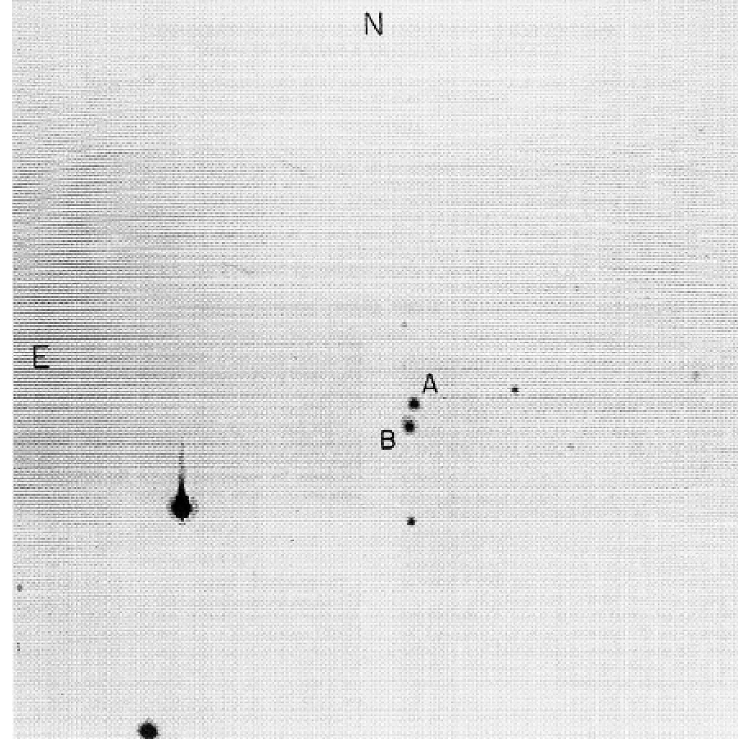


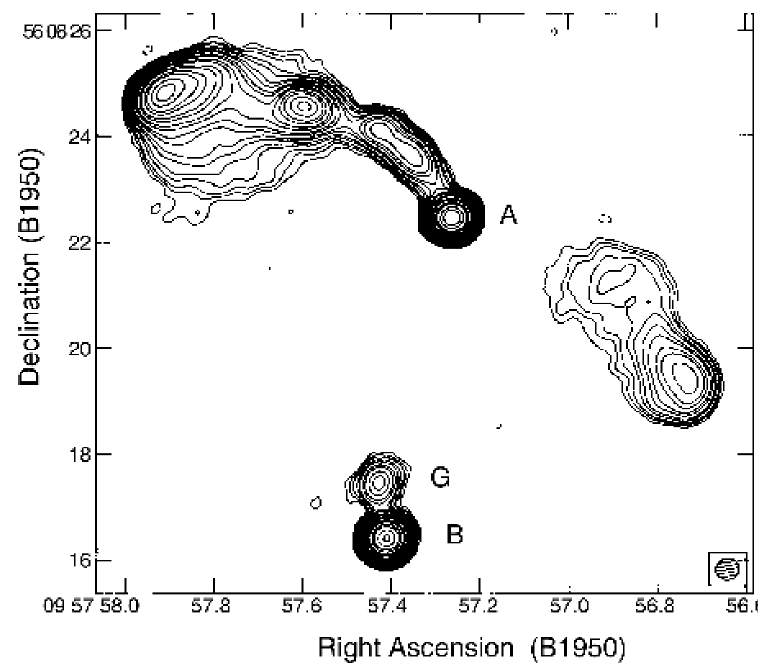
- Circles show relative angular size for lens versus source plane.
- Diamond-like curves in source plane show caustics ($\mu \rightarrow \infty$).
- Oblate curves in lens plane are critical curves.

First lens

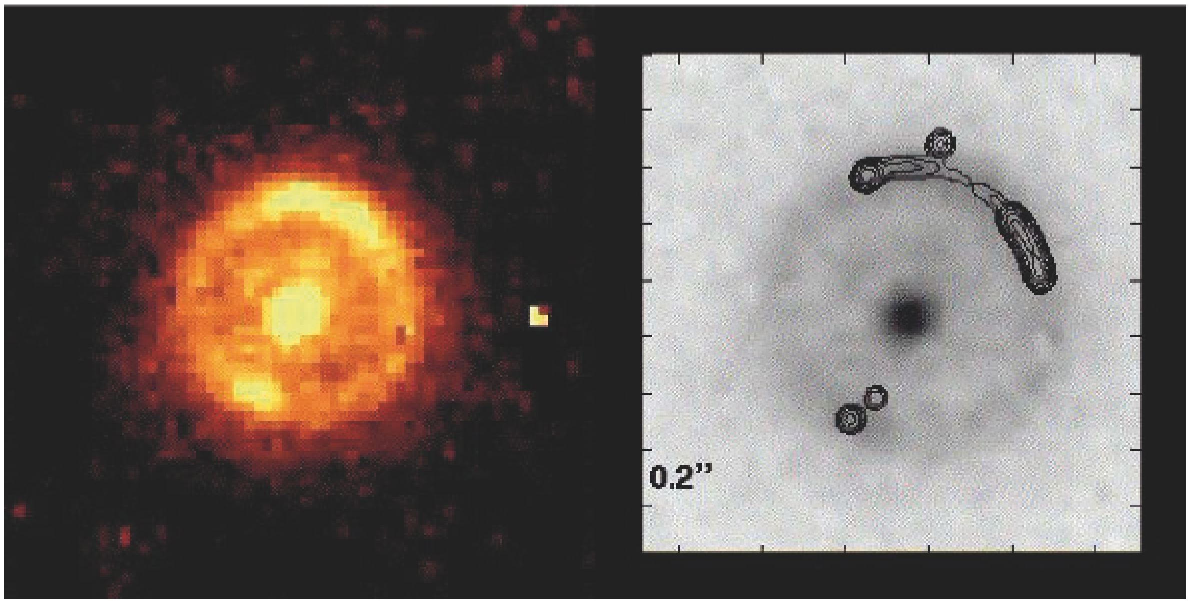
- First lens found in 1979.
- Two quasars with identical $z = 1.41$.
- Elliptical galaxy $z = 0.36$ between.

- Angle between images is 6.1 arcsec, larger than expected due to galaxy alone due to additional lensing from surrounding cluster.





Einstein ring



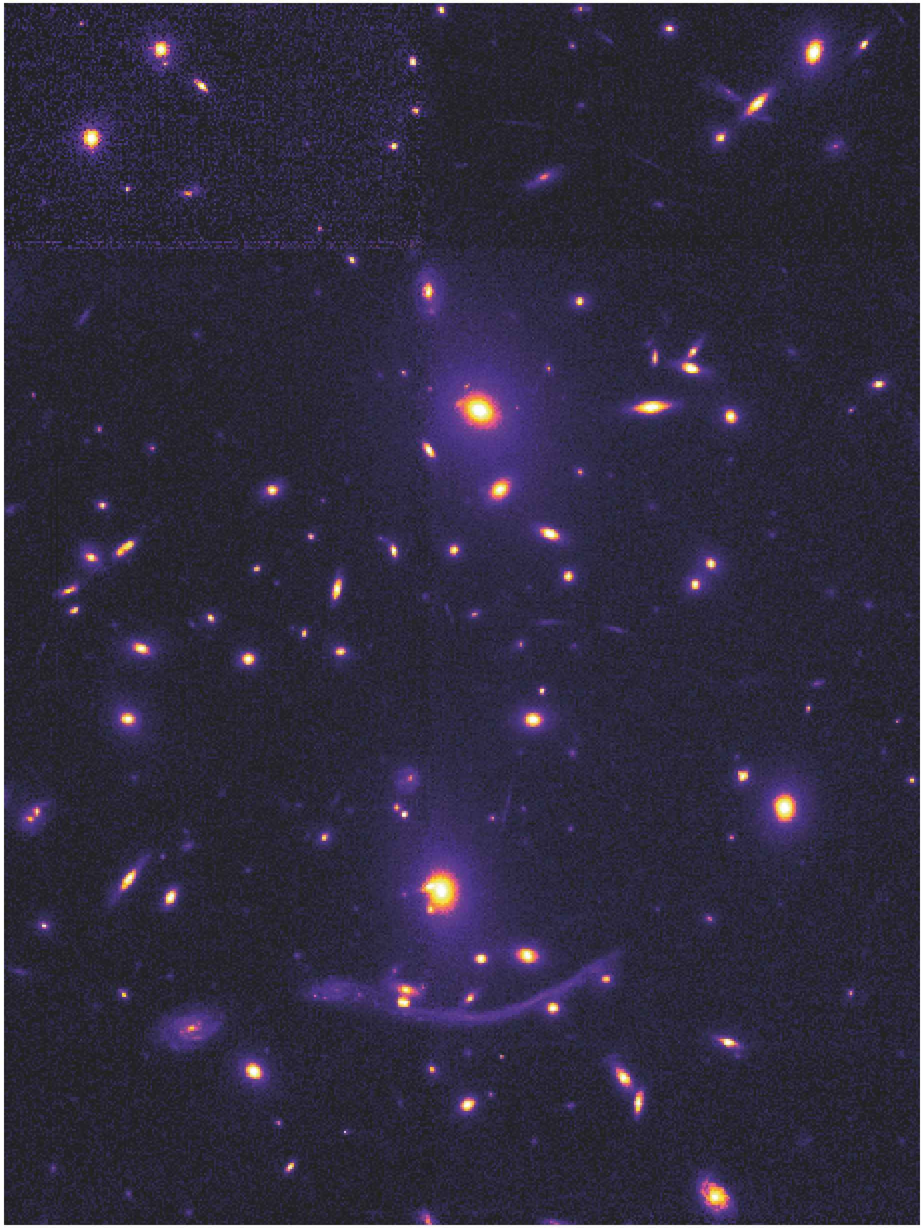
• Contours on right show multiple images and an arc in the radio.
• Color and gray-scale show infrared image with full Einstein ring and lens galaxy in the center.

• Can use properties of the lensed images to accurately measure the lens galaxy mass.

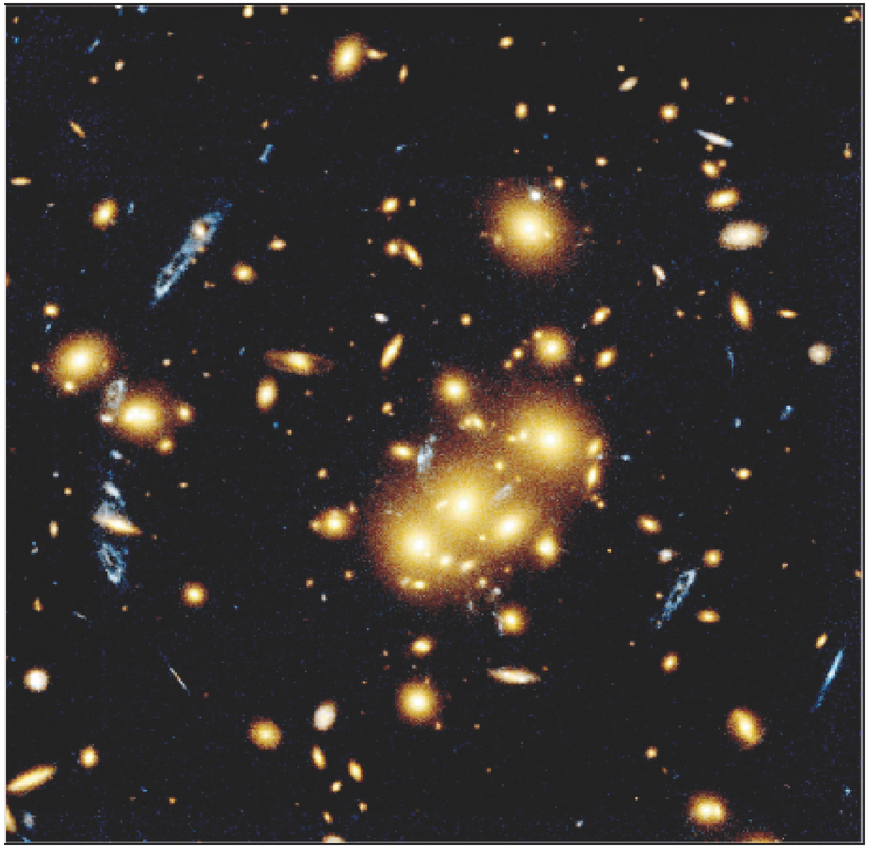
Cluster lenses

• First cluster lens found in 1986.
• "Giant luminous arcs" were first thought to be physical objects within the cluster.
• Measurement of redshifts of the arcs showed they were much more distant than the cluster.

• Gravitational lensing not expected from clusters because core densities were underestimated from ROSAT X-ray data.



Cluster lenses



• Using the singular isothermal sphere model for the cluster mass, one can derive a relation between the Einstein radius and the cluster velocity dispersion.

• Mass estimates using simplified models and clusters with few lens images are accurate to ~30% for regular clusters and worse for irregular clusters.

• Some clusters show many lens images, > 100. With large numbers of images, one can produce detailed maps of the cluster mass via iterative numerical modeling.

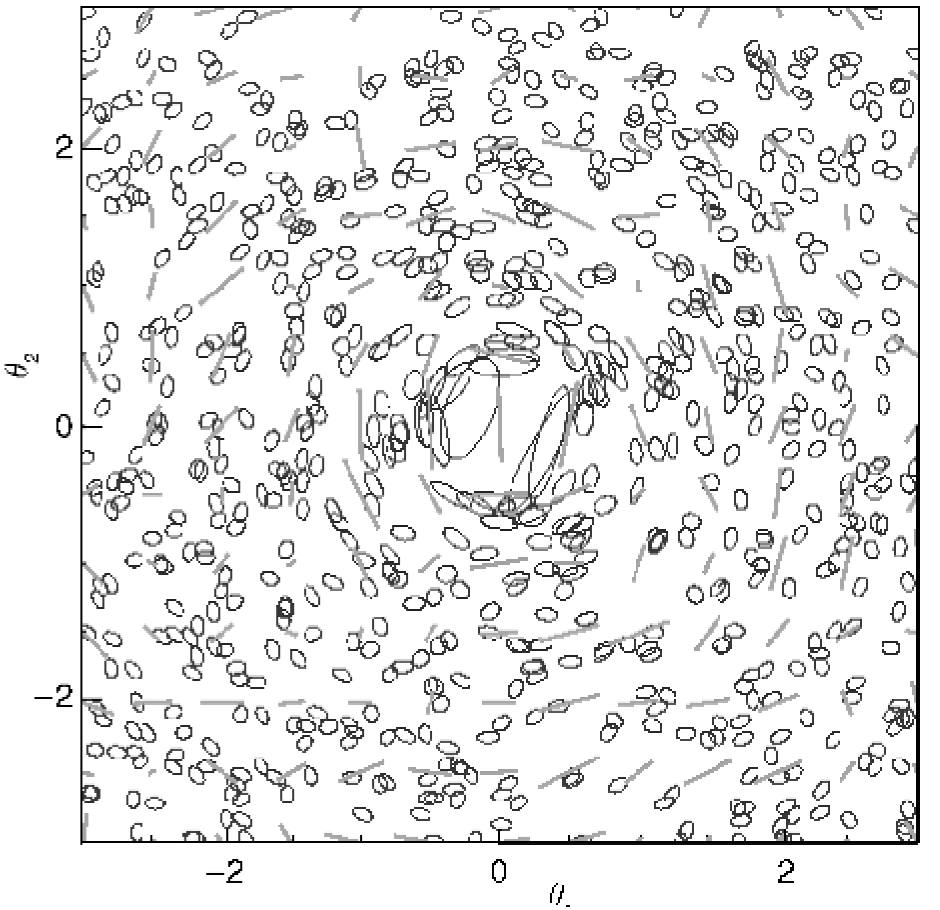
• The mass profiles for dynamically relaxed clusters with sharp central peaks in X-ray surface brightness match well between X-rays and lensing.

• For non-relaxed clusters, lensing shows smaller core radii than X-rays.

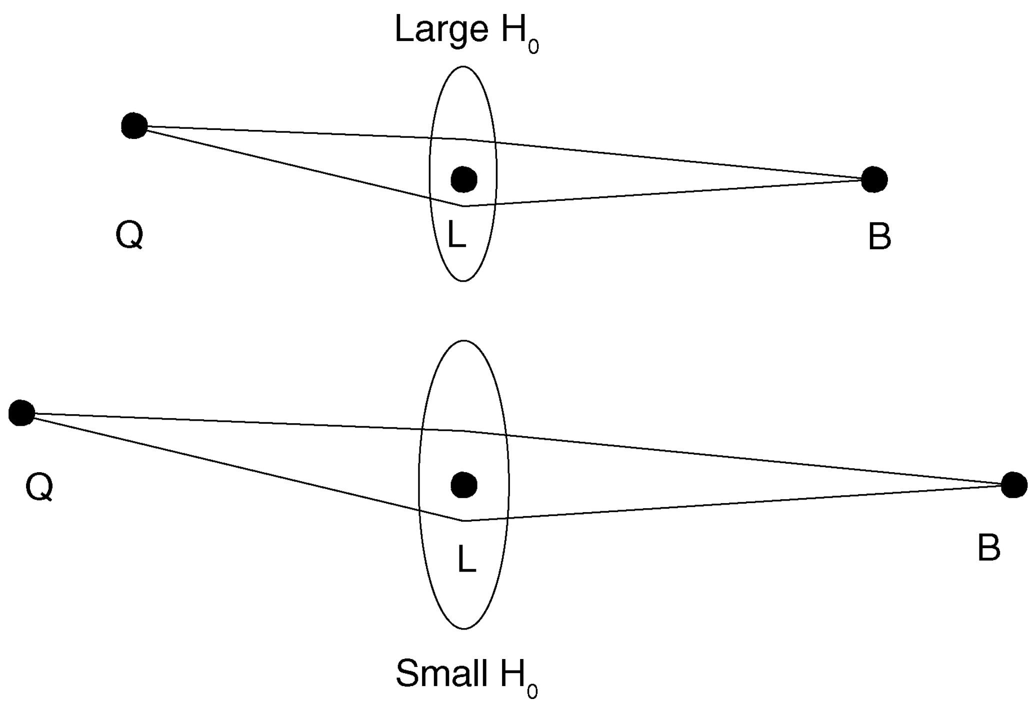
$$\theta_E = 28.8'' \left(\frac{\sigma_v}{1000 \text{ km/s}} \right)^2 \left(\frac{D_{ds}}{D_s} \right)$$

Weak lensing

• Lensing affects the observed shape of source galaxies, even if there are not strong deflections or multiple images.
• "Weak lensing" produces "shear" or slight distortions that are cylindrically symmetric around the cluster center of mass.
• Galaxies are, in general, not spherically symmetric, so measurement of shear requires samples of large numbers of galaxies.
• Weak lensing is being/will be used to measure dark matter and dark energy in surveys like BigBOSS and Euclid.



Lens time delays



- Angles in the lenses can be measured observationally.
- If source is variable, there will be time delays between different images due to differences in path length.
- If the mass distribution of the lens can be determined, then the time delays can be used to measure the path lengths, thus the distance to the source then the Hubble constant with a measurement of the source redshift.

For next class

- Read 7.1-7.4
- First draft of project papers will be due on October 29.

EARLY VOTING TIMES/LOCATIONS

Monday, 10/15: 2–8 pm at the Campus Recreation and Wellness Center and 10 am–5 pm at the Old Capitol Mall

Tuesday, 10/16: 10 am–5 pm at the Old Capitol Mall

Wednesday, 10/17: 10 am–5 pm at the Old Capitol Mall

Thursday, 10/18: 9 am–3 pm at the IMU, 2:30–8:30 pm at the Theater Building

Friday, 10/19: 11 am–5 pm at Mayflower, 12 pm–6 pm at Burge