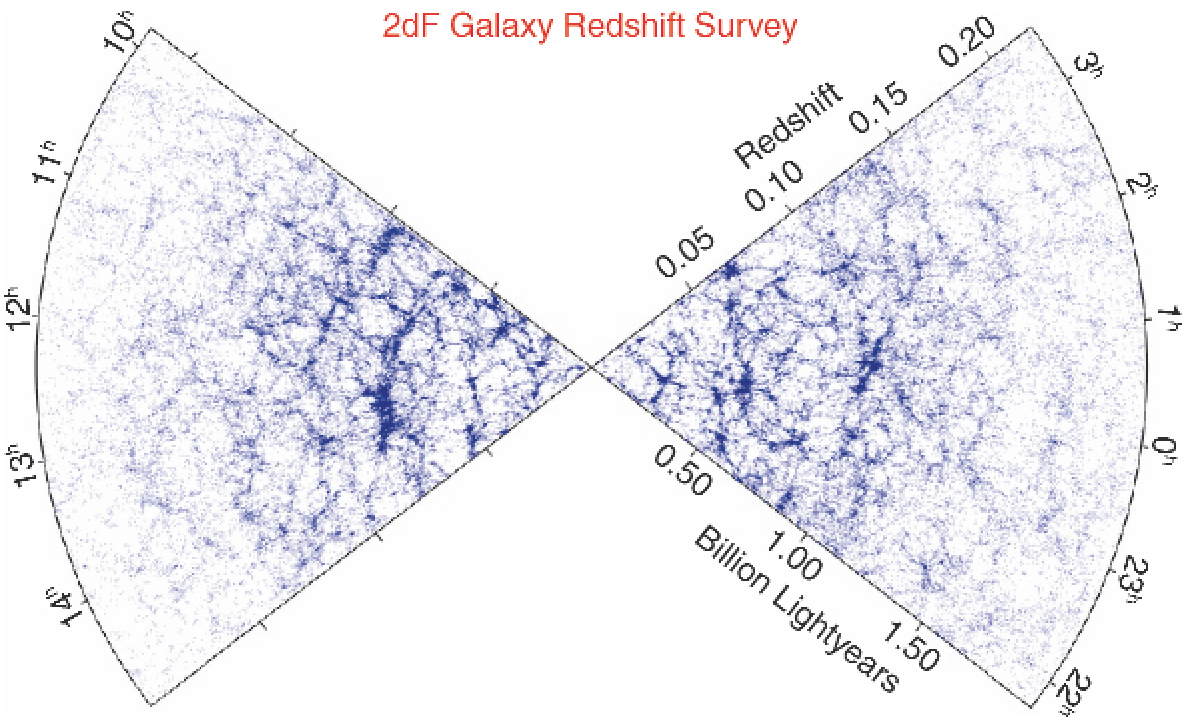
Bumps in the Night

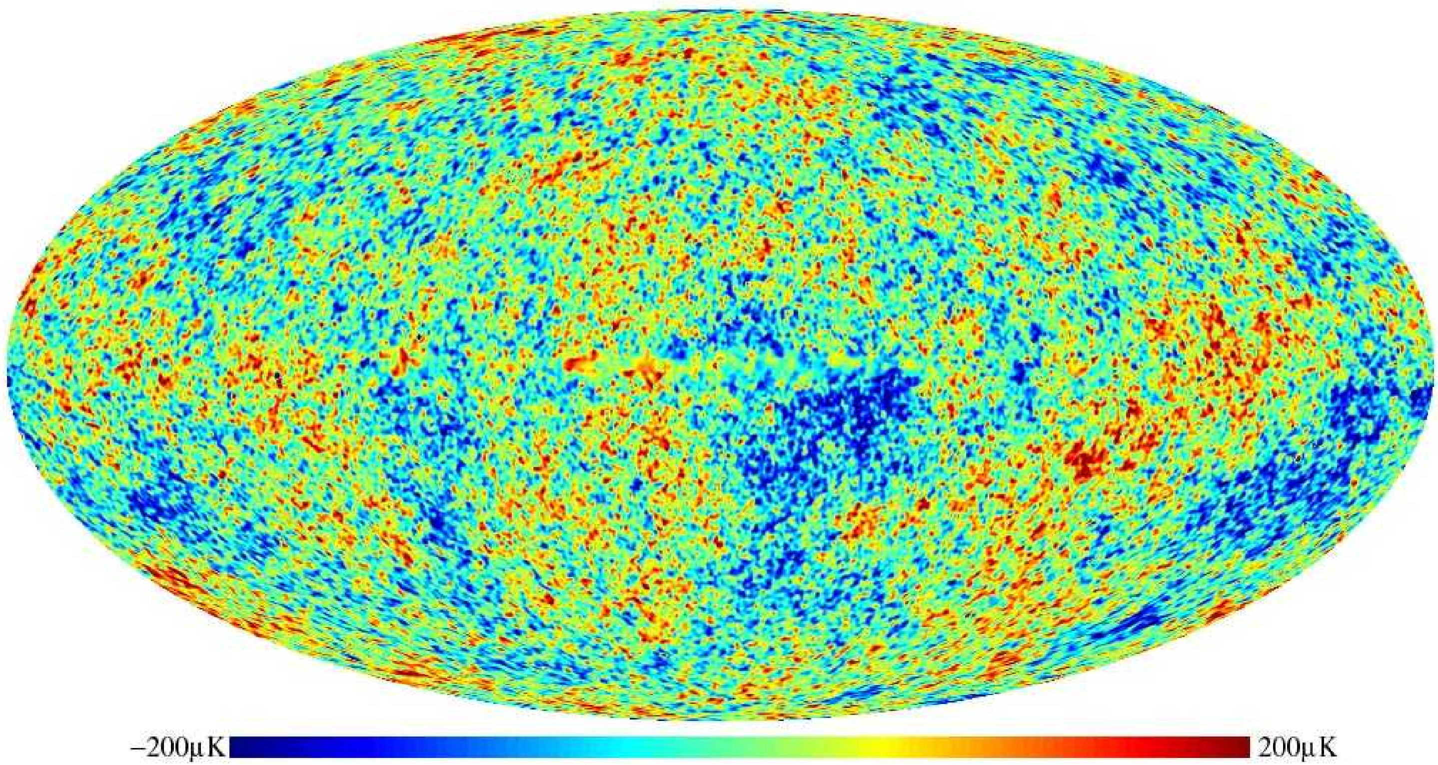
- (In)homogeneity of the universe
- Gravitational instability
- Density fluctuations
- Correlation functions and power spectra
- Evolution
- Non-linear evolution

Homogeneity at large scales



- Largest known structures have sizes ~ 70 Mpc
- This is ~3% of the Hubble radius of 2200 Mpc.
- Thus, the universe appears homogeneous at very large scales.

Inhomogeneity



- Define density contrast $\delta = (\rho-\bar{\rho})/\bar{\rho}$ where $\bar{\rho}$ = average matter density
- Fluctuations in CMB have $\Delta T/T \sim 10^{-5}$, suggest $\delta \sim 10^{-5}$ at $z \sim 1000$.
- For Earth at $z \sim 0$, $\delta \sim 10^{30}$. How?
- Gravity makes density fluctuations unstable.

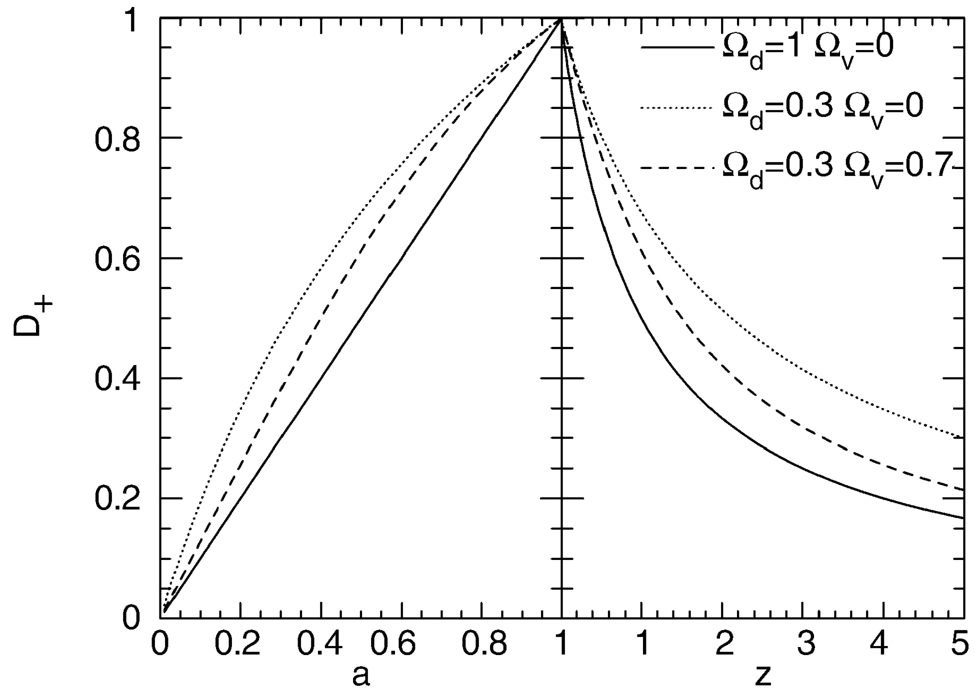
Linear perturbations

- Consider density perturbations in pressure-free matter, e.g. dust, in the linear regime.
- Use fluid approximation: density $\rho(\boldsymbol{r}, t)$ and velocity $\boldsymbol{v}(\boldsymbol{r}, t)$.
    - Have continuity equation (mass conservation): $\partial\rho/\partial t + \nabla\cdot(\rho\boldsymbol{v}) = 0$
    - And Euler equation (momentum conservation): $\partial\boldsymbol{v}/\partial t + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla\phi$
    - Gravitational potential: $\nabla^2\phi = 4\pi G\rho$
- For initial $\delta = (\rho-\bar{\rho})/\bar{\rho}$ one can calculate $\delta(t)$.

- In cosmology, one must consider the overall Hubble expansion $\boldsymbol{v}(\boldsymbol{r}, t) = H(t)\boldsymbol{r}$
- Rewrite in comoving coordinates, $\boldsymbol{r} = a(t)\boldsymbol{x}$, and peculiar velocity, $v = H(t)\boldsymbol{r} + \boldsymbol{u}$
- Equation for δ with only terms linear in δ and $\boldsymbol{u}$: $\partial^2\delta/\partial t^2 + (2H)\partial\delta/\partial t = 4\pi G\bar{\rho}\delta$
- No dependence on $\boldsymbol{x}$ due to Copernican principle, no spatial derivatives because terms beyond first order, e.g. $u\delta$ and $\boldsymbol{u}\cdot\boldsymbol{u}$, were ignored since $\delta \ll 1$.

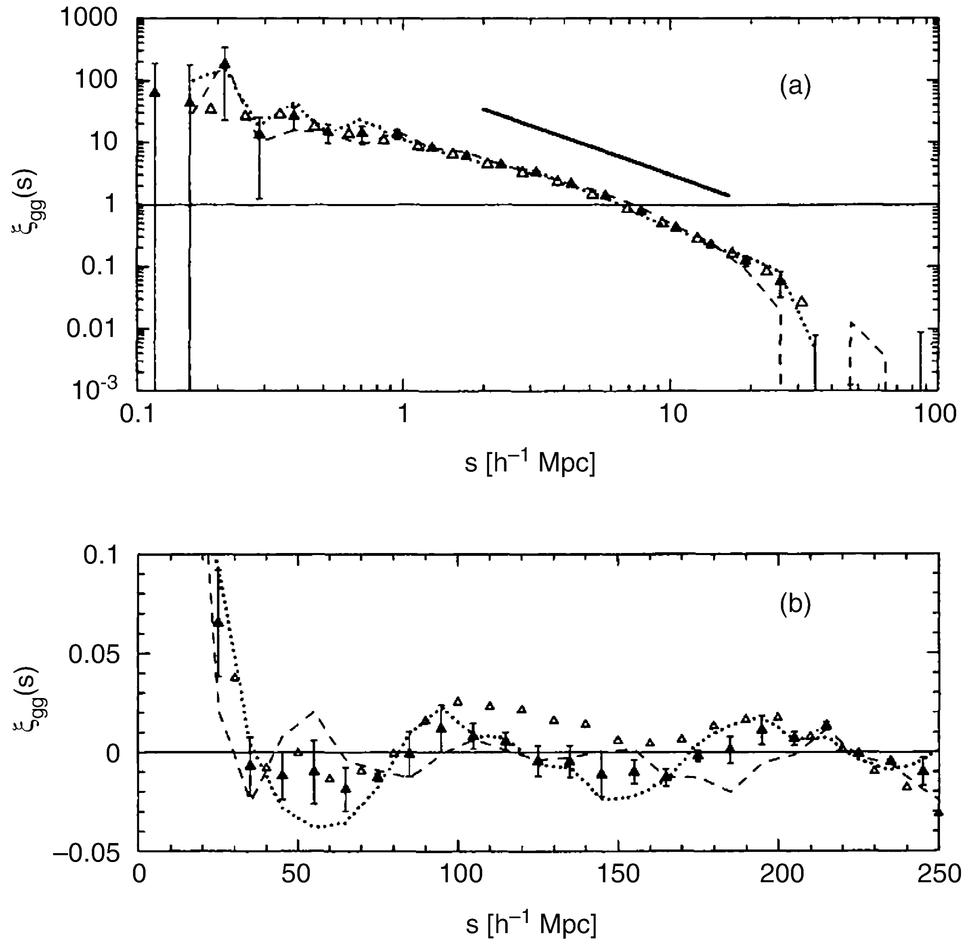
Linear perturbations

- Equation for δ with only terms linear in δ and ***u***: $\partial^2\delta/\partial t^2 + (2H)\partial\delta/\partial t = 4\pi G\bar{\rho}\delta$
- Write solutions as: $\delta(\boldsymbol{x}, t) = D(t)\,\delta_0(\boldsymbol{x})$, where $d^2D/dt^2 + (2H)\,dD/dt = 4\pi G\bar{\rho}D$
- In general, this equation has two linearly independent solutions: one increases with time, the other decreases. We are interested only in perturbations that increase with time. For this case, $D(t)$ is the "growth factor".
- For Einstein-de Sitter universe ($\Omega_m = 1$, $\Omega_\Lambda = 0$), solution is: $D(t) = (t/t_0)2/3 = a(t)$, growth factor equals scale factor. Expansion slows growth from exponential to linear.
  - Solution can be found for any cosmology and are relatively close to $D = a$, for realistic cosmologies.
  - Since $\delta > 1$ now, expect $\delta > 10^{-3}$ at $z \sim 1000$ when $a \sim 10^{-3}$
  - From CMB, $\delta \sim 10^{-5}$ for visible matter when $a \sim 10^{-3}$
  - Solution is dark matter – δ_{dark} not constrained by CMB, dark matter not smoothed by radiation.



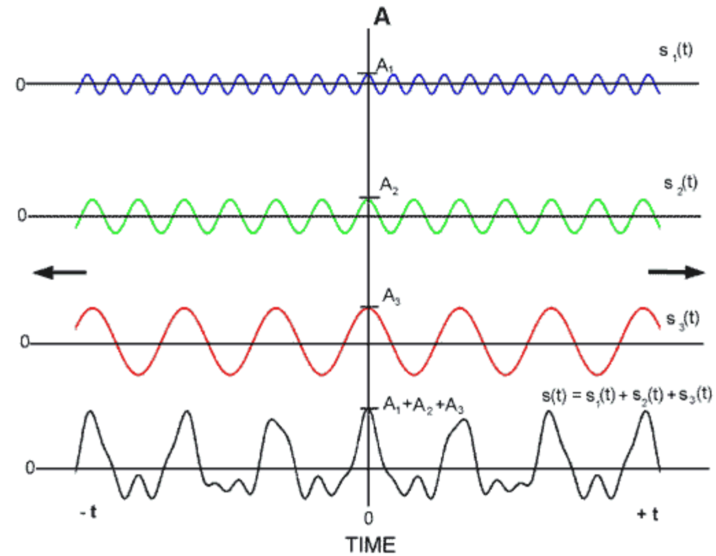
Correlation functions

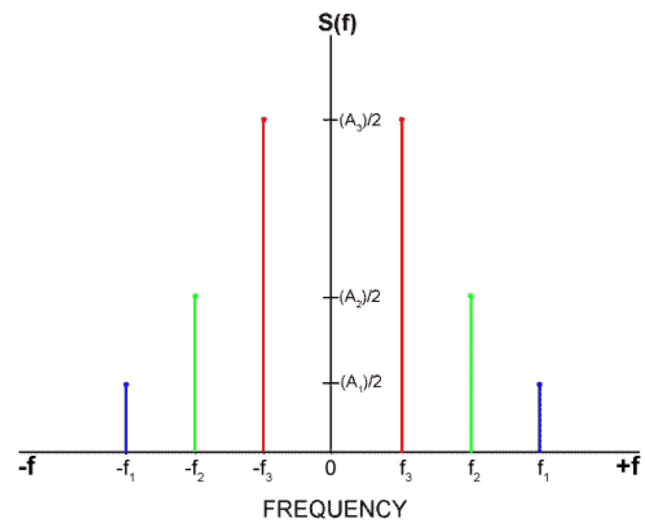
- How do we characterize δ?
- Use correlation function:
  $<\rho(\boldsymbol{x})\rho(\boldsymbol{y})> = \bar{\rho}^2<[1+\delta(\boldsymbol{x})][1+\delta(\boldsymbol{y})]>$
- Since $<\delta(\boldsymbol{x})> = 0$, then
  $<\rho(\boldsymbol{x})\rho(\boldsymbol{y})> = \bar{\rho}^2[1+<\delta(\boldsymbol{x})\delta(\boldsymbol{y})>])$
  $= \bar{\rho}^2([1+\xi(\boldsymbol{x},\boldsymbol{y})]$
- Correlation function $\xi(\boldsymbol{x},\boldsymbol{y}) = \xi(r)$ due to isotropy of universe.
- On scales of 2 to 20 Mpc, the correlation function is a power-law $\xi(\boldsymbol{r}) = (r/r_0)^{-\gamma}$, with $r_0 \sim 3$ Mpc and $\gamma \sim 1.8$.
- On scales larger than ~ 100 Mpc, ξ ~ 0.



Fourier transform

- Fourier transform converts a function of time to a function of (angular) frequency, $F(\omega) = \int f(t)e^{-i\omega t}\,dt$
- The transform, $F(\omega)$, reveals the frequencies at which there is significant structure in $f(t)$.
- The transform is additive: $\mathcal{F}(f+g) = \mathcal{F}(f) + \mathcal{F}(g)$
- The full Fourier transform of a function (in general complex) is a complete description of the original function.
- Fourier transforms can also be used to convert a function of position, $f(\mathbf{x})$, to a function of wavenumber, $F(\boldsymbol{k})$.
- The transform, $F(\boldsymbol{k})$, reveals the length scales, $L \sim 2\pi/k$, where there is significant structure in $f(\mathbf{x})$.
- Power is usually $F^{*}\cdot F$





Power spectrum

- Power spectrum $P(\boldsymbol{k})$ is the Fourier transform of the correlation function.
- $P(k) = 2\pi \int dr\, r^2\ \xi(r) \sin(kr)/kr$ where $\boldsymbol{k}$ is the wave number.
- Significant power $P(\boldsymbol{k})$ means there is structure at length scales $L \sim 2\pi/k$.

- Evolution of the power spectrum in the linear regime:

$$P(k,t) = D^2(t)P(k,t_0) = D^2(t)P_0(k)$$

- This is valid only for the evolution during the matter dominated phase of the universe (and only for small density fluctuations), so $P_0(k)$ is the power spectrum at the beginning of the matter dominated phase.

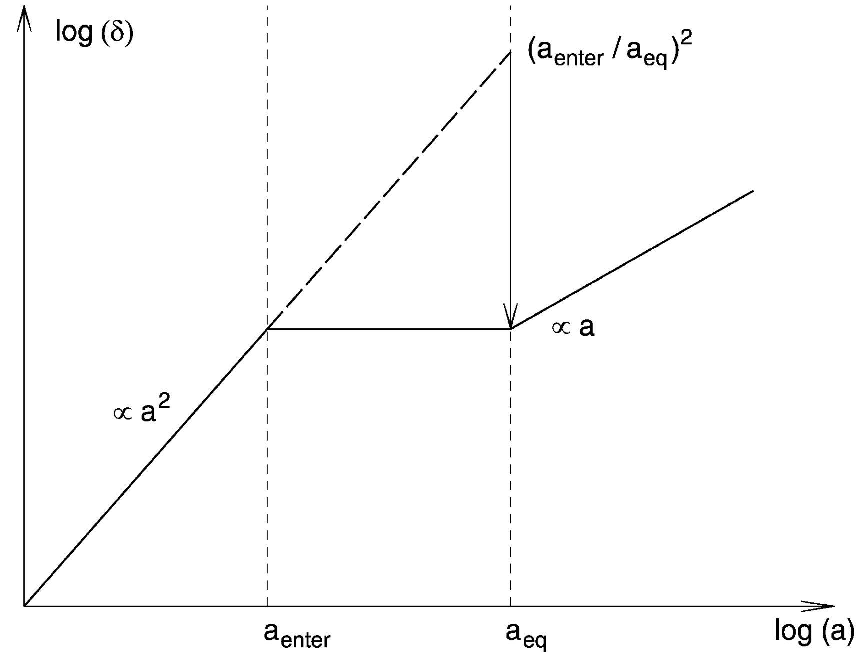
Initial power spectrum

- The initial spectrum of perturbations is thought to be a powerlaw, $P(k) \sim k^n$, because powerlaws are scale invariant, $f(ax) = a^b f(x)$.
- The initial power spectrum is modified during the radiation-dominated phase of the universe. These modifications are described by the "transfer function", $T(k)$.
- The power spectrum at the beginning of the matter-dominated phase is then

$$P_0(k) = \mathrm{A}\, k^n\, T^2(k)$$

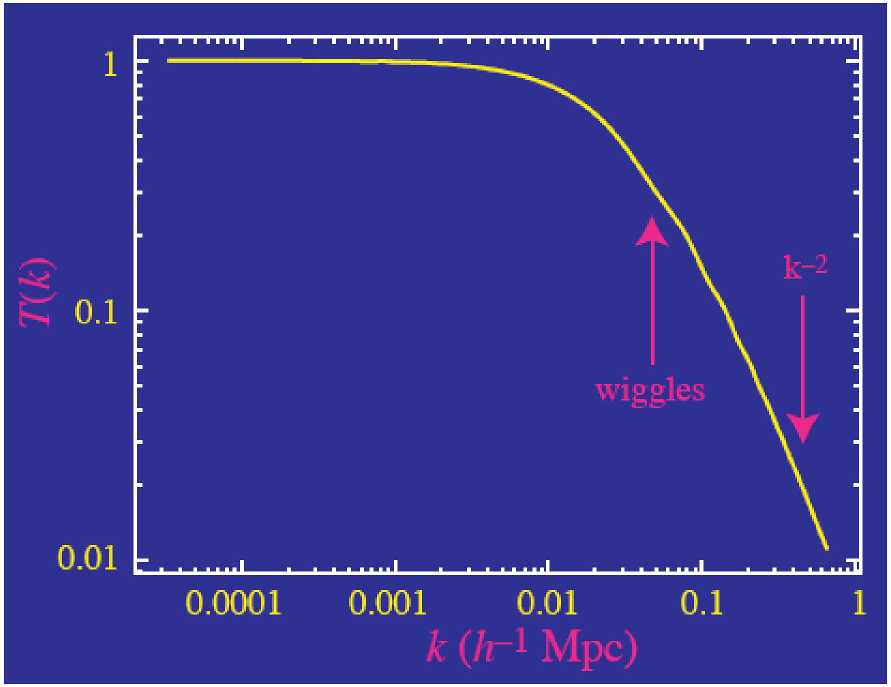
- The transfer function depends on the cosmological parameters and also on the nature of the dark matter. There are two major classes of dark matter depending on whether or not the particles were relativistic at t_{eq} when radiation and matter had equal densities.
- For hot dark matter $mc^2 << kT(t_{eq})$, for cold dark matter $mc^2 >> kT(t_{eq})$ (note $T \neq T$). The dividing mass works out to be ~ 1 eV/c^2.
- For HDM, small scale structure is suppressed because fast moving particles are not gravitationally confined in small potential wells "free streaming", $T(k) \sim e^{-k}$ for large k.
- In HDM, structure forms top-down, large structures form first and then fragment into smaller structures. This is inconsistent with observations of galaxies at $z > 6$.
- Thus, CMD is preferred.

Growth of fluctuations



- Perturbations can grow on length scales larger than the horizon.
- When radiation dominates, growth ~ a^2, when matter dominates, growth ~ a^1.
- Perturbations with length scale L "enter the horizon" when horizon > L.
- In radiation-dominated era, free-streaming and pressure of radiation impedes growth of perturbations with L < horizon. Therefore, the length scale L_0 = horizon at t_{eq} is important.
- Growth of perturbations with $L < L_0$ is suppressed from when perturbation enters the horizon until end of radiation-dominated era.
- Thus, $T(k) \sim 1$ for $k << 1/L_0$ and $T(k) \sim (kL_0)^{-2}$ for $k >> 1/L_0$ – perturbations with small length scales = large wavenumbers are suppressed.

Growth of fluctuations



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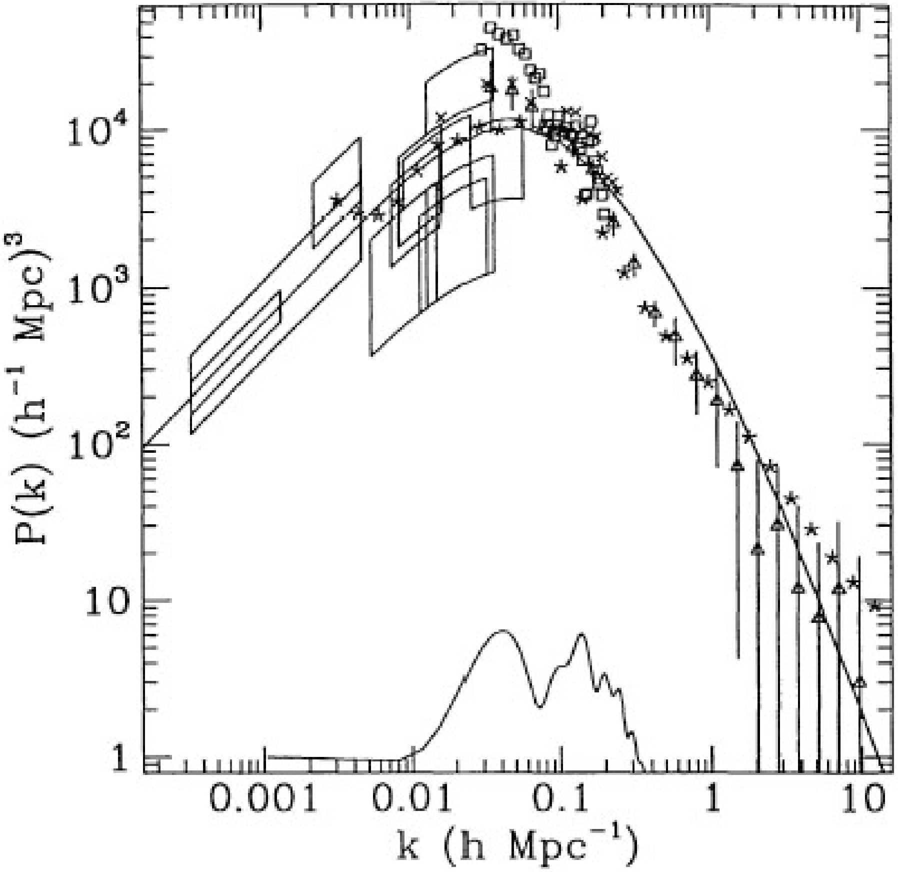
Power spectrum



- Power spectrum from White, Scott, and Silk (1994). Curve is CDM model, boxes are from CMB experiments, points from galaxy surveys, lower curve is radiation power spectrum.
- Distribution of baryons will differ from distribution of dark matter because baryons feel radiation pressure causing a smoother distribution. After recombination, baryons fall into the potential wells of dark matter and the distributions become similar.

For next class

- Read 7.5-7.7
- Homework #7 is due on October 22 (next class).
- Project work day scheduled for October 24.
- First draft of project papers will be due on October 29.

EARLY VOTING TIMES/LOCATIONS

Wednesday, 10/17: 10 am–5 pm at the Old Capitol Mall

Thursday, 10/18: 9 am–3 pm at the IMU, 2:30–8:30 pm at the Theater Building

Friday, 10/19: 11 am–5 pm at Mayflower, 12 pm–6 pm at Burge