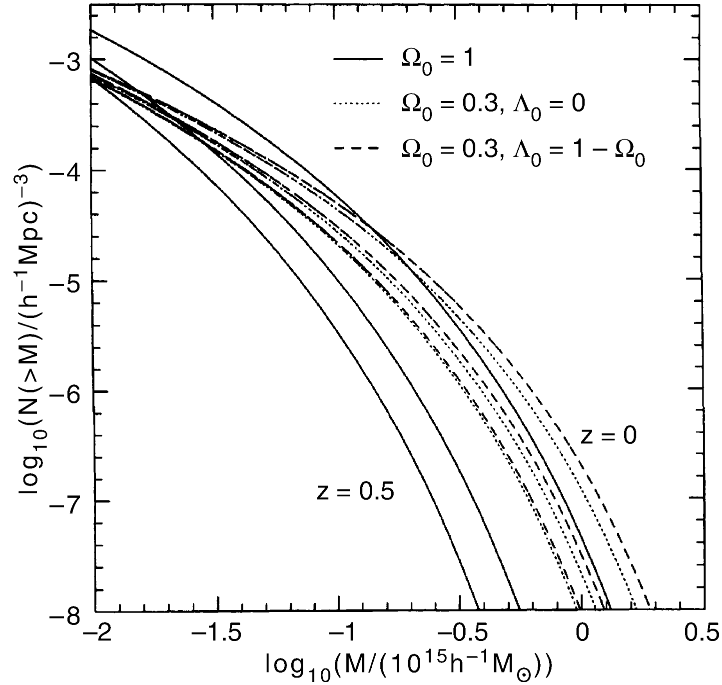
Bigger bumps in the Night

- Non-linear structure evolution
- Spherical collapse
- Dark matter halos
- Substructure problem
- Origin of density fluctuations

Spherical collapse

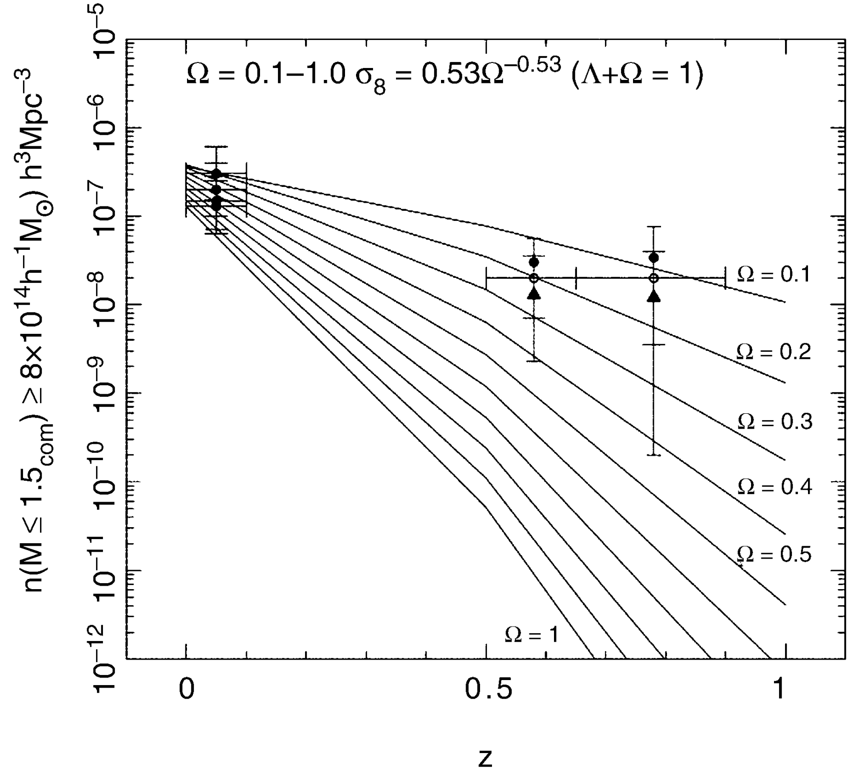
- In general, we can solve the evolution of density perturbations analytically only for small perturbation, $\delta << 1$. However, there are a handful of specific geometries that can be solved analytically for arbitrary δ. The simplest is a uniform sphere.
- Assume the initial density fluctuation is small, $\delta_i << 1$, then $M \approx (4\pi/3)R_{com}^{3}\, \rho_0$ where R_{com} is the comoving radius of the perturbation and ρ_0 is the current density.
- The equation describing the evolution of the sphere are the same as the Friedmann equations describing evolution of the universe, but with a higher density. The increased density slows the expansion, if the density if high enough, the expansion is eventually halted and perturbation collapses in on itself. Such a perturbation is called a "dark matter halo".
- At some point during the collapse, small scale fluctuation disturb the spherical symmetry and the system undergoes violent relaxation and becomes virialized. Further collapse is stopped because the kinetic motion can support the system against gravity.
- The density after virialization is $\langle\rho\rangle = (1+\delta_{vir})\, \bar{\rho}(t_{coll})$ where $(1+\delta_{vir}) \approx 178\, \Omega_m^{-0.6}$, equivalent to $\bar{\rho} \approx 200\, \rho_{cr}$ at epoch of collapse. Values in the literature range from 100 to 400, see http://arxiv.org/abs/1005.0411

Press-Schechter model



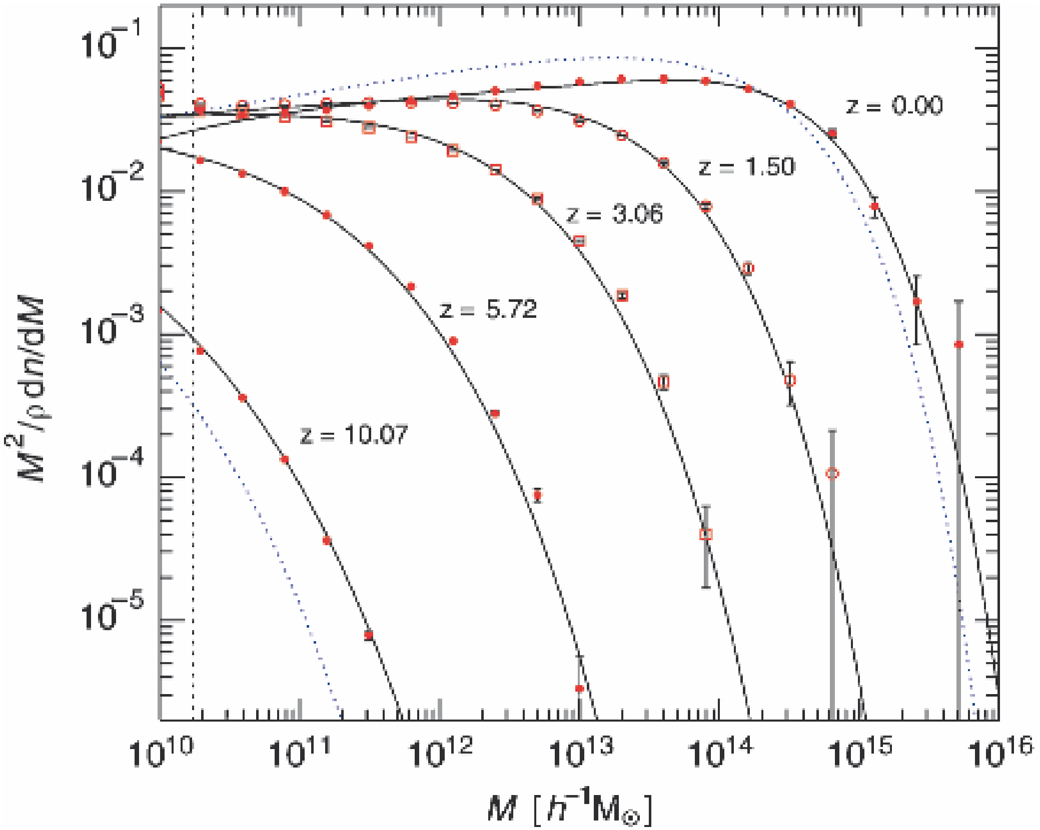
- Start with density fluctuations, $\delta_0(\boldsymbol{x})$, generated from a power spectrum $P_0(k)$.
- Smooth δ on a comoving length scale R. Find the peaks, then find the average density around each peak within R. If the average density is large enough, $\delta_R > \delta_{min}$, then that peak will form a dark matter halo. Note $\delta_{min} = \delta_{min}(z)$, denser halos will collapse (virialize) faster.
- The value of δ_{min} depends on how density fluctuations evolve (the growth factor as a function of time) and, thus, on the cosmological model. One can calculate this reasonably well using the uniform sphere model.
- The mass of the halo is $M \approx (4\pi/3)R^3\,\rho_0$. Thus, given an initial spectrum of perturbations, one can calculate the number density of dark matter halos versus mass (the "mass spectrum") for any cosmology.

Press-Schechter model



- The total mass of a cluster is a good approximation to the mass of the dark matter halo in which it is embedded. Thus, we can use the masses of clusters measured at various redshifts to constrain the cosmological parameters, particularly Ω_m.
- Figure above shows the number density of high mass clusters, $>10^{15}$ solar masses, versus redshift. Cosmologies with $\Omega_m > 0.5$ are ruled out.
- Mass spectrum ~ $(M/M^*(z))^{\gamma/2} \exp[-(M/M^*(z))^{\gamma}]$ where $\gamma \sim 0.5$ and $M^*(z) = M_0^*(1+z)^{-2/\gamma}$ is the characteristic mass scale that grows with time. Thus, structure formation is "hierarchical" or "bottom-up", small structures form first and merge to later form large structures.

Press-Schechter model



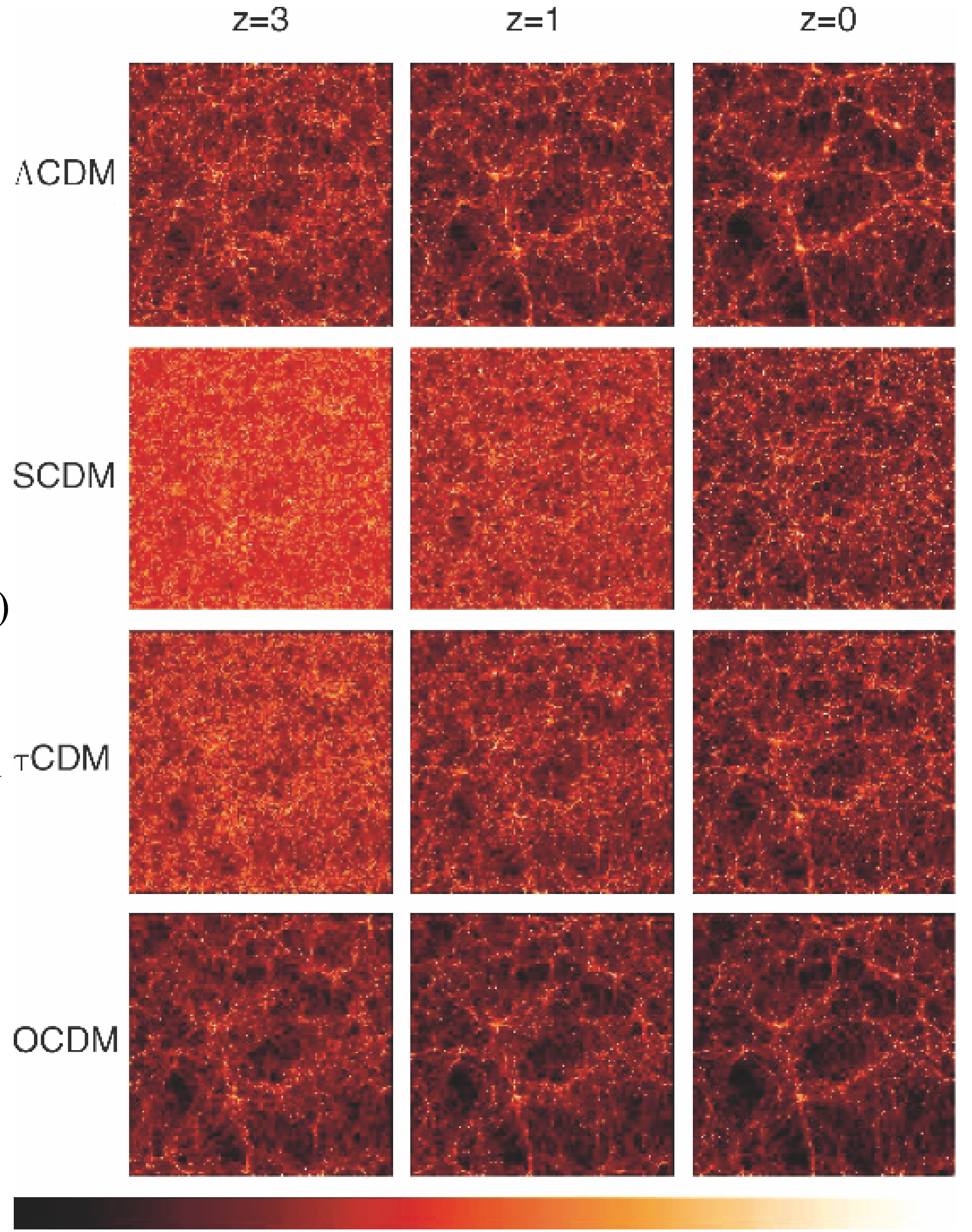
- The model can be improved by considering ellipsoidal collapse, which can also be done analytically.
- In the plot above, the "data points" are from a large numerical simulation and the dotted curves are the from the analytical model. The solid lines are fits to a different simulation.
- The Press-Schecter model is crude, but does a reasonable job.

Numerical simulations

- Consider only dark matter. Approximate dark matter with some number of particles, N, each has position and velocity. Initial distribution of particles arranged to match chosen power spectrum.
- Use the positions of the particles at time step t_i, to calculate the gravitational force on each particle from all the other particles, update the particle's velocity, and then update the particle's position. Do the calculation in some limited volume V.
- Given an average matter density ρ, the choice of N and V, means that each particle has an mass $M = \rho V/N$. Thus, simulation will not be accurate for mass scales $< M$.
- Make V large enough so that universe is homogeneous, e.g. $V = L^3$ and $L > 150$ Mpc.
- To handle particles on the edges, use periodic boundary conditions (like "Asteroids").
- Number of pairs in force calculation $\sim N^2$, so instead of calculating each pair, define a grid, add up the mass in each box in the grid, calculate force via Fast Fourier Transform (FFT).
- This doesn't work for particles close to each other, so do those using pairs. Pair-wise calculation is incorrect for particles very close to each other (one gets unrealistically strong collisions), so "soften" the $1/r^2$ law for small r. Softening scale length ~ mean particle separation. Simulation not accurate for shorter length scales.

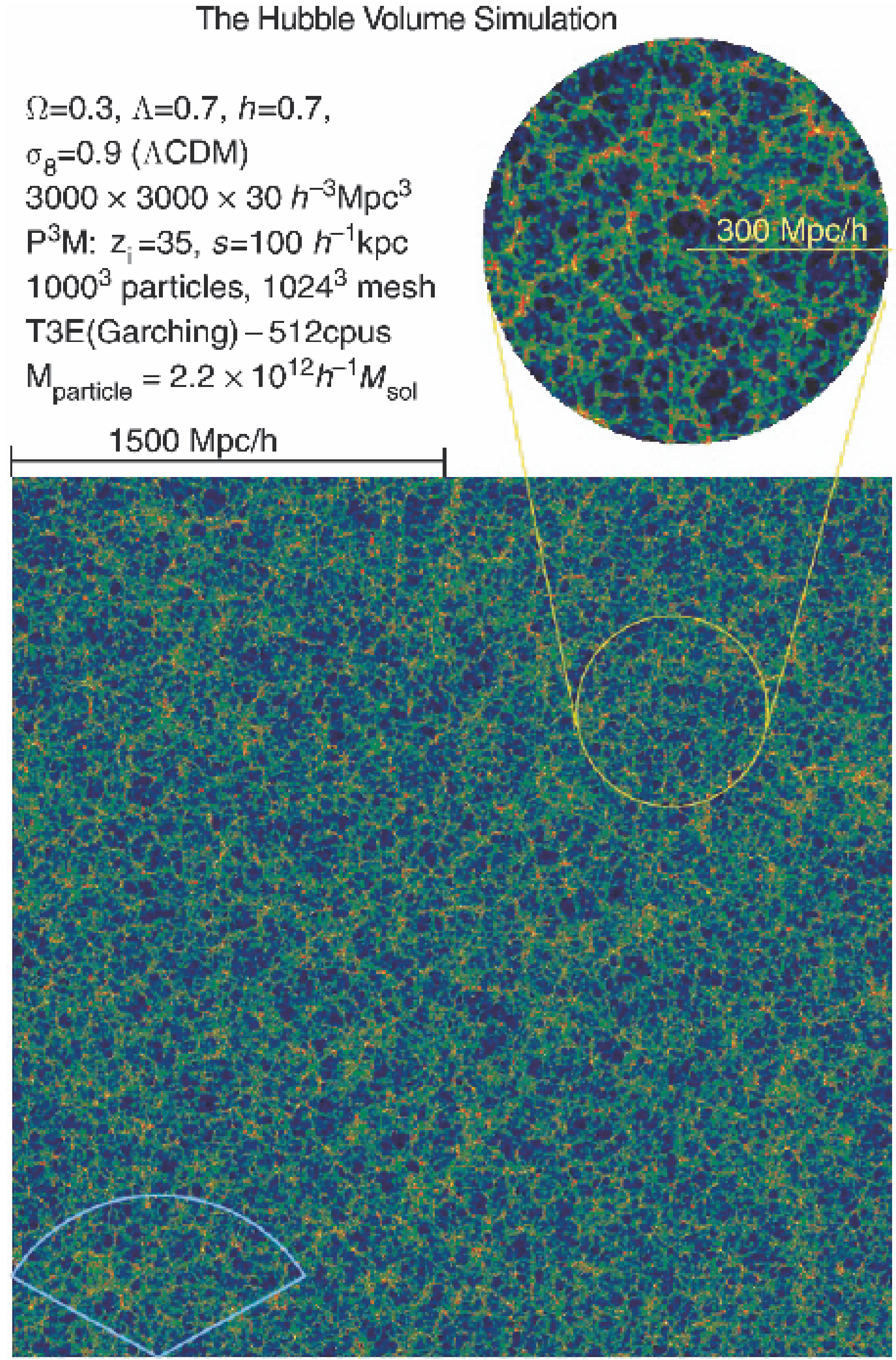
VIRGO simulation

- $N = 256^3$, $L = 170$ Mpc.
- ΛCMD $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$
- SCMD $\Omega_m = 1$, $\Omega_\Lambda = 0$
- τCMD $\Omega_m = 1$, $\Omega_\Lambda = 0$, different $P(k)$
- 0CMD $\Omega_m = 0.3$, $\Omega_\Lambda = 0$
- Modelers attempted to choose initial conditions so final state matches observed universe.
- Universes differ at high z.
- Note there are only ~5 clusters in this volume.

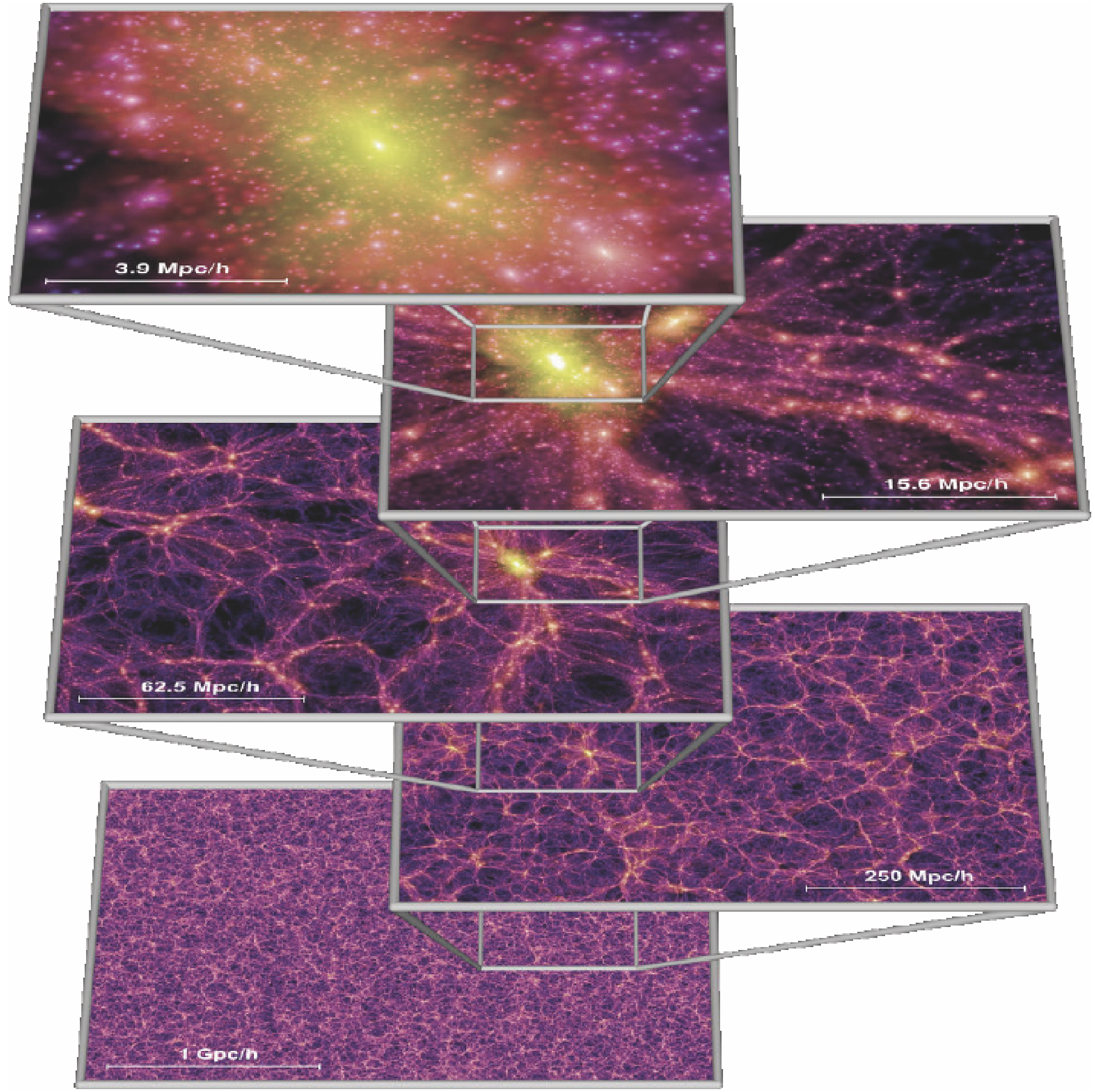


Hubble volume simulation

- ΛCMD $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$
- There are ~100 clusters.



Millenium simulation

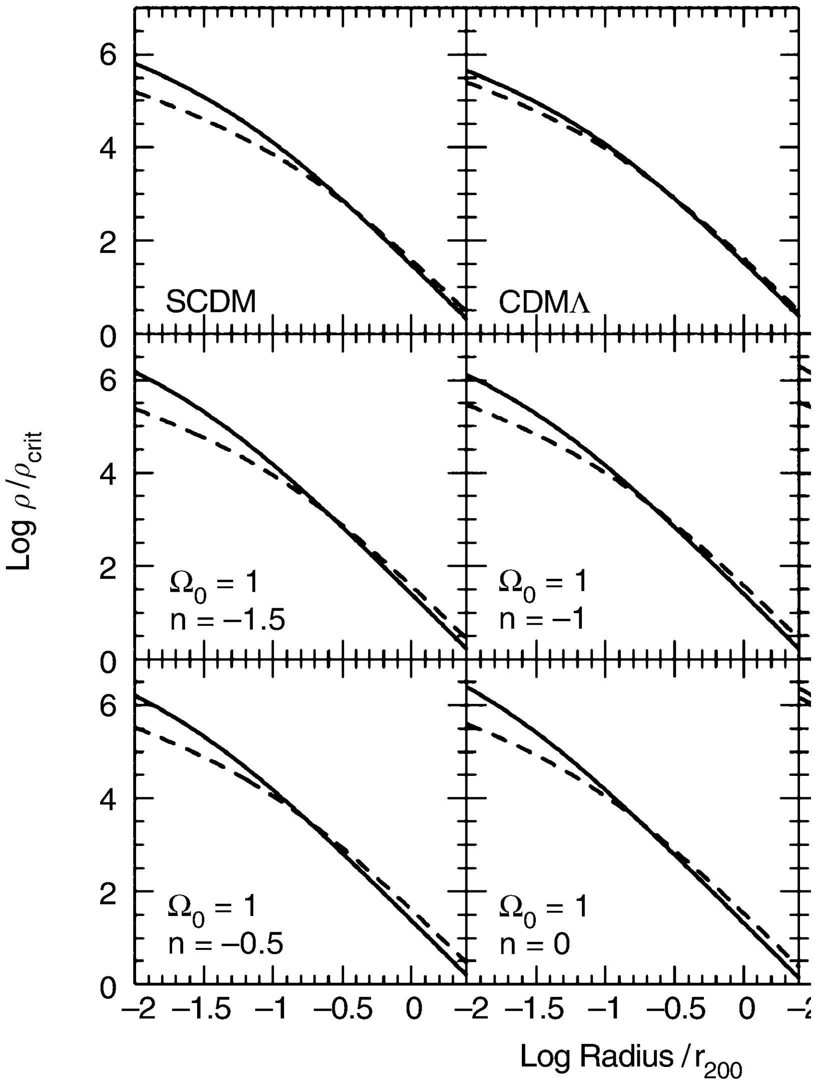


- $N = 2160^3 \sim 10^{10}$ $L = 360$ Mpc.
- ΛCMD $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$
- Spatial resolution ~ 4 kpc.
- There are ~50 clusters.
-
- Required a month on a 512 CPU machine.
- Produced 27 TB of data.
- How to look at the output?
  - Power spectrum
  - Mass spectrum
  - Properties of individual halos

Profiles of dark matter halos

- Define halo radius as r_{200} where $\bar{\rho} = 200\rho_{cr}$
- Halo mass is then defined by radius.
- Virial velocity is $V_{200}^2 = GM/r_{200}$.
- Then $M = V_{200}^3/10GH(z)$, $r_{200} = V_{200}/10H(z)$
- Halos are a 1-parameter family that varies with z.

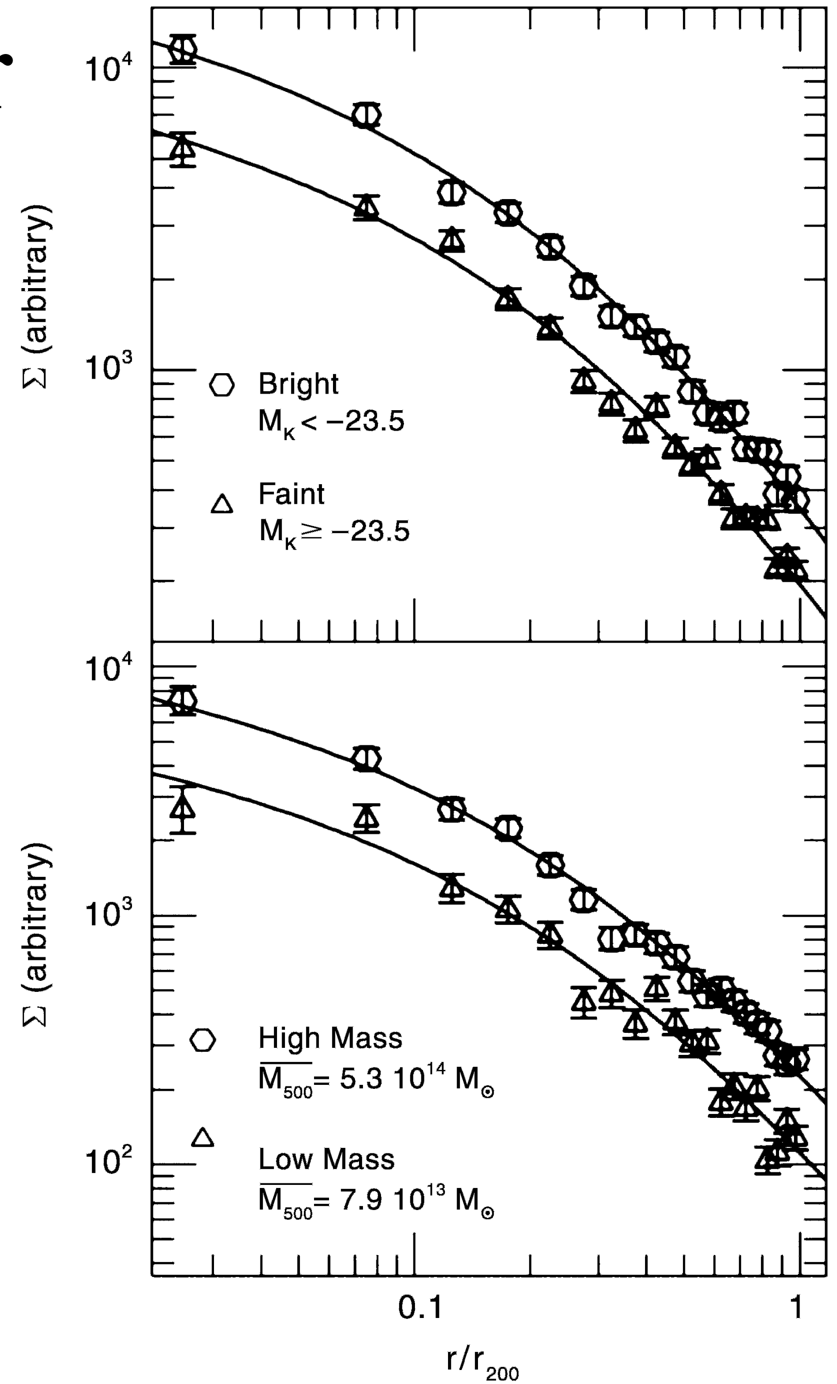
- Navarro, Frenk, and White (NFW) did numerical simulations and found that dark matter halos could be fitted $\rho_{cr}(r) = \rho_s/[(r/r_s)(1+r/r_s)^2]$ where ρ_s is normalization and r_s is characteristic radius.
- Can rewrite form in terms of "concentration index", $c = r_{200}/r_s$. Larger c, more concentrated.
- Find $c \sim M^{-1/9}/(1+z)$, again 1-parameter family.
- NFW profile is empirical.



Solid = low mass
Dashed = high mass

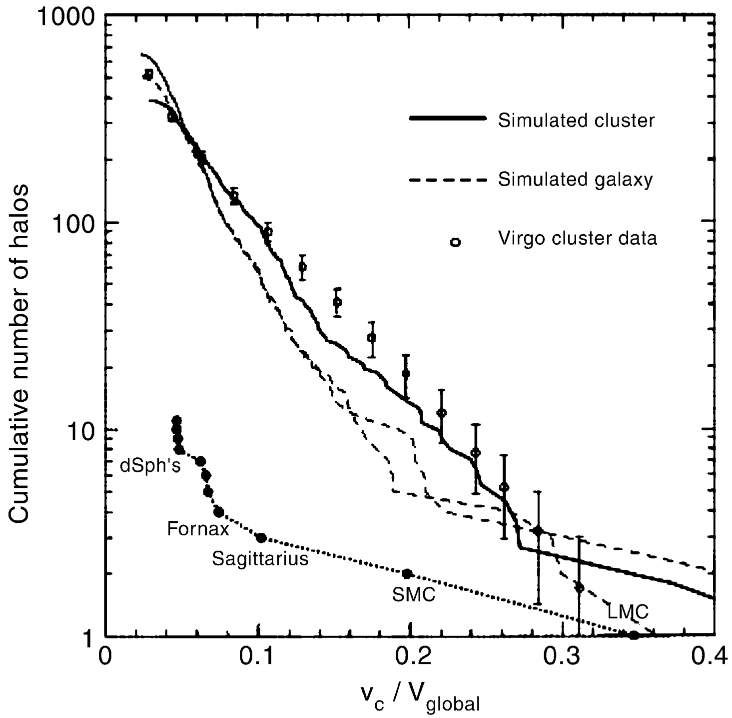
Profiles of dark matter halos

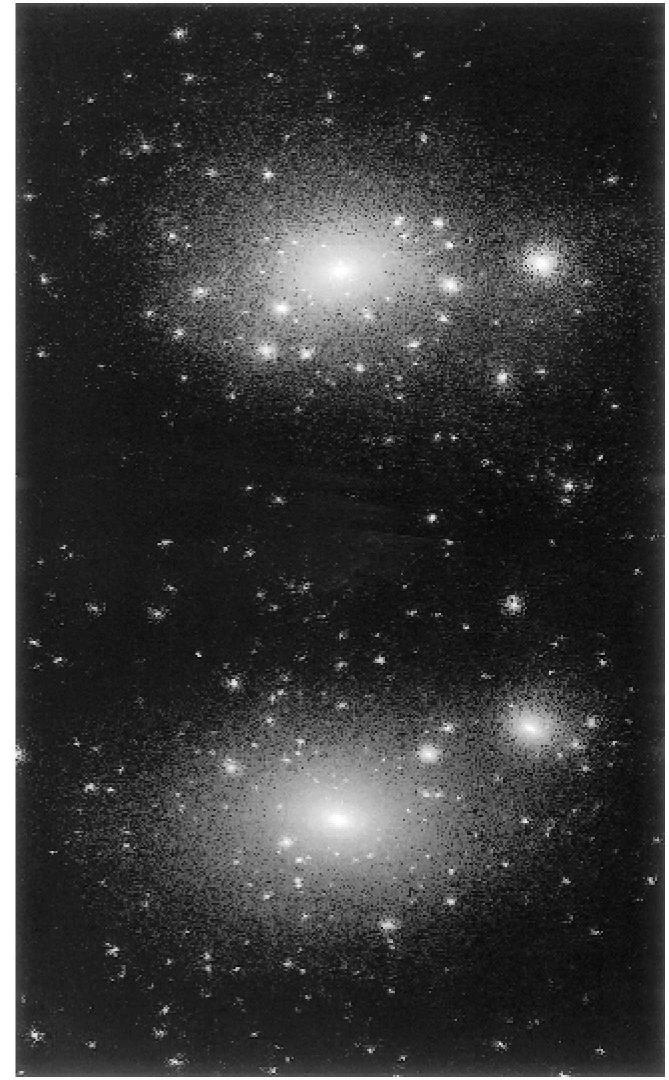
- Best comparisons with observations come from clusters.
- Averaging over many clusters leads to a galaxy surface density profile in good agreement with a fitted NFW profile.
- Find $c \sim 3$, independent of cluster mass.
- Simulations predict higher values of c and that c should vary with cluster mass.
- May indicate that galaxies are less strongly concentrated in clusters than is dark matter.
- Or may indicate modification of dark matter profile due to baryons.



Problems

- Rotation curves of low surface brightness galaxies do not match NFW profiles because they lack a central cusp.
- Simulations predict similar levels of substructure in high versus low mass halos.
- Substructure is seen in clusters (member galaxies) but not in low mass halos like the Milky Way.





Upper = $5 \times 10^{14}\ M_{sun}$

Lower = $2 \times 10^{12}\ M_{sun}$

Solutions?

- Halos are there but don't emit much light.
- Star formation does not proceed efficiently with low masses of baryons
- Models with smooth mass profiles accurate reproduce the positions of gravitational lenses of galaxies, but not the magnification factors. Positions are determined by average profile, while magnification is strongly affected by small scale bumps in profile, $\mu = (\partial\beta/\partial\theta)^{-1}$. This may suggest substructure in the galaxy mass profiles.
- Simulations likely over predict the amount of substructure for low mass halos because they neglect the effects of baryons. Radiation and pressure tend to smooth out structure and are relatively more important at low mass/short distance scales.

- Need better simulations including baryons, hydrodynamics, and radiation transport.
- People currently searching for extended, unidentified gamma-ray sources that could potentially be due to decaying particles in dark matter halos.

Origin of density fluctuations

- All simulations start with a spectrum of primordial density fluctuations.
- In inflation, there are quantum mechanical fluctuations that get amplified to cosmological scales during the inflationary epoch.
- Inflation, depending on the exact particle physics, predicts powerlaw spectra with index $n \sim 1$ or slightly smaller as observed.
- Inflation predicts a Gaussian distribution of fluctuations.
- Inflation predicts similar fluctuations in spacetime, that also have been amplified to cosmological scales. These fluctuations should persist today as very long wavelength gravitational waves. The gravitational waves should affect the polarization of the cosmic microwave background and it is possible to search for their signature.

For next class

- Project work day scheduled for October 24.
- Read 8.1-8.4 for October 29.
- First draft of project papers will be due on October 29.

EARLY VOTING TIMES/LOCATION

Monday-Friday, 10/22-26, 11am-7pm at the Iowa City Public Library