Outline

- Hand in, go over homework problem 3.6
- Rates of nuclear reactions
- Nuclear energy production in Sun

Nuclear Reaction Rates

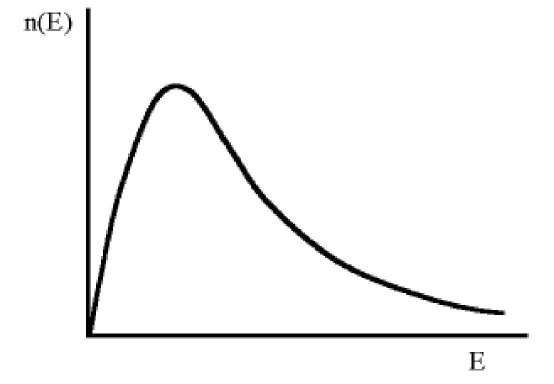
- One two protons are close enough for strong interactions, the probability of an interaction occurring depends on the nuclear cross section
- Most cross sections $\sigma = S_0/E$, why?
- Cross section is a measure of the size of a particle.
- What is the quantum mechanical size of a particle?
  - The deBroglie wavelength, $\lambda = h\,(2mE)^{-1/2}$
  - $\sigma \propto \text{size}^2 \propto \lambda^2 \propto E^{-1}$
- S_0 is a constant or weak function of energy.
  - S_0 is either measured or calculated from theory.
- Cross section including Coulomb screening $\sigma = (S_0/E)\exp(-(E_G/E)^{1/2})$

Nuclear Reaction Rates

- Number of reactions of nucleus A as it travels a distance dx
  - $dN_{\mathrm{A}} = n_{\mathrm{B}}\, \sigma_{\mathrm{AB}}\, dx$
- To find rate of reactions (per unit volume), divide by dt, multiply by n_{A}
  - $R_{\mathrm{AB}} = n_A\, dN_{\mathrm{A}}/dt = n_A\, n_{\mathrm{B}}\, \sigma_{\mathrm{AB}}\, dx/dt = n_A\, n_{\mathrm{B}}\, \sigma_{\mathrm{AB}}\, v_{\mathrm{AB}}$
- To find power per unit mass, multiply by energy of each reaction = Q, divide by density ρ,
  - $\varepsilon = n_A\, n_{\mathrm{B}}\, \sigma_{\mathrm{AB}}\, v_{\mathrm{AB}}\, Q/\rho$
- We could re-write this in terms of abundances X_{A}, X_{B}, but that just makes it harder to type in the equations.

Nuclear Reaction Rates

- An ideal gas contains particles with a distribution of velocities, given by the Maxwell-Boltzmann distribution. Holds for relative velocities if one uses the reduced mass.



$$P(v) = 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} v^2 \exp\left(\frac{-\mu v^2}{2kT}\right)$$

- Need to find average

$$\langle \sigma v \rangle = \int_0^\infty \sigma_{AB} v_{AB} P(v_{AB})\, dv_{AB}$$

- Change integration variable to $E = 0.5\mu v^2$, $dE = v\mu\, dv$,

$$\langle \sigma v \rangle \;=\; \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty e^{-E/kT} e^{-\sqrt{E_G/E}}\, dE$$

- Call $\quad f(E) \;=\; e^{-E/kT} e^{-\sqrt{E_G/E}}$

Nuclear Reaction Rates

- Integrand $f(E) = e^{-E/kT} e^{-\sqrt{E_G/E}}$

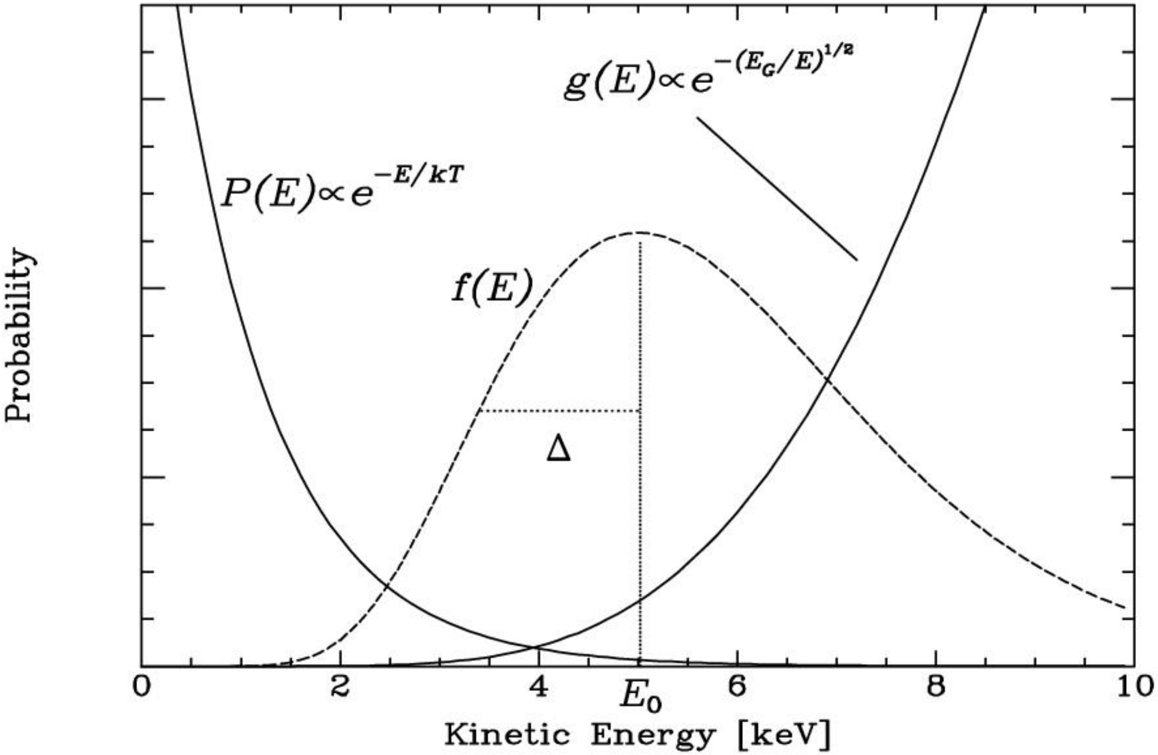
- Maximum at $E_0 = (kT/2)^{2/3} E_G^{1/3}$.

- Approximate as

  Gaussian ~ $\exp(-E^2/2\sigma^2)$

  with $\sigma = 3^{-1/2} 2^{1/6} E_G^{1/6} (kT)^{5/6}$

- Then integrate Gaussian.



Nuclear Reaction Rates

- Energy production per unit mass

$$\epsilon = 2^{5/3}\sqrt{\frac{2}{3}}\frac{\rho X_A X_B}{m_H^2 A_A A_B \sqrt{\mu}} Q S_0 E_G^{1/6} (kT)^{-2/3} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

- For $p + p \rightarrow d + e^+ + \nu_e$,
  - $S_0 = 4\times10^{-46}$ cm^2 keV, $E_G = 500$ keV
  - Q = 26.2 MeV for full reaction, less neutrinos (plus electrons)
- In core of Sun, $\rho = 150$ g/cm^3, $X = 0.5 = X_A = X_B$, $kT = 1$ keV
  - $\varepsilon = 10$ erg/s/gm
- Mass of core ~ 0.2 $M_{sun} = 4\times10^{32}$ gm, power = $\varepsilon\times$mass = 4×10^{33} erg/s.
  - Match solar luminosity, neutrino flux at Earth.
- 1/rate per proton = 6×10^{17} seconds = 2×10^{10} years > age of Sun.

CNO cycle

- First step: $p + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$
  - How does this differ from p-p interaction?
- Subsequent steps:
  - ${}^{13}\text{N} \rightarrow {}^{13}\text{C} + e^{+} + \nu_e$
  - $p + {}^{13}\text{C} \rightarrow {}^{14}\text{N} + \gamma$
  - $p + {}^{14}\text{N} \rightarrow {}^{15}\text{O} + \gamma$
  - ${}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^{+} + \nu_e$
  - $p + {}^{15}\text{N} \rightarrow {}^{12}\text{C} + {}^{4}\text{He}$
- Coulomb barrier is higher than p-p, but first step is not a weak interaction. CNO dominates over p-p at higher core temperatures.

Homework

- For next class:
  - Problem 3-7