Outline

- Hand in, go over homework problem 3.8
- Equations of stellar structure
- How to solve them

Equations of Stellar Structure

$$\frac{dP(r)}{dr} = \frac{-GM(r)\rho(r)}{r^2} \qquad \frac{dT(r)}{dr} = \frac{-3L(r)\kappa(r)\rho(r)}{16\pi r^2 a c T^3(r)}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \qquad \frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

- These are four coupled first-order differential equations.
- Need four boundary conditions to specify a unique solution:
- $M(0) = 0$, $L(0) = 0$, $P(\mathrm{r}_*) = 0$, $M(r_*) = M_*$.
- Note that r_* is not known, so really last is $M(\infty) = M_*$.
  - Not knowing r_* in advance creates troubles for numerical solution.

Equations of Stellar Structure

- Only variable with known the range of at beginning of calculations is *M*. Recast the equations to be functions of *M* instead of *r*.

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \quad \rightarrow \quad \frac{dr(M)}{dM} = \frac{1}{4\pi r^2 \rho(r)}$$

- Multiply other equations by dr/dM.
- Now independent variable is M, runs over fixed range 0 to M_*.
- Boundary conditions: $r(M=0) = 0$, $L(M=0) = 0$
- How to translate boundary conditions at r*?
  - $T(M=M_*) = T_e$, $P(M=M_*) = P_e$ conditions at photosphere.
- Due to Vogt-Russell, T_e and P_e are determined by M.
- These are trial boundary conditions, adjusted to get a good solution.

Integration Method

- How to solve first order diff eq numerically?

$$\frac{dy}{dx} = f(x, y), \qquad y(0) = y_0$$

- Evaluate dy/dx at $x = 0$, $dy/dx = f(0, y_0)$
- Take step size h, evaluate y at $x = h$,
  - $y(h) = y(0) + h \times dy/dx = y_0 + \mathrm{h} \times \mathrm{f}(0, y_0)$
- Keep looping – Euler's method.
- Produces 'truncation' error ~ h^2.
- Can be improved to ~ h^4 with more evaluations of f at each step.
- Because boundary conditions are specified at both ends, integration method is usually run in both directions.

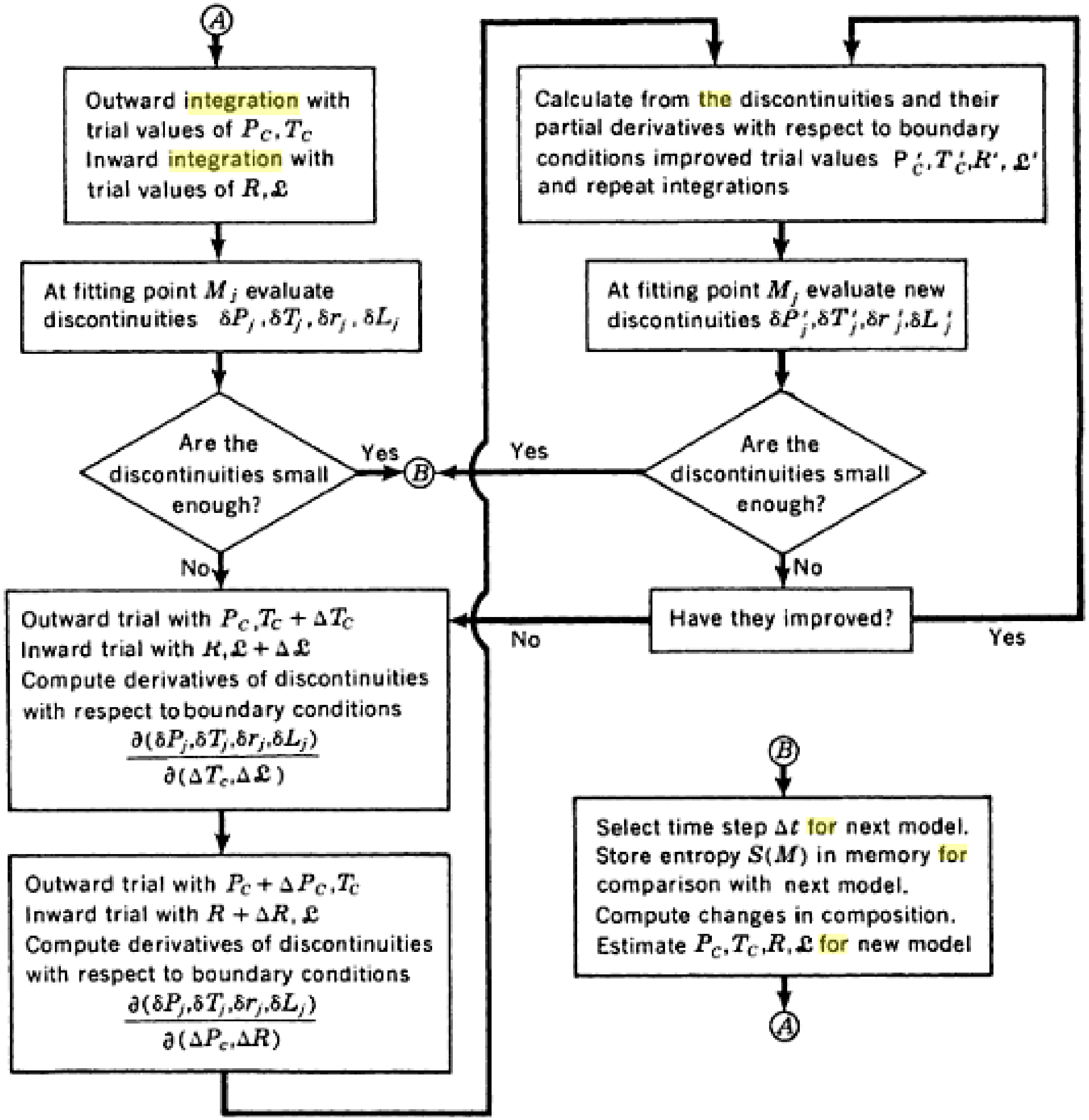


Fig. 6-2 Flow diagram for the integration method. The sequence of logical decisions and steps employed in the compution of the stellar structure is indicated by the arrows. [*Adapted from R. L. Sears and R. R. Brownlee, Stellar Evolution and Age Determinations, in L. H. Aller and D. B. McLaughlin (eds.), "Stellar Structure," The University of Chicago Press, Chicago, 1965, by permission of The University of Chicago Press. Copyright 1965 by The University of Chicago.*]

Relaxation Method

- How to solve first order diff eq numerically?

$$\frac{dy}{dx} = f(x, y), \qquad y(0) = y_0$$

- Generate a trial function $y(x)$, specified at each step.
- At each step evaluate $g = dy/dx - f(x,y)$
- Should be zero if y is a solution.
- Adjust values of y to decrease g, repeat until the solution is good enough.
- Finding how to adjust values of y is equivalent to finding the roots of a function. Simplest method is Newton's.

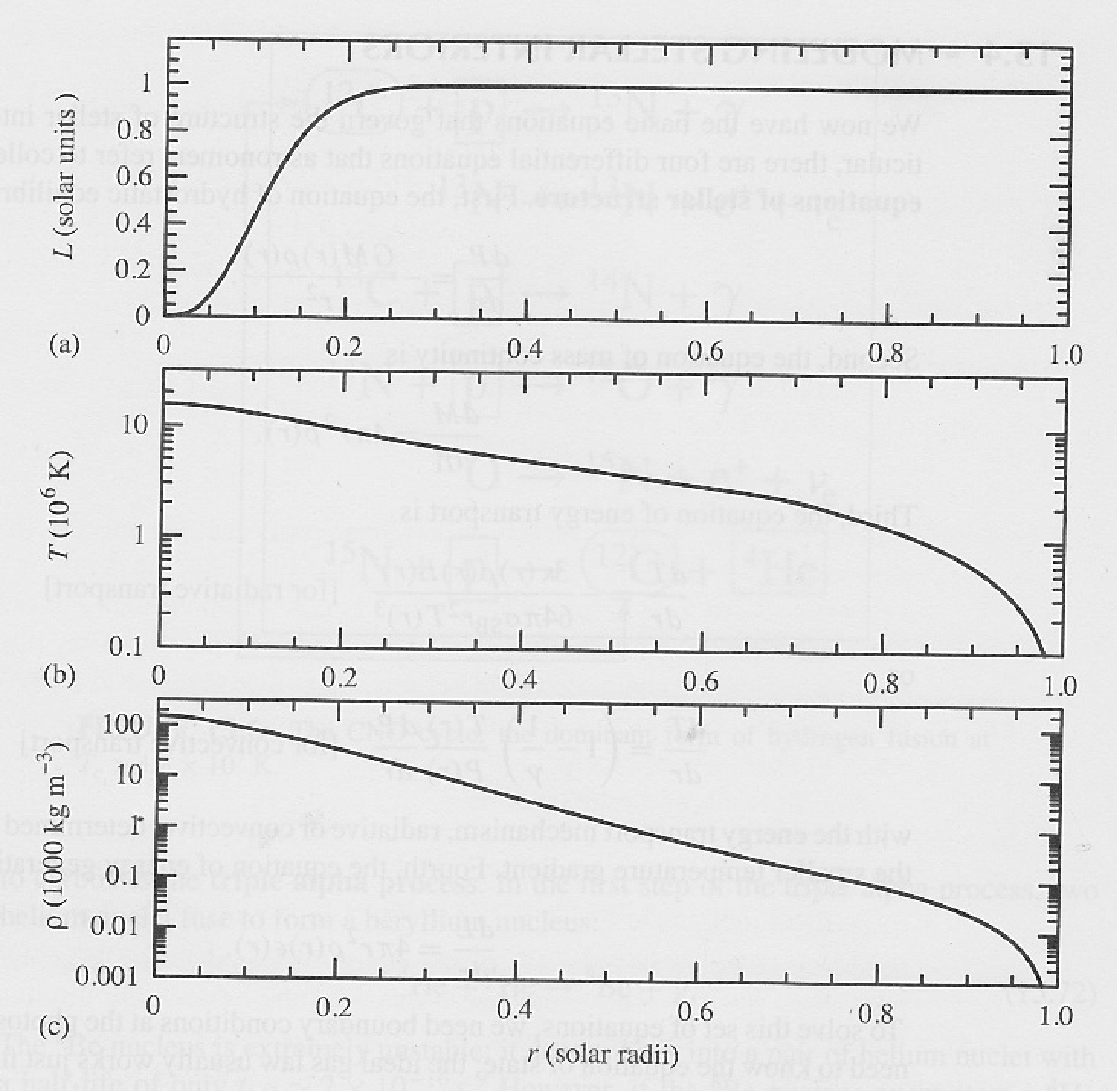
Solutions for the Sun

Table 6-6 Model of the sun at 4.5×10^9 years†

$M(r)$, *solar masses*	r, 10^{11} *cm*	T, 10^6 °K	ρ, g/cm^3	$L(r)$, 10^{33} *ergs/sec*	ϵ, *ergs* g^{-1} sec^{-1}	κ, cm^2/g	X_H
0.0	0.00	15.7	158	0.00	17.5	1.09	0.36
0.05	0.06	13.8	103	1.30	10.0	1.32	0.52
0.1	0.08	12.8	83	2.13	6.8	1.48	0.58
0.2	0.10	11.3	59	3.09	3.3	1.78	0.65
0.3	0.13	10.1	43	3.55	1.6	2.09	0.68
0.4	0.15	9.0	31.5	3.77	0.7	2.42	0.69
0.5	0.17	8.1	22.4	3.86	0.3	2.79	0.70
0.6	0.20	7.1	15.2	3.90	0.06	3.2	0.70
0.7	0.23	6.2	9.4	3.90	0.02	3.8	0.71
0.8	0.26	5.1	5.0	3.90	0.00	4.5	0.71
0.9	0.32	3.9	1.84	3.90	0.00	6.0	0.71
0.95	0.38	3.0	0.74	3.90	0.00	7.4	0.71
0.99	0.48	1.73	0.117	3.90	0.00	9.6	0.71
0.99955	0.62	0.66	0.0063	3.90	0.00	Conv.	0.71
1.0	0.694			3.90			0.71

† B. Strömgren, Stellar Models for Main-sequence Stars and Subdwarfs, in L. H. Aller and D. B. McLaughlin (eds.), "Stellar Structure." By permission of The University of Chicago Press. Copyright 1965 by The University of Chicago. Initial composition $X = 0.71$, $Y = 0.27$.

Solutions for the Sun



Stellar Solutions

- First solutions calculated are usually for a specific elemental composition taken to be uniform through the star.
  - “Zero-age main sequence” or ZAMS
- Discuss how stars age in next section.

Homework

- For next class:
  - Problem 3-9
  - E-mail topics for review