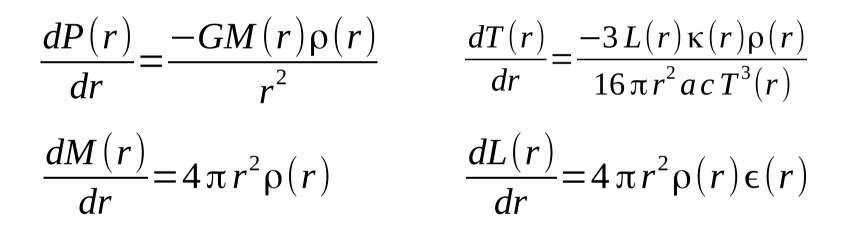
# Outline

- Properties of white dwarfs
- White dwarf cooling
- Minimum stellar mass (brown dwarfs)

### Equations of Stellar Structure

• Which are valid, relevant for white dwarfs?



# **Scaling Relations**

- Assume  $P(r) \propto r^{\beta}$ , same for M(r),  $\rho(r)$ , then  $dP/dr \propto P/r$ , etc...
- Find scaling relations from equations of stellar structure

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \rightarrow M \propto r^3 \rho$$
$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \rightarrow P \propto \frac{GM\rho}{r} \propto \frac{GM^2}{r^4}$$

• Equation of state for degenerate electron gas

$$P = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20\,m} m_p^{-5/3} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3} = b \rho^{5/3} \propto b \frac{M^{5/3}}{r^5}$$

• Equate pressures to find that radius decreases with mass

$$r \propto \frac{b}{G} M^{-1/3}$$
  $r = 2.3 \times 10^9 \text{ cm} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{M}{M_{Sun}}\right)^{-1/3}$ 

# Limiting mass of White Dwarf

As mass increases, radius decreases towards zero.
At what point does this break down?

#### Pressure

- In calculating pressure, assumed v = p/m.
- This breaks down as  $v \rightarrow c$ . Instead,  $v \approx c$ .

$$P \neq \frac{1}{3} \int_{0}^{\infty} n(p) p v dp$$
 instead  $P = \frac{1}{3} \int_{0}^{\infty} n(p) p v dp = \frac{8 \pi c}{3 h^{3}} \frac{p_{f}^{4}}{4}$ 

• Then equation of state is

$$P = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}$$

- Note different exponent on density.
- Why no dependence on mass of electron?

# **Scaling Relations**

- Scaling relations from equations of stellar structure  $P \propto \frac{GM^2}{r^4}$
- Equation of state for degenerate electron gas is  $P \propto \rho^{(4+\epsilon)/3}$  with  $\epsilon = 1$  for non-relativistic gas,  $\epsilon = 0$  for ultra-relativistic gas.
- Equate pressures to find dependence of radius on mass

$$r \propto M^{(\epsilon-2)/3\epsilon}$$
 as  $\epsilon \rightarrow 0$ ,  $r \rightarrow M^{-\infty} = 0$ 

• When electrons are ultra-relativistic, pressure increases too slowly with density to support the star against collapse as mass increases. Maximum mass, the "Chandrasekhar mass",

$$M \approx \left(\frac{hc}{G}\right)^{3/2} m_p^{-2} = \left(\frac{hc}{G m_p^2}\right)^{3/2} m_p$$

• Accurately calculated value is 1.4 solar masses.

### White Dwarf Cooling

• Thermal energy of star while still supported by thermal pressure is

$$E_{th} \approx \frac{GM^2}{2r} = \frac{3}{2}NkT \approx \frac{M}{m_p}kT \Rightarrow kT \approx \frac{GMm_p}{r}$$

- For *M* and *r* of typical white dwarf, find  $kT \sim 10$  keV,  $T \sim 10^8$  K.
- White dwarf radiates as blackbody with it surface temperature.
- Electrons are good conductors in most of star, leading to a near uniform temperature, but a thin layer of normal matter on the surface insulates the star and slows cooling. Ignore this layer to estimate cooling time, which will then be an upper limit.

$$\frac{-dE_{th}}{dt} = L \rightarrow 4\pi r^2 \sigma T^4 = \frac{3}{8} \frac{Mk}{m_p} \frac{dT}{dt} \rightarrow dt = \frac{3Mk}{32\pi\sigma m_p r^2} T^{-4} dT$$

• Solution  $T \sim t^{1/3}$ . Cooling time is on order of 10<sup>9</sup> years.

### Brown Dwarfs

- As a protostar collapses, fusion turns when the core temperature gets high enough. Hydrogen fusion requires  $T > 10^7$  K.
- We have assumed that radius will keep contracting, until a sufficiently high temperature is reached. But, what is degeneracy pressure stops contraction first?
- Use same "white dwarf" radius calculated before. Now Z/A = 1 since gas is hydrogen.

$$r = 2.3 \times 10^9 \text{ cm} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{M}{M_{Sun}}\right)^{-1/3} \qquad kT \approx \frac{GMm_p}{r} = 10^7 \text{ K}$$

- Equate *r* and solve for  $M = 0.07 M_{Sun}$ .
- Nuclear fusion in stars of this mass or smaller never turns on. They are "brown dwarfs".

### Homework

- For next class:
  - Problem 4-2