Outline

- Go over problem 4-3, exam problem #1
- Discovery of pulsars
- Pulsar emission mechanisms

Little Green Men

- First pulsar was discovered in 1967 with pulse period of 1.33 s.
- What can make extremely regular pulsations?
- Discoverers named object "LGM-1" for "Little Green Men".
- Additional pulsars discovered within a year, periods 0.03 s to several seconds.

Neutron Stars

- Period is too short to be orbital.
- Fastest period at which an object can rotate is limited by break up due to centrifugal forces. Need force of gravity > centripetal acceleration:

$$\frac{GMm}{r^2} > m\omega^2 r \rightarrow \frac{M}{r^3} > \frac{\omega^2}{G} \rightarrow \bar{\rho} = \frac{3M}{4\pi r^3} > \frac{3(2\pi/\tau)^2}{4\pi G}$$

- For period of 1.33 s, need $\rho > 8 \times 10^7 \text{ g/cm}^3$
- Densest known objects at the time were white dwarfs $\rho \sim 10^6$ g/cm³
- Conclude that pulsating sources were a new type of very dense star.

Spin up of neutron star

If the Sun (spin rate 1/25 days, radius 7×10^{10} cm) were to collapse to a neutron star with a radius of 12 km, how fast would it be spinning?

Angular momentum of sphere where M is mass, R is radius, ω is angular frequency:

$$L = I \,\omega = \frac{2}{5} M R^2 \,\omega$$

$$\omega_f = \omega_i \left(\frac{R_i}{R_f}\right)^2 = 3 \times 10^{-6} \,\mathrm{s}^{-1} \left(\frac{7 \times 10^{10} \,\mathrm{cm}}{1.2 \times 10^5 \,\mathrm{cm}}\right)^2 = 1 \times 10^6 \,\mathrm{s}^{-1}$$

Very high rotation rates can be reached simply via conservation of angular momentum.

This is faster than any known (or possible) neutron star. Mass and angular momentum are lost during the collapse.

Crab Pulsar



Spin down of a pulsar

Energy
$$E = \frac{1}{2}I\omega^2$$

Power
$$P = -\frac{dE}{dt} = I \omega \frac{d\omega}{dt}$$

- For Crab pulsar, power ~ $(1-2) \times 10^{39}$ erg/s.
- Over a year, the spin rate changes by 0.04%.
- Total luminosity of nebula is ~ 5×10^{38} erg/s.

Magnetic Field

If a solar type star collapses to form a neutron star, while conserving magnetic flux, we would naively expect

$$R_{sun}^2 B_{sun} = R_{ns}^2 B_{ns} \Rightarrow \frac{B_{ns}}{B_{sun}} = \left(\frac{7 \times 10^{10}}{10^6}\right)^2 \approx 5 \times 10^9$$

For the sun, $B \sim 100 G$, so the neutron star would have a field of magnitude $\sim 10^{12} G$.

Magnetosphere

Neutron star rotating in vacuum:



Electric field induced immediately outside NS surface. $E \simeq \frac{V}{C}B$

The potential difference on the scale of the neutron star radius:

$$\Phi = ER \sim 10^{18} V$$

Dipole Radiation

Even if a plasma is absent, a spinning neutron star will radiate if the magnetic and rotation axes do <u>not</u> coincide:



$$L = \frac{1}{6c^3} B^2 r^6 \omega^4 \sin^2 \theta \propto \omega^4$$

Equate this to loss of rotational energy:

$$P = I \,\omega^4 \frac{d \,\omega}{dt} \propto \omega^4 \rightarrow \frac{d \,\omega}{dt} = C \,\omega^3$$

Separate variables and integrate:

$$t_{pulsar} = \int dt = \frac{\omega^3}{2\dot{\omega}} \left(\frac{1}{\omega^2} - \frac{1}{\omega_i^2} \right)$$

For Crab, get 1260 years from measured ω and $d\omega/dt$, assuming $\omega_i = \infty$. Matches age of 950 years.

Braking Index

In general, the slow down may be expressed as

 $\dot{\omega} = -k\omega^n$ where *n* is referred to as the <u>braking index</u>

The time that it takes for the pulsar to slow down is

$$t = -(n-1)^{-1} \omega \dot{\omega}^{-1} \left[1 - (\omega/\omega_i)^{n-1} \right]$$

If the initial spin frequency is very large, then

$$t = -(n-1)^{-1} \omega \dot{\omega}^{-1} = (n-1)^{-1} P \dot{P}^{-1}$$

For dipole radiation, n=3, we have

$$t = \frac{P}{2 \dot{P}}$$
 Characteristic age of the pulsar

Homework

- For next class:
 - Problem 4-4
 - Exam problem #2