Outline

- Go over problem 4-7, exam problem #3
- Accretion disks
- Accretion rate and Eddington luminosity
- Evolution of interacting binaries

Accretion Disks

- To model accretion disk, assume:
  - particles move on circular (Keplerian) orbits
  - particles have viscous interactions with particles on nearby orbits that cause them to lose energy and angular momentum
  - frictional heat is radiated away with each disk annulus acting as a blackbody of given temperature
- Note that nature of the viscosity is not well understood.

- How does energy change when mass dM moves from $r+dr$ to r?

$$dE_g = GMdM\left(\frac{1}{r} - \frac{1}{r+dr}\right) \approx GMdM\frac{dr}{r^2}$$

Accretion Disks

- This is only gravitational potential energy. Need to also consider kinetic energy – particle needs higher velocity to orbit closer in.
- Virial theorem: $E_{total} = E_k + E_g = \frac{E_g}{2}$
  - So, need a factor of 1/2.
- Energy change when mass moves at a rate *dM/dt* though the disk:

$$\frac{dE}{dt} = \frac{1}{2} GM \frac{dr}{r^2} \frac{dM}{dt} = dL = 2(2\pi r)\sigma T^4 dr$$

- Energy is then radiated in an annulus, surface area = $2(2\pi r)dr$.
- Solve for $T(r)$

$$T(r) = \left(\frac{GM\dot{M}}{8\pi\sigma}\right)^{1/4} r^{-3/4}$$

- Inner parts of disk are hotter and produce most of luminosity.
- Disk must also transport angular momentum outwards, this creates outflows.

Accretion Disks

- Integrate over r to find total luminosity:

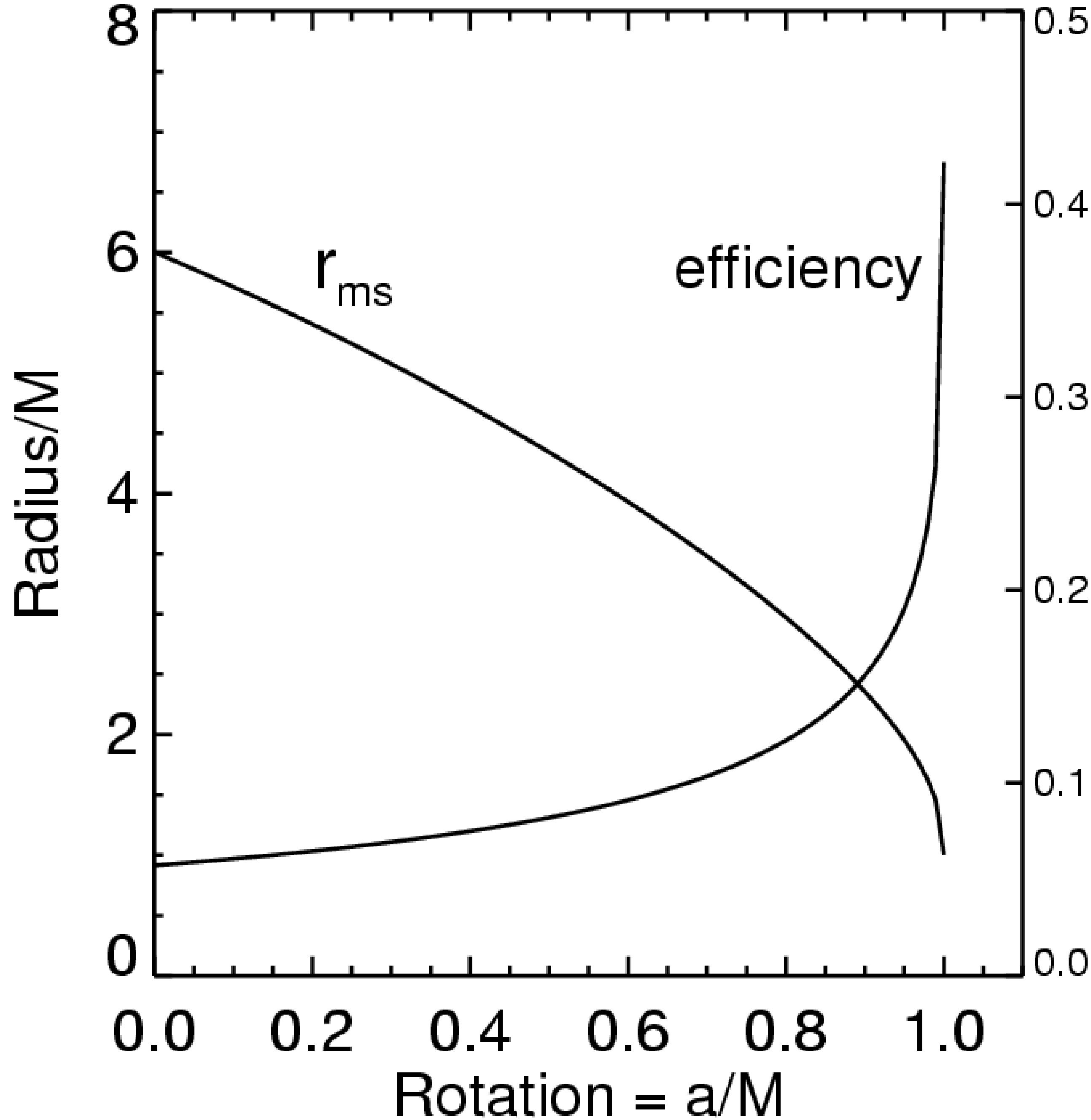
$$L = \int_{r_{\rm in}}^{r_{\rm out}} 2(2\pi r)\sigma T^4(r)\,dr = \frac{1}{2}GM\dot{M}\left(\frac{1}{r_{\rm in}} - \frac{1}{r_{\rm out}}\right) \approx \frac{1}{2}\frac{GM\dot{M}}{r_{\rm in}}$$

- Radiative efficiency is fraction of rest mass energy of accreted matter that is radiated

$$\eta = \frac{L}{\dot{M}c^2} = \frac{1}{2}\frac{GM}{r_{\rm in}c^2}$$

- For neutron star $M = 1.4\ M_{\rm Sun}$ and $r_{\rm in} = 10$ km, then $\eta = 0.10$.
- For non-rotating black hole, stable orbits are not possible inside of the “innermost stable circular orbit” at $r_{\rm in} = 3\ r_{\rm s}$ and $\eta = 0.057$.
- For maximally-rotating black hole, $r_{\rm in} = 0.5\ r_{\rm s}$ and $\eta = 0.42$.
- Note accretion is much more efficient than nuclear burning, $\eta = 0.007$.

Orbits around Black Holes



Accretion Disks

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Accretion Disks

- Typical mass transfer rate ~ $10^{-9}\ M_{sun}$/year.
- White dwarf accretor, mass ~ 1 M_{sun}, radius ~ 10^4 km.
  - L ~ 4×10^{33} erg/s
  - T ~ 5×10^4 K

- Neutron star accretor M = 1.4 M_{Sun}, radius = 10 km.
  - L ~ 10^{37} erg/s
  - T ~ 10^7 K

Accreting White Dwarfs

- Novae – mass that transfers through disk builds up on WD surface, eventually undergoes nuclear fusion.
  - Typical energy ~ 10^{46} erg.
  - Duration ~ 1 month, typical luminosity ~ 4×10^{39} erg/s.

- Type Ia supernovae – mass builds up on WD until mass exceeds Chandrasekhar limit. WD fuses to iron-group elements and explodes.
  - Typical energy ~ 10^{51-52} erg/s
  - Typical radiated energy ~ 10^{49-50} erg/s
  - Duration ~ 1 month, typical luminosity ~ 10^{43-44} erg/s ~ 10^{10} L_{Sun}.

Eddington Luminosity

- Consider radiation pressure from object of luminosity L acting on ionized, inflowing gas. Dominant interaction will be Thomson scattering.
- Look at one electron. Rate at which it scatters photons depends on the number of photons per unit area, Σ, at the position, r, of the photon

$$R_{\text{scat}} = \sigma_T \Sigma$$

- Consider photons of a specific frequency. Number of photons per unit area, Σ, is energy flux at that frequency, f_ν, divided by energy of individual photon $h\nu$:

$$\Sigma = \frac{f_\nu}{h\nu} = \frac{L_\nu}{4\pi r^2 h\nu} \rightarrow R_{\text{scat}} = \sigma_T \frac{L_\nu}{4\pi r^2 h\nu}$$

- Force on electron is dp/dt of scattered photons, $p = h\nu/c$,

$$F_\nu = R_{\text{scat}} \frac{h\nu}{c} = \frac{L_\nu \sigma_T}{4\pi r^2 c}$$

- Integrate over ν

$$F_{\text{rad}} = \frac{L \sigma_T}{4\pi r^2 c}$$

Eddington Luminosity

- If the electron gets too far from its proton, then electrostatic forces would build up in the accretion flow. So, the effective gravitational force on the electron is actually that on the proton

$$F_{\mathrm{g}} = \frac{G\,M\,m_p}{r^2}$$

- The accretion flow will stop if Frad > Fg, since the net force on matter in the flow would then be outward. The maximum accretion rate and maximum luminosity occurs when radiation pressure exactly balances gravity. This is the "Eddington luminosity"

$$\frac{L_E\,\sigma_T}{4\pi\,r^2\,c} = \frac{G\,M\,m_p}{r^2} \rightarrow L_E = \frac{4\pi\,c\,G\,M\,m_p}{\sigma_T}$$

$$L_E = 1.3\times10^{38}\,\mathrm{erg/s}\,\frac{M}{M_{\mathrm{Sun}}} = 3.3\times10^{4}\,L_{\mathrm{Sun}}\,\frac{M}{M_{\mathrm{Sun}}}$$

- This applies for steady state accretion. Novae and supernovae exceed Eddington (and have strong, radiation-driven outflows).

Orbital Evolution

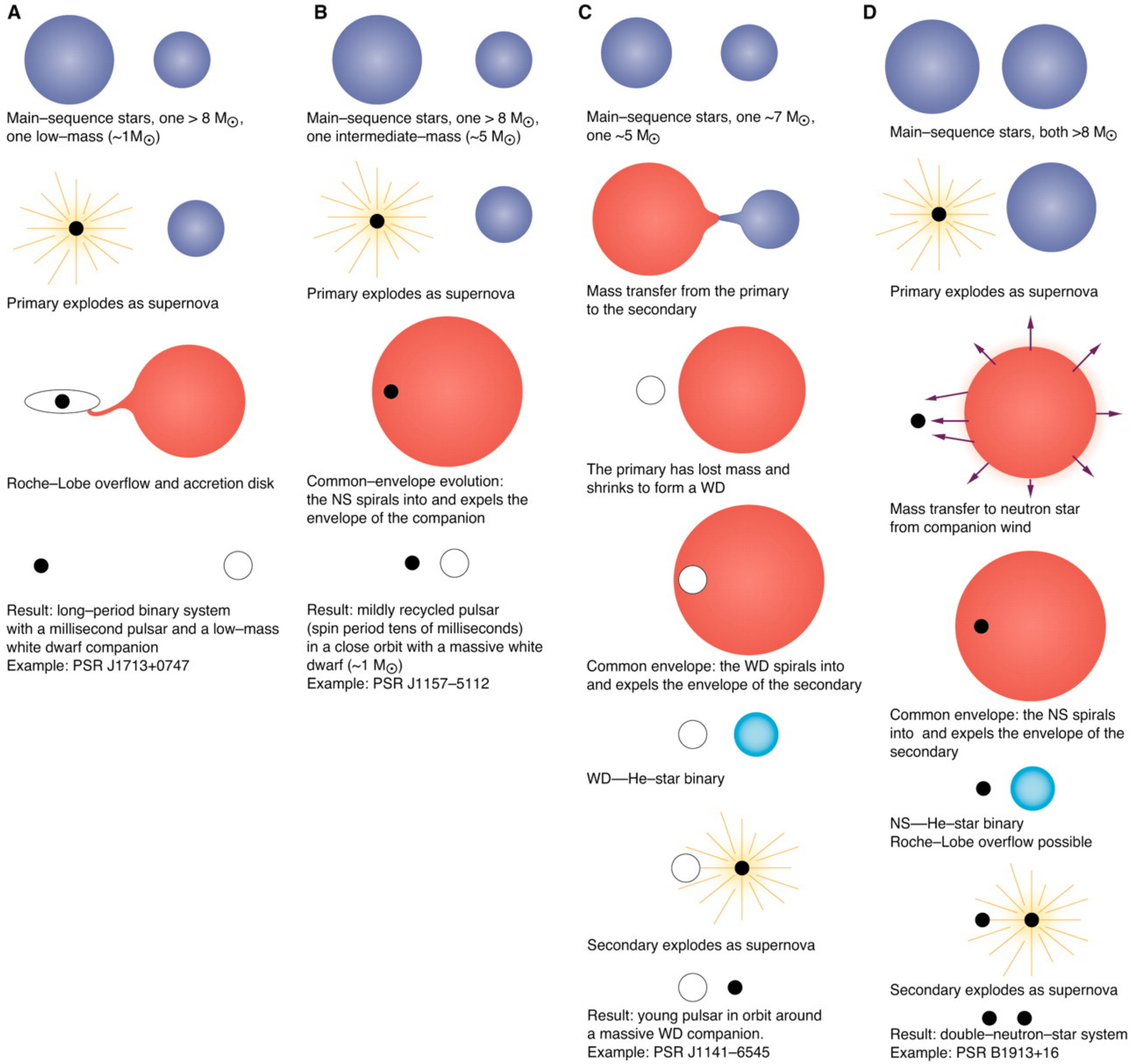
- The transfer of material between stars in a binary changes the orbit.
- For simplicity, consider only orbital angular momentum and ignore losses due to winds.
- Angular momentum $J = \mu a^2\omega$, a = separation, ω = orbital frequency, and reduced mass

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

- Dynamics are determined by Kepler's law and conservation of angular momentum

$$\omega^2 = \frac{G(M_1 + M_2)}{a^3} \quad \text{and} \quad \frac{dJ}{dt} = 0 \quad \text{note} \quad J = \mu\sqrt{G(M_1 + M_2)a}$$

$$\text{Note } \dot{M}_1 = -\dot{M}_2 \quad \text{and find} \quad \frac{1}{a}\frac{da}{dt} = 2\dot{M}_1 \frac{M_1 - M_2}{M_1 M_2}$$



A
Main–sequence stars, one > 8 M☉, one low–mass (~1M☉)
Primary explodes as supernova
Roche–Lobe overflow and accretion disk
Result: long–period binary system with a millisecond pulsar and a low–mass white dwarf companion
Example: PSR J1713+0747
B
Main–sequence stars, one > 8 M☉, one intermediate–mass (~5 M☉)
Primary explodes as supernova
Common–envelope evolution: the NS spirals into and expels the envelope of the companion
Result: mildly recycled pulsar (spin period tens of milliseconds) in a close orbit with a massive white dwarf (~1 M☉)
Example: PSR J1157–5112
C
Main–sequence stars, one ~7 M☉, one ~5 M☉
Mass transfer from the primary to the secondary
The primary has lost mass and shrinks to form a WD
Common envelope: the WD spirals into and expels the envelope of the secondary
WD—He–star binary
Secondary explodes as supernova
Result: young pulsar in orbit around a massive WD companion.
Example: PSR J1141–6545
D
Main–sequence stars, both >8 M☉
Primary explodes as supernova
Mass transfer to neutron star from companion wind
Common envelope: the NS spirals into and expels the envelope of the secondary
NS—He–star binary
Roche–Lobe overflow possible
Secondary explodes as supernova
Result: double–neutron–star system
Example: PSR B1913+16

Homework

- For next class:
  - Problem 4-9, 4-10
  - Exam problem #4