Outline

- Hand in, go over homework problems 1.1, 1.2
- Use of cgs units
- Blackbody radiation
- Atomic spectra
- Stellar spectra
  - Photosphere
  - Absorption lines
  - Spectral types

Units cgs vs SI

- Maxwell's equations in cgs contain *c* and do not contain ε_0 nor μ_0.
- Charge in cgs in measured in statcoulombs with units of $g^{1/2}\, cm^{3/2}\, s^{-1}$.
- 'erg' = unit of energy.
- cgs units fit in your pocket.

Name	Gaussian units	SI units
Gauss's law (macroscopic)	$\nabla \cdot \mathbf{D} = 4\pi\rho_f$	$\nabla \cdot \mathbf{D} = \rho_f$
Gauss's law (microscopic)	$\nabla \cdot \mathbf{E} = 4\pi\rho$	$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$
Gauss's law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction):	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampère–Maxwell equation (macroscopic):	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J}_f + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$
Ampère–Maxwell equation (microscopic):	$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$	$\nabla \times \mathbf{B} = \mu_0\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

Other basic laws [edit]

Name	Gaussian units	SI units
Lorentz force	$\mathbf{F} = q\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)$	$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$
Coulomb's law	$\mathbf{F} = \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$	$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$
Electric field of stationary point charge	$\mathbf{E} = \frac{q}{r^2}\hat{\mathbf{r}}$	$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$
Biot–Savart law	$\mathbf{B} = \frac{1}{c}\oint \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$	$\mathbf{B} = \frac{\mu_0}{4\pi}\oint \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$

Blackbody Radiation

- Energy density inside a cavity filled with blackbody radiation with the walls of the cavity at temperature T is

$$u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}$$

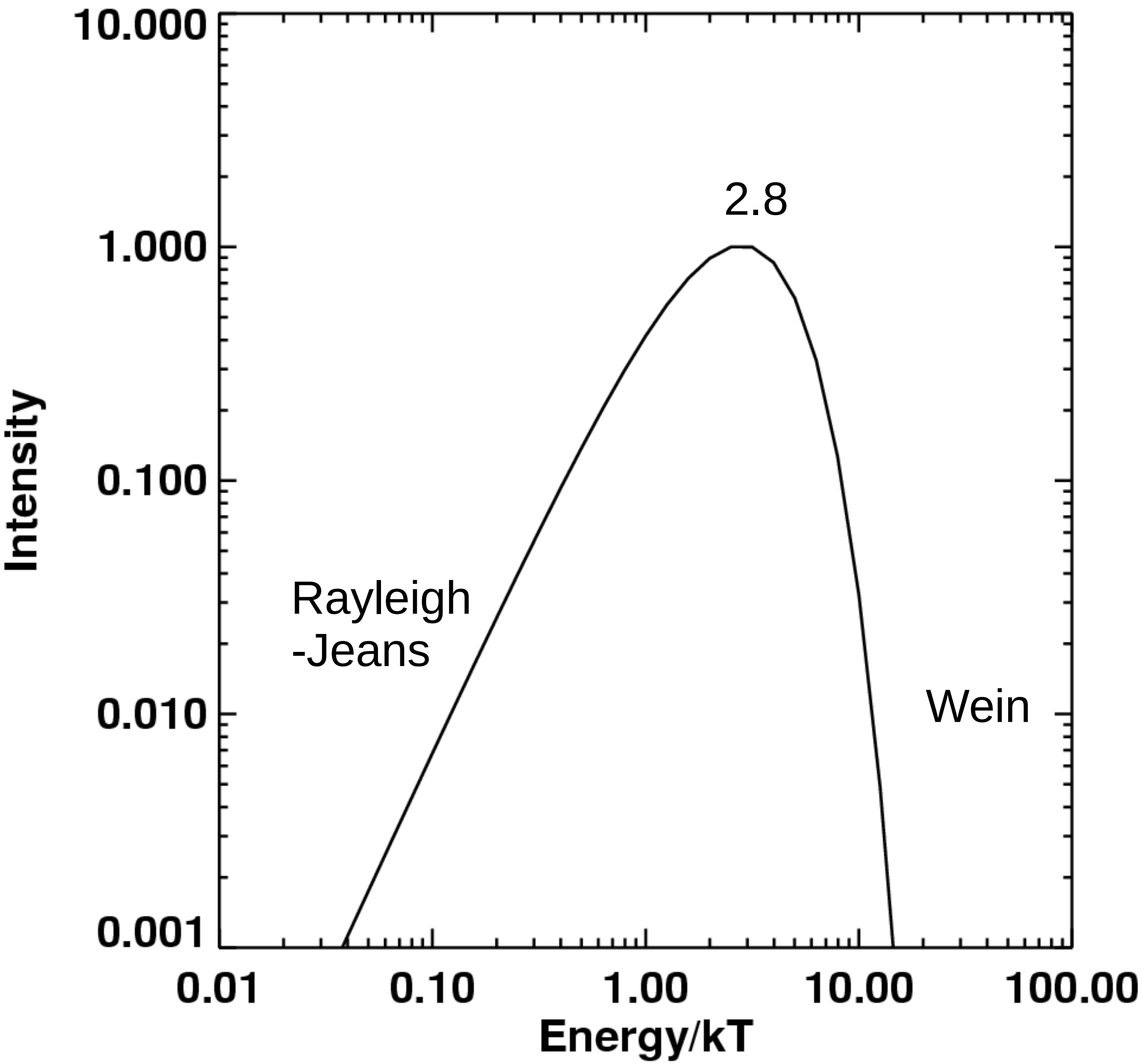
- If we stick a little frame into the cavity and look at the radiation passing through the screen per unit area and unit time over a given range of photon angles, then the intensity of radiation is

$$I_\nu = c\frac{du_\nu}{d\Omega} = c\frac{u_\nu}{4\pi} = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} = B_\nu$$

  Note that blackbody radiation is isotropic.

- Units of I_ν are erg s^{-1} cm^{-2} Hz^{-1} steradian^{-1}. Does that make sense?

Blackbody Radiation



- Intensity of blackbody radiation is

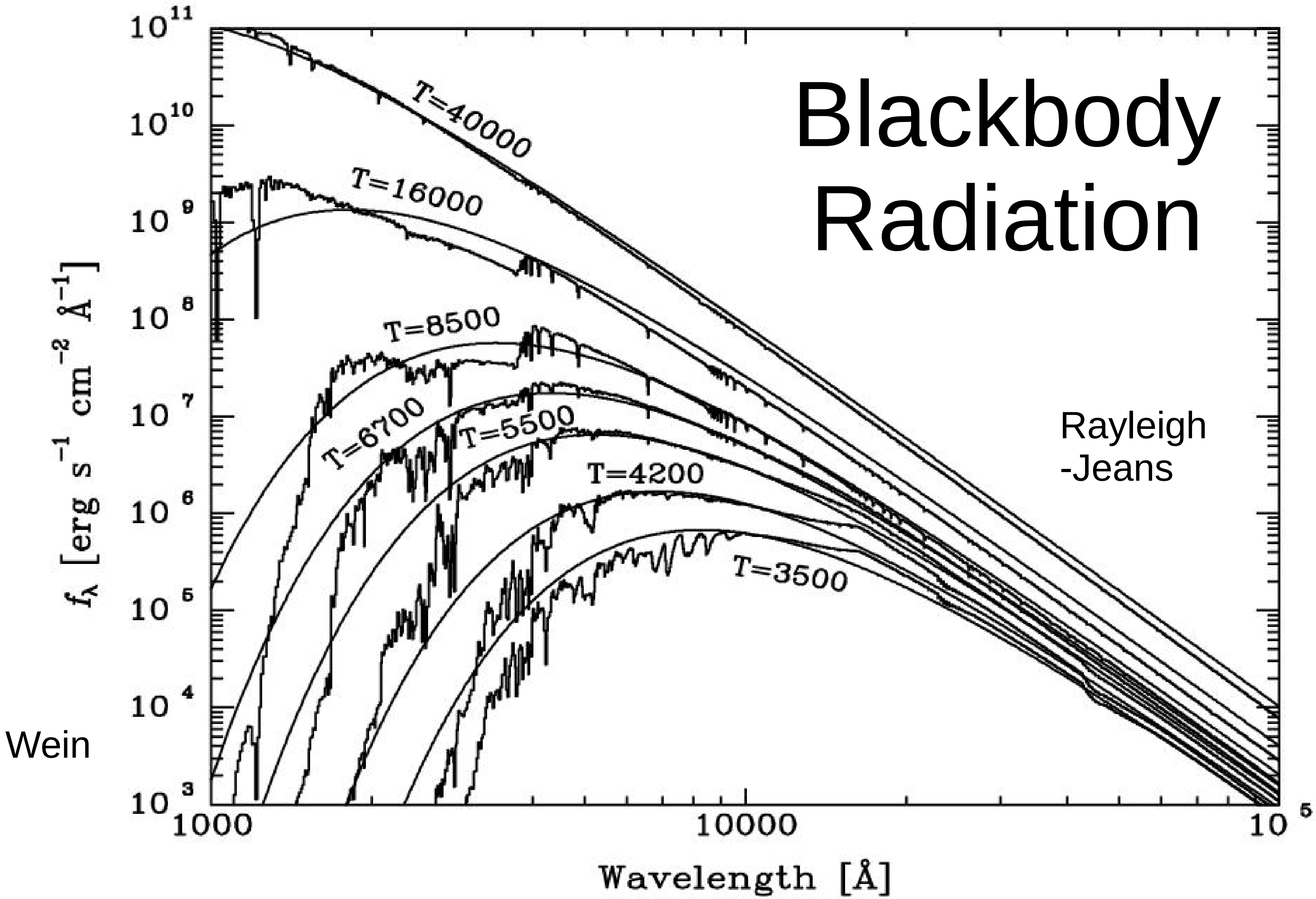
$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

- Falls off at high frequencies = high energies as exp(-$h\nu/kT$). This is the 'Wein tail'.

- Falls off at low frequencies = low energies as ν^2. This is the Rayleigh-Jeans law.

- Spectrum peaks at
  $h\nu_{max}$ = 2.8 kT = 2.4 eV $T/10^4$ K
  $\lambda_{max}\ T$ = 0.29 cm K
  (How to find peak?)

Blackbody Radiation



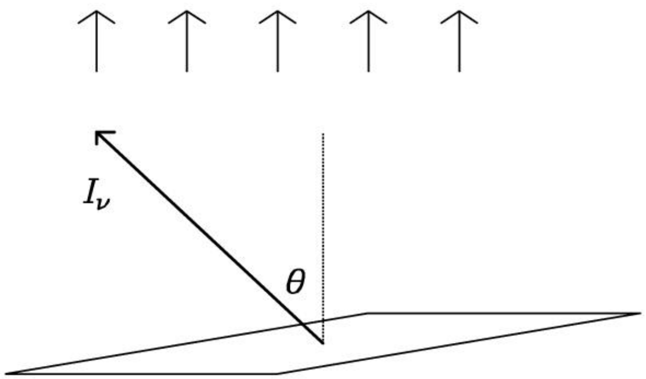
- Optical astronomers look at their spectra backwards, i.e. as a function of wavelength

$$B_\lambda = B_\nu \left| \frac{d\nu}{d\lambda} \right| = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Blackbody Radiation

- Now let's look at emission from the surface of a spherical, isotropically emitting object.
- We need to average over all outgoing angles θ. The flux density is then

$$f_\nu = \int_{\theta=0}^{\theta=\pi/2} I_\nu \, d\Omega = \frac{1}{2} I_\nu 2\pi = \frac{2\pi h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$



- Flux density describes flux at a particular frequency.

- To find the luminosity density, we integrate over the surface of the star:

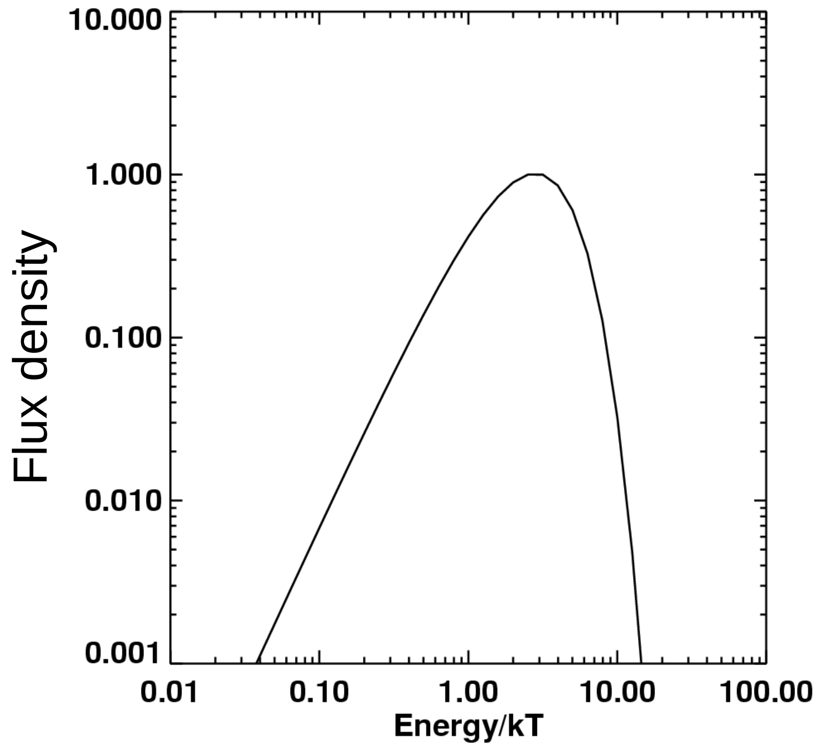
$$L_\nu = f_\nu(r_*) 4\pi r_*^2$$

- Note that we have been talking only about the astrophysical object with no reference to the observer. The observed flux density for an observer at a distance d from a star with radius r_* is:

$$f_\nu(d) = f_\nu(r_*) \frac{r_*^2}{d^2}$$

Blackbody Radiation

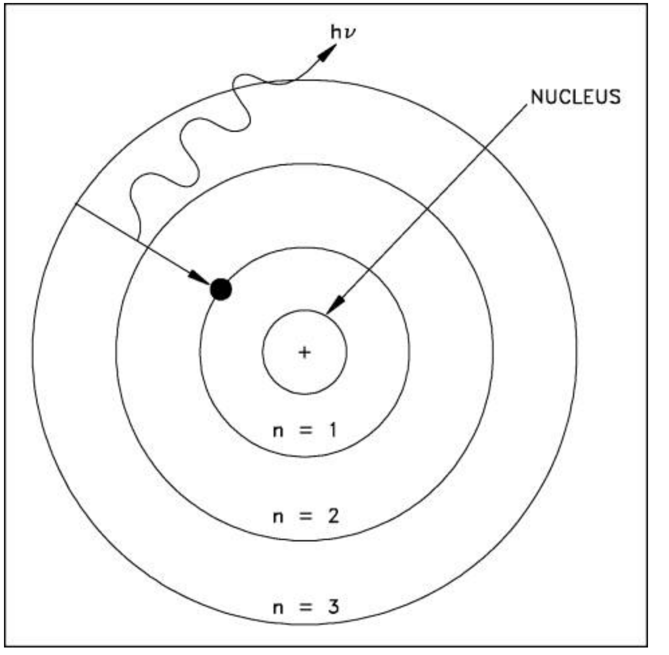
- Flux density describes flux at a particular frequency.
- Flux = flux density integrated over some frequency range.
- Bolometric flux = flux density integrated over all frequencies.



- Have luminosity density and luminosity in the same way. Sometimes, people are loose about 'luminosity' versus 'bolometric luminosity' – assuming that all radiation comes in their measurement band.
- Stefan-Boltzman law relates bolometric luminosity, surface area, and temperature:

$$L = 4\pi r_*^2 \sigma T^4$$

Atomic Spectra

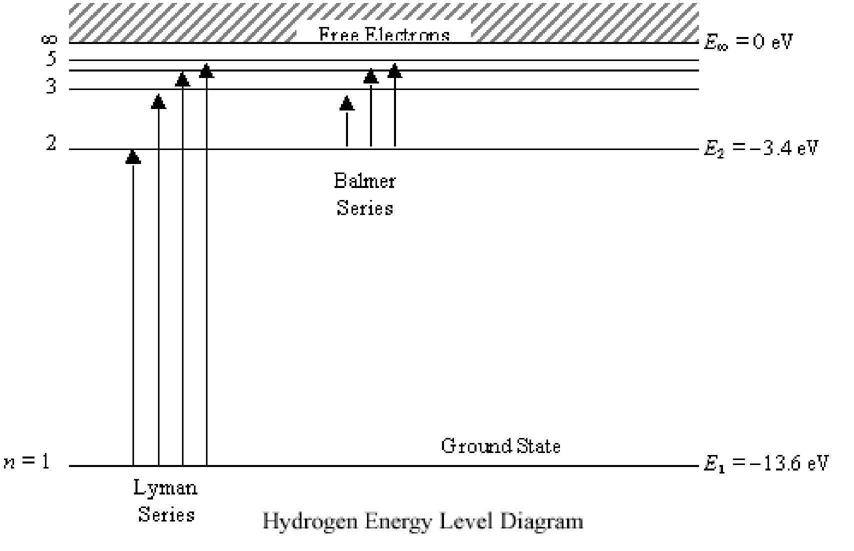


- Electrons orbit around the nucleus only at fixed energy levels. Thus, the emission and absorption spectrum of any atom consists of a discrete set of lines.

- Energy levels in hydrogen: $E_n = -13.6\,\mathrm{eV}\,\frac{1}{n^2}$

- Energy difference between levels = photon energy:

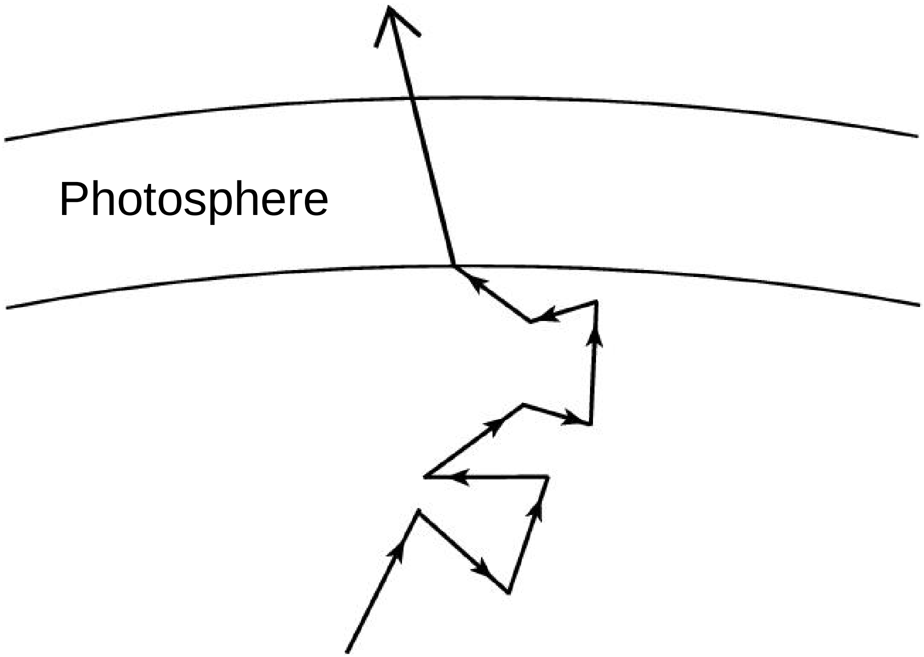
$$E_{n,m} = 13.6\,\mathrm{eV}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$



- Series of lines are named by *m*:
  Lyman *m* = 1, Balmer *m* = 2, Paschen *m* = 3, …

- Balmer lines are in optical, so get special names:
  Hα = 3 ↔ 2 = 6563 Å
  Hβ = 4 ↔ 2 = 4861 Å
  Hγ = 5 ↔ 2 = 4340 Å
  Balmer contiuum = ∞ ↔ 2 < 3646 Å

Photosphere

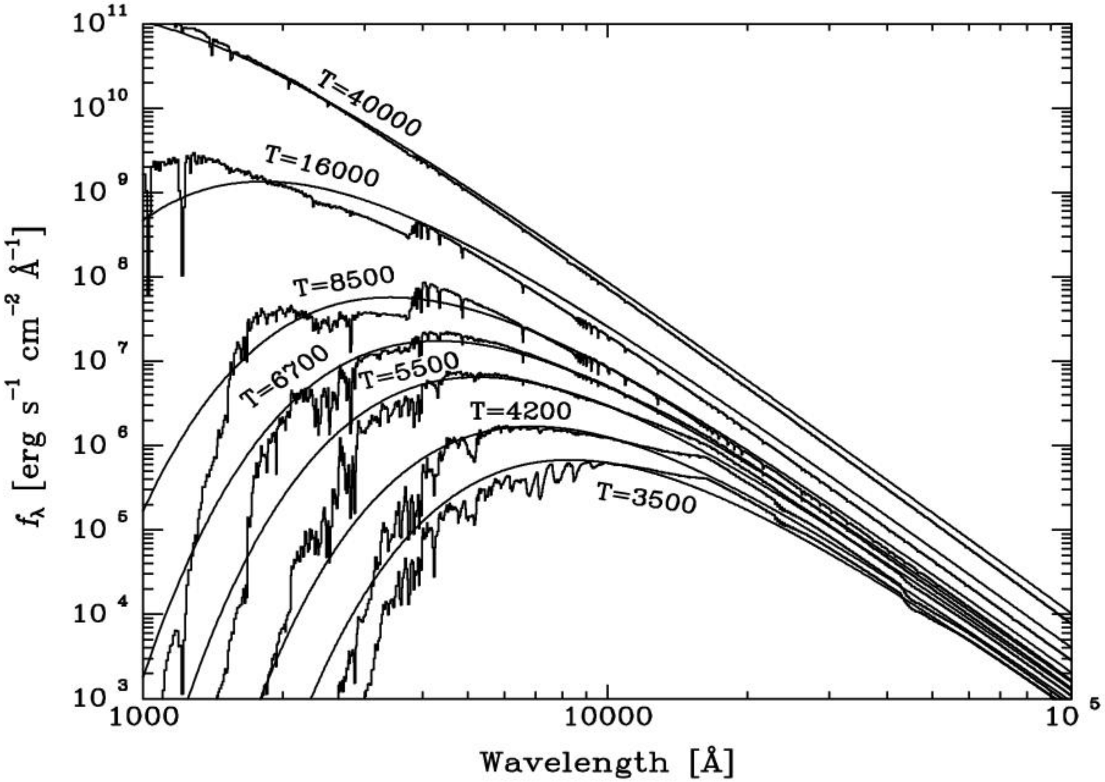


- The temperature of a star varies with radius.
- A photon produced in the interior undergoes many scatterings before it finally leaves the star.
- The surface of a star is not opaque – one can see to some depth into the star.

- The photosphere is the region where photons can escape without further scattering.
- The depth of the photosphere depends on the photon wavelength since the scattering cross-section does.

- The 'color temperature' of a star is the temperature of the blackbody that best fits the stellar spectrum.

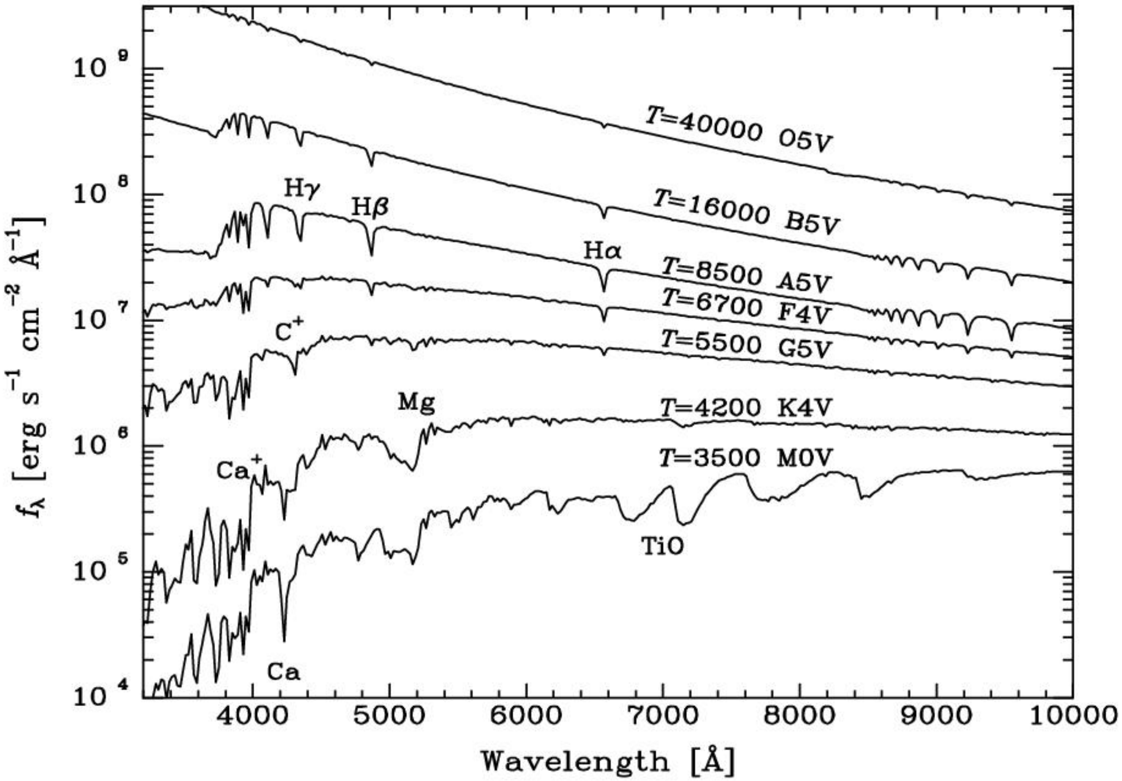
Spectral Types



- Stars are classified according to their surface or color temperature.
- Spectral types are OBAFGMK with a digit 0 to 9 in order from hottest (O1) to coolest (K9).

- A Roman numeral is added to the classification to indicate size:
  I = giant to V = dwarf.

Spectral Types



- Atomic spectral lines produced in the photosphere also depend on the temperature of the photosphere and provide another means to classify stars.

- A stars have strong Balmer absorption lines. Cooler and hotter stars do not. Why?

Homework

- For next class:
  - Problem 2-1