Outline

• Hand in, go over homework problems 1.1, 1.2
• Use of cgs units
• Blackbody radiation
• Atomic spectra
• Stellar spectra
  – Photosphere
  – Absorption lines
  – Spectral types
Units cgs vs SI

- Maxwell's equations in cgs contain $c$ and do not contain $\varepsilon_0$ nor $\mu_0$.

- Charge in cgs measured in statcoulombs with units of $g^{1/2} \text{ cm}^{3/2} \text{ s}^{-1}$.

- 'erg' = unit of energy.

- cgs units fit in your pocket.

<table>
<thead>
<tr>
<th>Name</th>
<th>Gaussian units</th>
<th>SI units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss's law (macroscopic)</td>
<td>$\nabla \cdot \mathbf{D} = 4\pi \rho_f$</td>
<td>$\nabla \cdot \mathbf{D} = \rho_t$</td>
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<td>Gauss's law (microscopic)</td>
<td>$\nabla \cdot \mathbf{E} = 4\pi \rho$</td>
<td>$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0$</td>
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<tr>
<td>Gauss's law for magnetism:</td>
<td>$\nabla \cdot \mathbf{B} = 0$</td>
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<td>Maxwell–Faraday equation (Faraday's law of induction):</td>
<td>$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$</td>
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<td>$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{J}_f + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}$</td>
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Other basic laws

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<tr>
<td>Lorentz force</td>
<td>$\mathbf{F} = q \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right)$</td>
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<td>Coulomb's law</td>
<td>$\mathbf{F} = \frac{q_1 q_2}{r^2} \hat{r}$</td>
<td>$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$</td>
</tr>
<tr>
<td>Electric field of stationary point charge</td>
<td>$\mathbf{E} = \frac{q}{r^2} \hat{r}$</td>
<td>$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$</td>
</tr>
<tr>
<td>Biot-Savart law</td>
<td>$\mathbf{B} = \frac{1}{c} \oint \frac{I d\mathbf{l} \times \hat{r}}{r^2}$</td>
<td>$\mathbf{B} = \frac{\mu_0}{4\pi} \oint \frac{I d\mathbf{l} \times \hat{r}}{r^2}$</td>
</tr>
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</table>
Blackbody Radiation

• Energy density inside a cavity filled with blackbody radiation with the walls of the cavity at temperature $T$ is
  \[ u_\nu = \frac{8 \pi \nu^2}{c^3} \frac{h \nu}{e^{h \nu/kT} - 1} \]

• If we stick a little frame into the cavity and look at the radiation passing through the screen per unit area and unit time over a given range of photon angles, then the intensity of radiation is
  \[ I_\nu = c \frac{du_\nu}{d \Omega} = c \frac{u_\nu}{4 \pi} = \frac{2 h \nu^3}{c^2} \frac{1}{e^{h \nu/kT} - 1} = B_\nu \]

Note that blackbody radiation is isotropic.

• Units of $I_\nu$ are erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ steradian$^{-1}$. Does that make sense?
Blackbody Radiation

- Intensity of blackbody radiation is
  \[ B_\nu = \frac{2 h \nu^3}{c^2} \frac{1}{e^{h \nu/kT} - 1} \]

- Falls off at high frequencies = high energies as \( e^{-h \nu/kT} \). This is the 'Wein tail'.

- Falls off at low frequencies = low energies as \( \nu^2 \). This is the Rayleigh-Jeans law.

- Spectrum peaks at
  \[ h \nu_{\text{max}} = 2.8 kT = 2.4 \text{ eV} \ T/10^4 \text{ K} \]
  \[ \lambda_{\text{max}} = 0.29 \text{ cm} \ K \]
  (How to find peak?)
Optical astronomers look at their spectra backwards, i.e. as a function of wavelength:

\[ B_\lambda = B_\nu \left| \frac{d\nu}{d\lambda} \right| = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \]
Blackbody Radiation

- Now let's look at emission from the surface of a spherical, isotropically emitting object.
- We need to average over all outgoing angles $\theta$. The flux density is then

$$f_\nu = \int_{\theta=0}^{\theta=\pi/2} I_\nu \, d\Omega = \frac{1}{2} I_\nu 2 \pi \frac{2 \pi \hbar \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

- Flux density describes flux at a particular frequency.

- To find the luminosity density, we integrate over the surface of the star:

$$L_\nu = f_\nu (r_*) 4 \pi r_*^2$$

- Note that we have been talking only about the astrophysical object with no reference to the observer. The observed flux density for an observer at a distance $d$ from a star with radius $r_*$ is:

$$f_\nu (d) = f_\nu (r_*) \frac{r_*^2}{d^2}$$
Blackbody Radiation

- Flux density describes flux at a particular frequency.
- Flux = flux density integrated over some frequency range.
- Bolometric flux = flux density integrated over all frequencies.
- Have luminosity density and luminosity in the same way. Sometimes, people are loose about 'luminosity' versus 'bolometric luminosity' – assuming that all radiation comes in their measurement band.
- Stefan-Boltzman law relates bolometric luminosity, surface area, and temperature:

\[ L = 4\pi r_\star^2 \sigma T^4 \]
Atomic Spectra

- Electrons orbit around the nucleus only at fixed energy levels. Thus, the emission and absorption spectrum of any atom consists of a discrete set of lines.

- Energy levels in hydrogen: $E_n = -13.6 \text{ eV} \frac{1}{n^2}$

- Energy difference between levels = photon energy:
  
  $$E_{n,m} = 13.6 \text{ eV} \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

- Series of lines are named by $m$:
  Lyman $m = 1$, Balmer $m = 2$, Paschen $m = 3$, ...

- Balmer lines are in optical, so get special names:
  
  Hα = 3 ↔ 2 = 6563 Å
  Hβ = 4 ↔ 2 = 4861 Å
  Hγ = 5 ↔ 2 = 4340 Å
  Balmer continuum = $\infty$ ↔ 2 < 3646 Å
The temperature of a star varies with radius.

A photon produced in the interior undergoes many scatterings before it finally leaves the star.

The surface of a star is not opaque – one can see to some depth into the star.

The photosphere is the region where photons can escape without further scattering.

The depth of the photosphere depends on the photon wavelength since the scattering cross-section does.

The 'color temperature' of a star is the temperature of the blackbody that best fits the stellar spectrum.
Spectral Types

- Stars are classified according to their surface or color temperature.
- Spectral types are OBAFGMK with a digit 0 to 9 in order from hottest (O1) to coolest (K9).
- A Roman numeral is added to the classification to indicate size: I = giant to V = dwarf.
Spectral Types

- Atomic spectral lines produced in the photosphere also depend on the temperature of the photosphere and provide another means to classify stars.

- A stars have strong Balmer absorption lines. Cooler and hotter stars do not. Why?
Homework

• For next class:
  - Problem 2-1