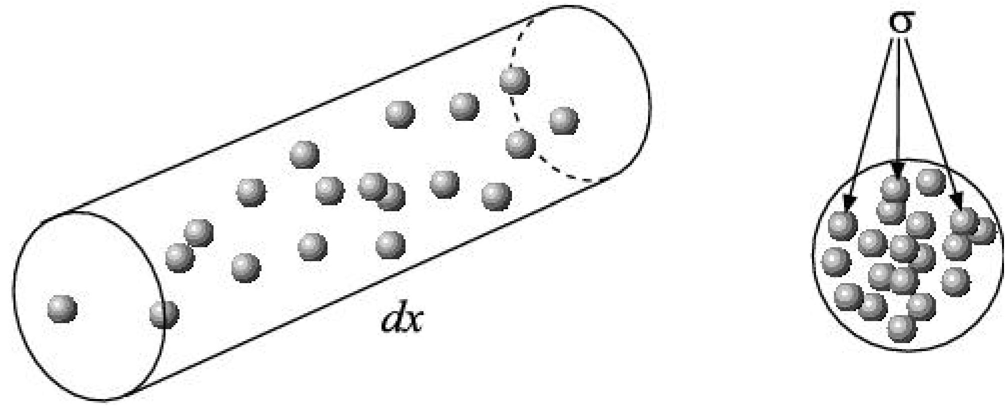
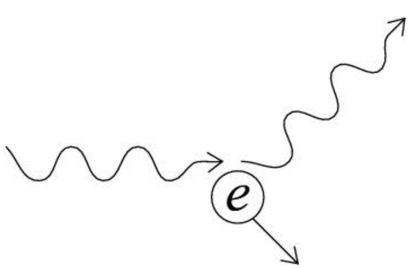
Outline

- Hand in, go over homework problem 3.1
- Proof of virial theorem
- Radiative Energy Transport

Cross Section

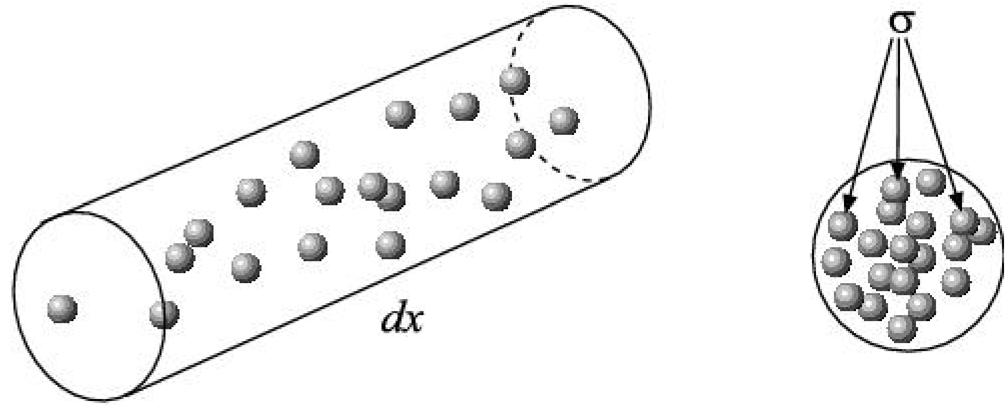


- Think about scattering of a point particle off of spherical targets.
  - Scattering is more likely for larger targets, probability $\propto$ area.
- We characterize the targets via their “cross section” = σ, units of cm^2.
- Note that cross section usually has nothing to do with the physical size of the particle, but instead with the strength of the interaction.
- The cross section for a photon scattering on a free electron is the Thomson cross section:



$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.7\times 10^{-25}\,\mathrm{cm}^2$$

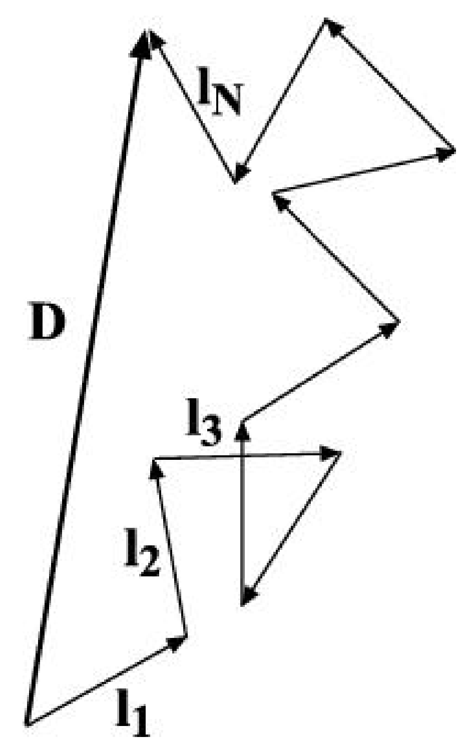
Mean Free Path



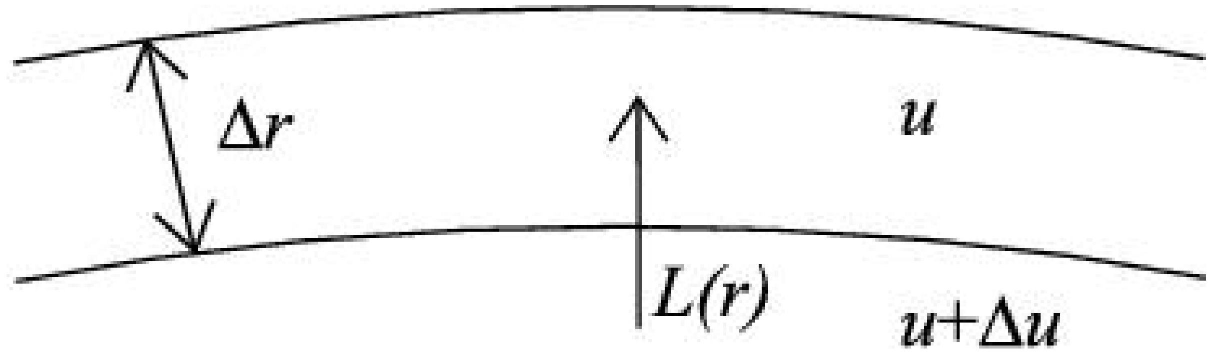
- If the number density of scatters is *n*, then the typical number of interactions that a particle will undergo while traversing a distance *dx* is # interactions = $n\,\sigma\,dx$.
- We define the mean free path as the distance over which there is typically one interaction: $$l = \frac{1}{n\sigma}$$
- For multiple scatterers, add up the contributions: $$l = \frac{1}{\sum n_i \sigma_i} = \frac{1}{\rho\kappa}$$
- The opacity, κ, relates density to scattering, has units of $cm^2\ g^{-1}$, and can depend on the local element abundance, temperature, and density.

Escape of Photons from the Sun

- For the Sun, we can make a very rough guess at the mean free path.
  - Assume all gas is hydrogen and all electrons are free (fully ionized).
  - Then $n \sim \rho/m_H$ and $l \sim 1/\sigma n \sim m_H/\rho\sigma_T \sim 2$ cm
- Density varies strongly with radius, so the mean free path also varies strongly with radius. There are more scatterings in denser regions, so the 'average' mean free path is more like $l \sim 1$ mm.
- Photons undergo a 'random walk', since each scattering sends the photon in a roughly random direction. The distance from the origin increases as $D \sim l \cdot \mathrm{sqrt}(N)$. For Sun $N \sim (R_{Sun}/l)^2 \sim 10^{24}$, time for escape $\sim Nl/c \sim R_{Sun}{}^2/lc \sim 10^{12}$ s.



Radiative Diffusion



- Since mean free path is small, each radial shell of the Sun is essentially a blackbody with $T = T(r)$.
- Energy density $u = aT^4$, so higher energy density at smaller radii.
- Rate of energy flow through a shell = L(r) = energy difference between shells at r versus $r+\Delta r$ divided by time for energy to flow distance Δr:

$$L(r) \approx \frac{-4\pi r^2 \Delta r \, \Delta u}{(\Delta r)^2 / lc} = -4\pi r^2 l c \frac{\Delta u}{\Delta r}$$

- Converting to differentials and doing the proper average over photon directions:

$$\frac{L(r)}{4\pi r^2} = -\frac{c\, l}{3} \frac{du}{dr}$$

A diffusion equation:
flux = - coefficient × gradient

Radiative Energy Transport

- Translate this into an equation for $T(r)$.
- Energy density $u = aT^4$:
$$\frac{du}{dr} = \frac{du}{dT}\frac{dT}{dr} = 4aT^3\frac{dT}{dr}$$
- Write mean free path in terms of opacity: $l = 1/\kappa\rho$.
- Then radiation diffusion equation can be rewritten as:
$$\frac{dT(r)}{dr} = \frac{-3L(r)\kappa(r)\rho(r)}{16\pi r^2 a c T^3(r)}$$
- This is our second equation of stellar structure.

Homework

- For next class:
  - Problem 3-3