Outline

- Hand in, go over homework problem 3.3
- Mass continuity
- Energy conservation
- Equations of stellar structure
- Equation of state, average particle mass, radiation pressure

Mass Conservation

- How does mass vary versus radius?
- Think of building up mass in radial shells,
  - amount of mass = volume of shell density at radius.
- Assume no net flow of material.

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

Energy Conservation

- How does energy flow through star?
- Define ε(r) as power produced per unit mass of stellar material.
- Assume net flow of energy is outward.

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \epsilon(r)$$

Equations of Stellar Structure

$$\frac{dP(r)}{dr} = \frac{-GM(r)\rho(r)}{r^2} \qquad \frac{dT(r)}{dr} = \frac{-3L(r)\kappa(r)\rho(r)}{16\pi r^2 a c T^3(r)}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \qquad \frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \epsilon(r)$$

- These are four coupled first-order differential equations.
- Need four boundary conditions to specify a unique solution:
- $M(0) = 0$, $L(0) = 0$, $P(\mathrm{r}_*) = 0$, $M(r_*) = M_*$.
- This is the Vogt-Russell theorem: the properties and evolution of a star are set by its (initial) mass (and chemical abundances).

Ancillary Equations

- The four equations of stellar structure do not provide enough information to solve the problem. We also need to understand the physics of how the gas (plasma) in the Sun behaves. Note these are all local equations and, in principle, can be measured via experiments done on Earth.
  - $P = P(\rho, T, \text{composition})$ – equation of state
  - $\kappa = \kappa(\rho, T, \text{composition})$ – opacity
  - $\varepsilon = \varepsilon(\rho, T, \text{composition})$ – energy generation
- The composition is the abundance of each element. This is usually simplified to H, He, and everything else (called metals):
  - $X \equiv \rho_H/\rho$, $Y \equiv \rho_{He}/\rho$, $Z \equiv \rho_{metals}/\rho$.
- What assumptions have we made about stellar structure?

Equation of State

- The (classical, nonrelativistic) ideal gas law is a good approximation for the equation of state in most normal stars:
  - $P = nkT = (\rho/\overline{m})\, kT$
- Need to calculate average mass of particles. This depends on composition
  - $\overline{m} = \sum n_i m_i \,/\, \sum n_i = \rho/n$
  - For fully ionized H gives 2 particles, He gives 3 particles.
  - For a heavy element with atomic number = Z, there are Z+1.
  - For heavy elements, $Z \approx A/2$, where A = atomic mass.

Equation of State

- Use $X+Y+Z = 1$ to find:

$$n = 2n_H + 3n_{He} + \sum \frac{A}{2} n_A = \frac{\rho}{m_H}\left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right) = \frac{\rho}{2m_H}\left(3X + \frac{Y}{2} + 1\right)$$

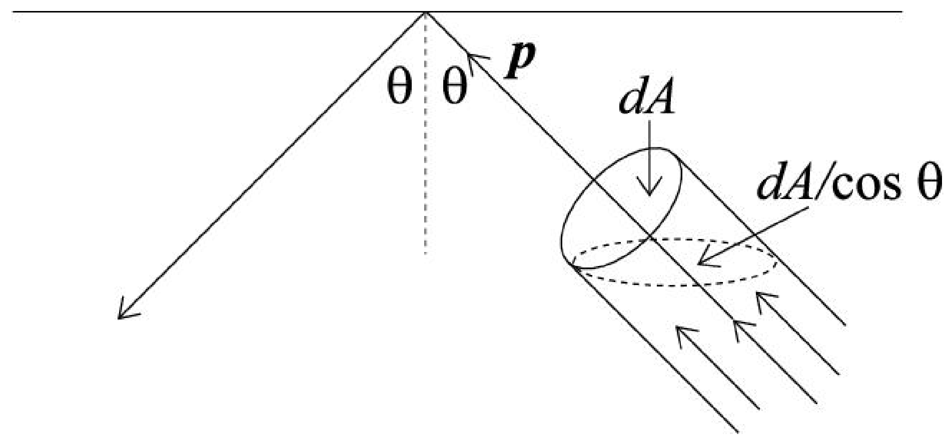
- Then for a fully ionized gas:

$$\frac{\bar{m}}{m_H} = \frac{\rho}{nM_H} = \frac{2}{1+3X+0.5Y}$$

- Pure H (X = 1, Y = 0, Z = 0) → $\bar{m}/m_{\mathrm{H}}$ = 0.5.
- Solar abundances (X = 0.71, Y = 0.27, Z = 0.02) → $\bar{m}/m_{\mathrm{H}}$ = 0.61.
- Core of Sun (X = 0.34, Y = 0.64, Z = 0.02) → $\bar{m}/m_{\mathrm{H}}$ = 0.85.

Radiation Pressure

- Since the gas is the star locally emits as a blackbody, the blackbody photons will produce radiation pressure.
- Radiation intensity integrated over all frequencies and directions is *B*.
- Find the momentum imparted to a perfectly reflecting surface of area *A*.
- For a photon, $p = E/c$, incident at angle θ, the photon surface density is reduced by a factor of cos(θ). The momentum transfer is $\Delta p = (2E/c)$ cos(θ).



$$P = \frac{F}{A} = \frac{dp/dt}{A} = \frac{2}{c}\int B\cos^2\theta\, d\Omega = \frac{2}{c}B(2\pi)\int_0^{\pi/2}\cos^2\theta\sin\theta\, d\theta = \frac{4\pi}{3c}B$$

Radiation Pressure

- We can relate this to the energy density and the temperature using the relations derived for blackbody radiation:

$$P_{rad}=\frac{4\pi}{3c}B=\frac{1}{3}u=\frac{1}{3}aT^4$$

- The total pressure is then

$$P=P_{gas}+P_{rad}=\frac{\rho kT}{\bar{m}}+\frac{1}{3}aT^4$$

- In normal stars, the gas (“kinetic”) pressure usually dominates.

Homework

- For next class:
  - Problems 3-2, 3-4