Outline

- Finish scaling relations on main sequence
- Hand in, go over homework problems 3.5
- Nuclear energy production

Scaling Relations

$$\frac{L}{L_\odot} \approx .23\left(\frac{M}{M_\odot}\right)^{2.3} \qquad (M < .43M_\odot)$$

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{4} \qquad (.43M_\odot < M < 2M_\odot)$$

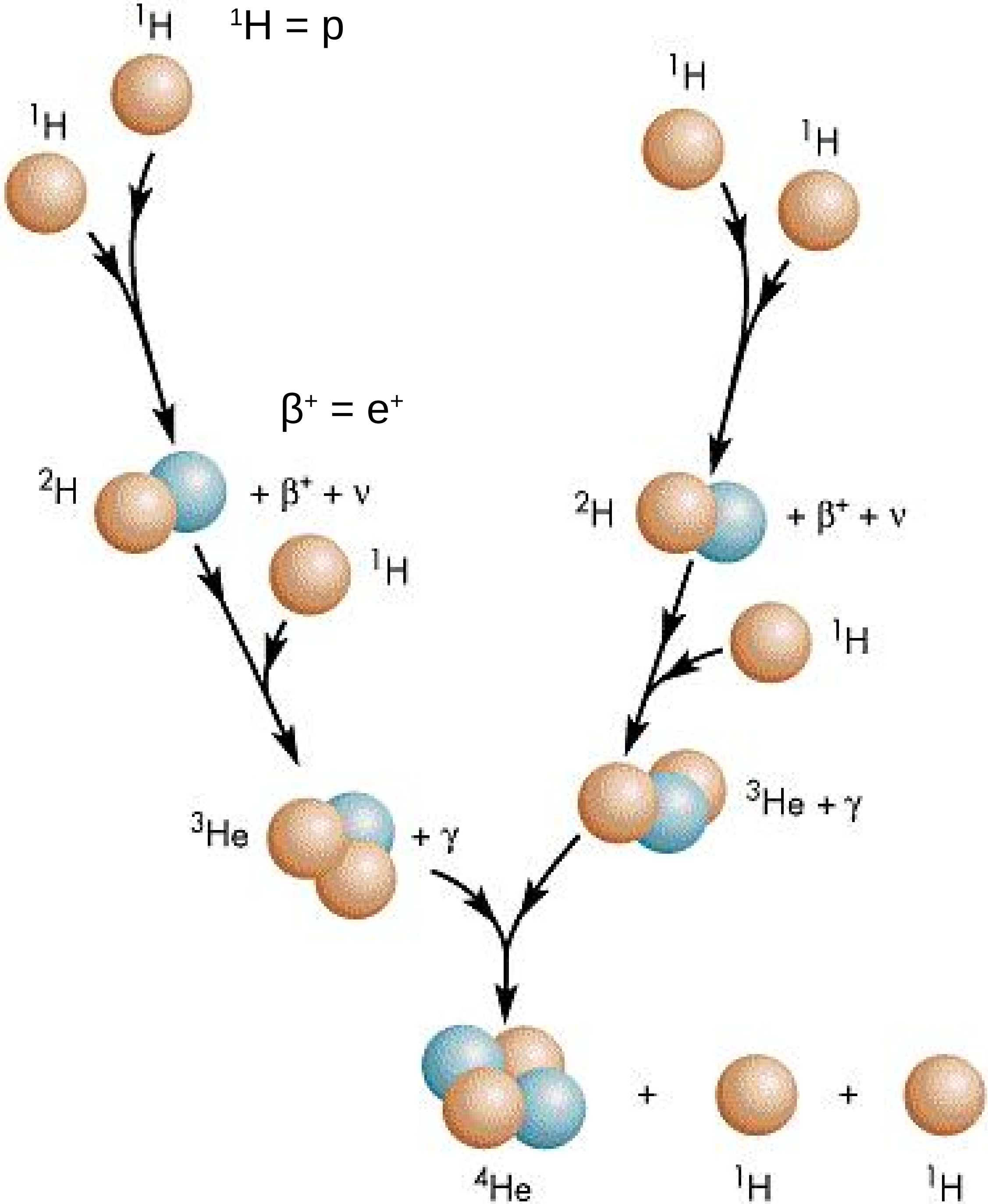
$$\frac{L}{L_\odot} \approx 1.5\left(\frac{M}{M_\odot}\right)^{3.5} \qquad (2M_\odot < M < 20M_\odot)$$

$$\frac{L}{L_\odot} \approx 3200\frac{M}{M_\odot} \qquad (M > 20M_\odot)$$

Nuclear Energy Production

- Last equation of stellar structure is energy production
- Release of gravitational energy makes a negligible contribution
  - Calculate how long Sun could shine at current luminosity via release of gravitational energy
  - $E \sim GM^2/R$
  - $T \sim E/L \sim 2\times10^7$ yr ("Kelvin-Helmholtz time scale")

p-p chain



Produces 99% of energy in the Sun

p-p chain

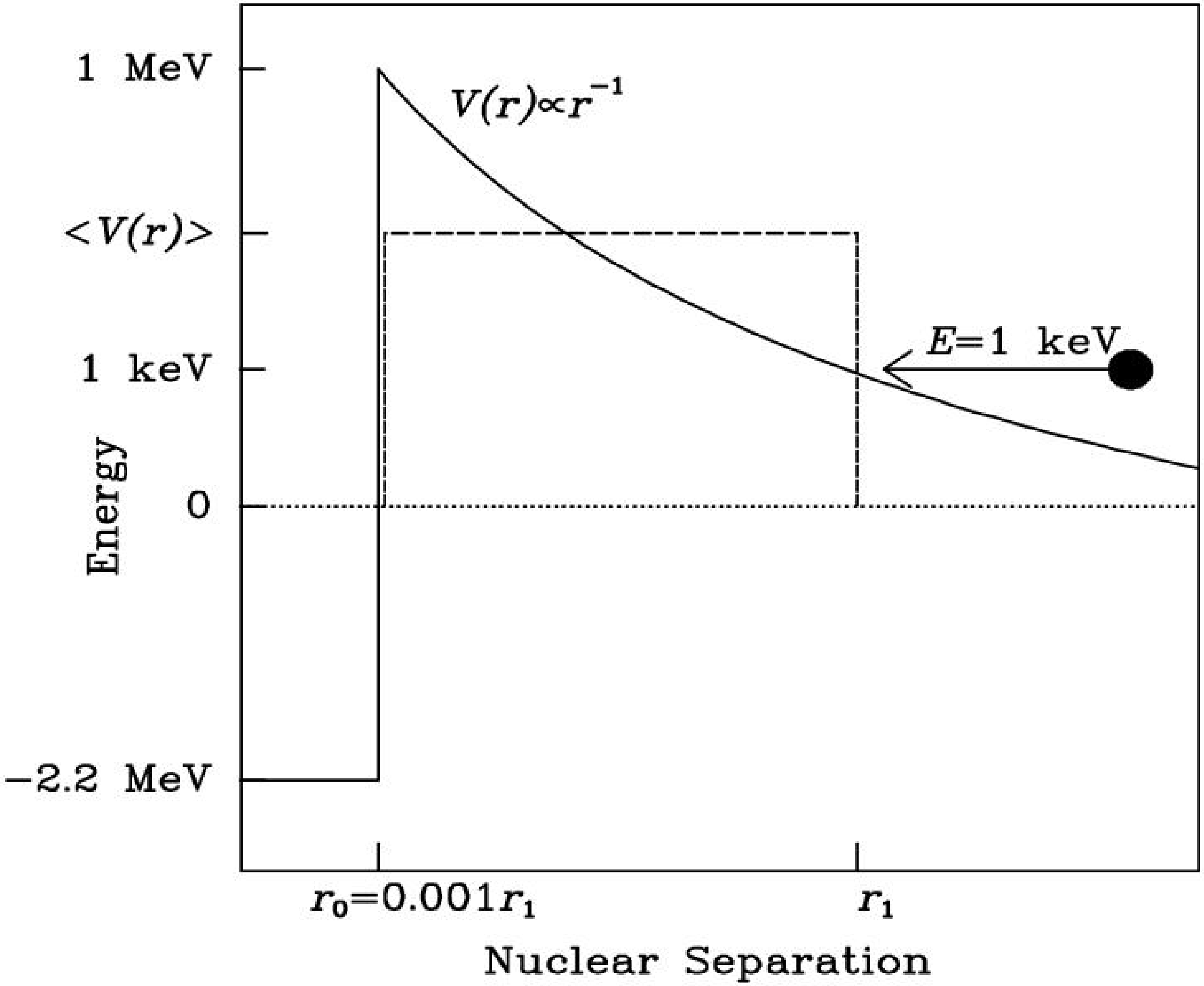
- First step: $p + p \rightarrow d + e^{+} + \nu_e$
  - Weak interaction – time scale ~ 10^{10} yr in Sun's core
  - Energy released = 0.425 MeV
  - Neutrino escapes Sun, average energy = 0.26 MeV
  - Positron quickly annihilates on ambient electron
    - $e^{+} + e^{-} \rightarrow \gamma + \gamma$, each γ has energy of ~0.511 MeV
- Second step: $p + d \rightarrow {}^{3}\mathrm{He} + \gamma$
  - Electromagnetic interaction, time scale ~ 1s in Sun's core
  - Energy released = 5.49 MeV
- Third step ${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + p + p$
  - Strong interaction, time scale ~ 300,000 yr in Sun's core
  - Energy released = 12.86 MeV

p-p chain

- Net energy release (in MeV)
  - $2 \times 0.425 + 4 \times 0.511 - 2 \times 0.511 + 2 \times 5.49 + 12.86 = 25.71$
  - This equals the difference in rest mass energy between 4 protons and one ^{4}He nucleus
  - This is 0.7% of the rest mass of four protons
  - Efficiency of mass to energy conversion is 0.7%

- Assume Sun can fuse 10% of its mass
  - Energy available = $0.1 \times 0.007 \times Mc^2 = 1.3 \times 10^{51}$ erg
  - Time scale $T \sim E/L \sim 10^{10}$ yr

p-p chain



Forces acting on nuclei

- Strong force is attractive, but operates only over short distances,

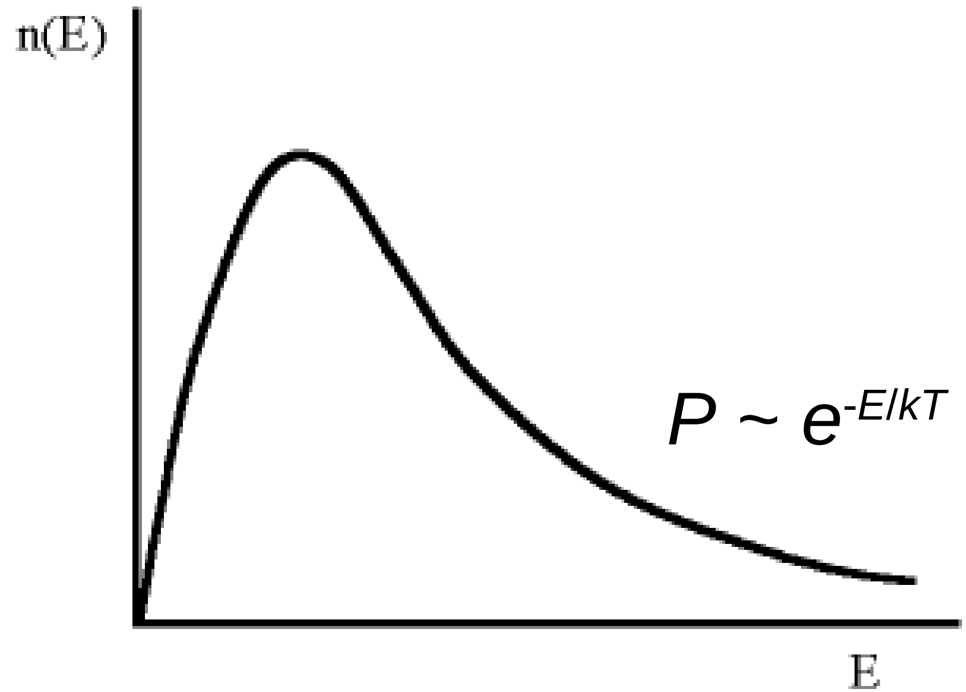
  $r_0 \approx 1.4\times10^{-13}$ cm.

- Coulomb force is repulsive, acts over large distances.

- Coulomb energy barrier $E_{coul} = Z_A Z_B e^2/r$
- Closest approach $r_1 = Z_A Z_B e^2/E_k \approx 10^{-10}$ cm for $E_k \approx 1$ keV ($T \approx 10^7$ K)
- Need 1000× average energy to get close enough for strong force.

p-p chain

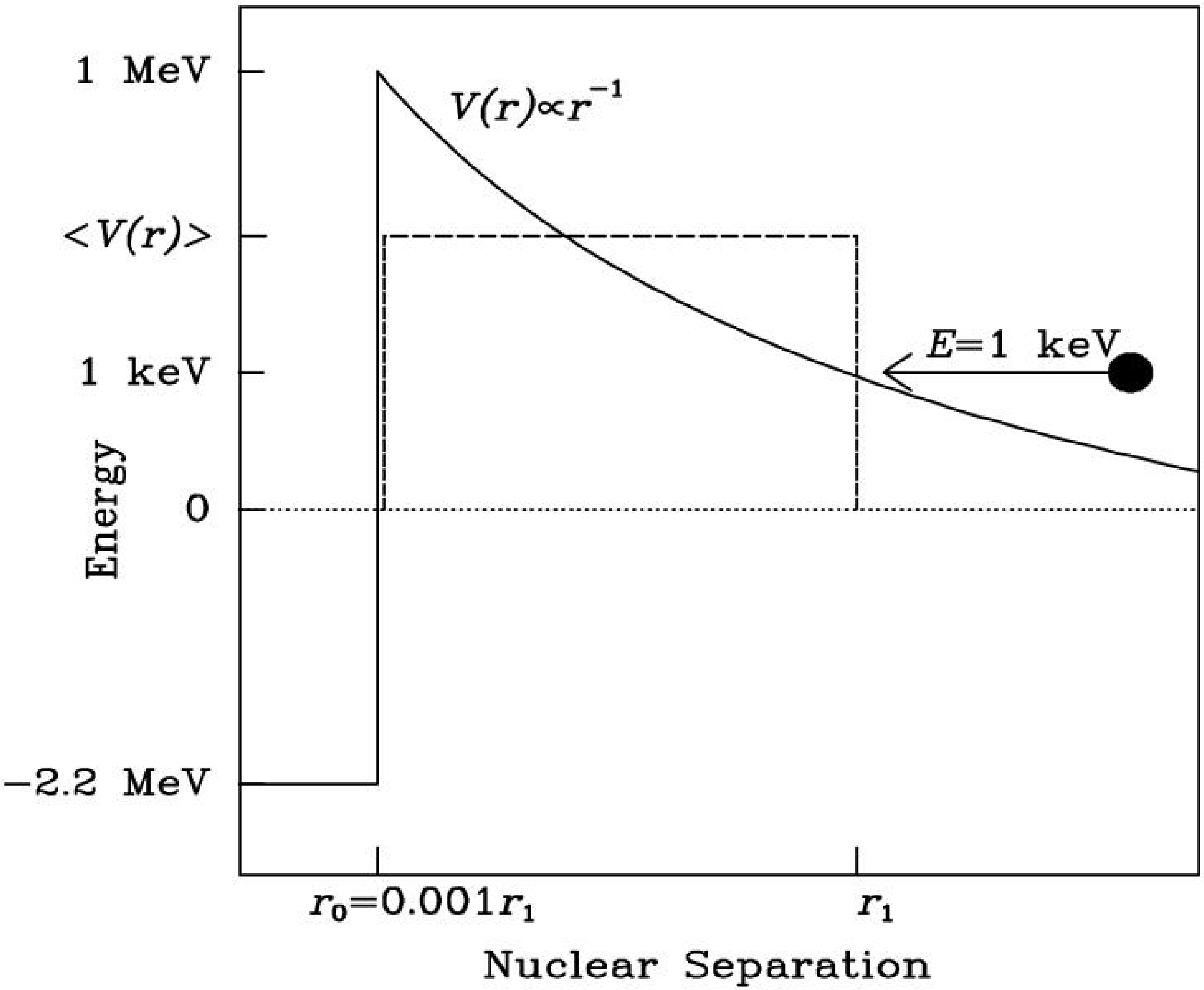
- Need 1000× average energy to get close enough for strong force.
- Maxwell-Boltzmann distribution does have a tail extending to high energies, but probability of state of energy E being occupied at a temperature T is $P \sim e^{-E/kT}$
  - $e^{-1000} \sim 10^{-434}$
- # protons in Sun ~ 10^{57}
- Hence, there are zero protons in the Sun with sufficient energy to undergo fusion.



Maxwell-Boltzmann distribution

p-p chain

- Quantum-mechanical tunneling.
- Approximate barrier as rectangular, $V \sim 3/2\ V(r_1)$
- Solve Schrödinger equation for tunneling through a rectangular barrier.



- For two protons in core of Sun, tunneling probability ~ 10^{-10}.
- Recall number of protons in Sun ~ 10^{57}

Homework

- For next class:
  - Problem 3-6