Outline

- Go over problem 5-4.
- Do problems on following pages.
- Exam will be draw from material in Chapters 4 and 5. There will also probably be a dimensional analysis problem.
- Note that the homework problems covered some topics not covered in the attached problems: tides, orbital evolution of binaries, dealing with distributions like dN/dm, Boltzmann factor, ...

White Dwarf

- A typical white dwarf has a density of 10^6 g/cm^3, $Z/A \sim 0.5$, and a temperature of 10^7 K.
- Estimate the classical thermal pressure within the white dwarf.
- Estimate the pressure due to degenerate electrons within the white dwarf.
- Equating the classical thermal pressure with the pressure due to degenerate electrons, derive an expression for the conditions under which electrons become degenerate. Compare with the condition we derived in class based on when the separation of particles is comparable to their de Broglie wavelength.

Pressure of a classical gas

- Use the Maxwell-Boltzmann distribution

$$n(p)\,d^3\vec{p} \quad = \quad 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{p^2}{m^3}\,e^{-p^2/2mkT}\,dp$$

- to calculate the pressure of a classical idea gas

$$P \;=\; \frac{1}{3}\int_0^{\infty} n(p)\,p\,v\,dp$$

and derive the ideal gas law $P = nkT$.

- Hints: write integrals in terms of the dimensionless $x = p^2/2mkT$,

Note $n \;=\; \int_0^{\infty} n(p)\,dp$

integrate by parts to find $\int_0^{\infty} x^{3/2} e^{-x}\,dx = \frac{3}{2}\int_0^{\infty} x^{1/2} e^{-x}\,dx$

Degenerate gas

- Show that the equation of state for an ultra-relativistic degenerate electron gas is

$$P = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4 m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}$$

Degenerate gas

- Starting from the virial theorem and the equation of state for an ultra-relativistic degenerate electron gas, find an expression for the maximum mass of a white dwarf (the Chandrasekhar mass).

Supernova

- The 1 solar mass core of a 10 solar mass star collapses, creating a supernova. Assume 1% of the gravitational potential energy released by the collapsing core is transferred to the outer envelope of the star and ejects the outer 9 solar masses of material to infinity.
- Calculate the maximum possible final radius of the 1 M_{sun} core that would provide enough energy to power the ejection.
- Clearly state any additional assumption you need to make.

Supernova

- The neutrino flux (integrated over time) from SN 1987A was 1.3×10^{10} cm^{-2} and the average neutrino energy was 4.2 MeV.
- Estimate the amount of energy released in neutrinos by SN 1987A.
- Calculate the gravitational binding energy of a 1.4 M_{sun} neutron star with a radius of 12 km. Compare to the energy released in neutrinos by SN 1987 A.

Pulsars

- Given that the rate of energy loss, L, of a rotating dipole magnet is proportional to the fourth power of the spin frequency, $L \sim \omega^4$.
- A) Find a differential equation for ω as a function of time.
- B) Integrate the equation to find the age of the pulsar in terms of its current spin frequency, ω, and its initial spin frequency, ω_i.

Black holes

- Calculate the average density of black holes with the following masses: 1 Earth mass, 1 solar mass, 10^6 solar masses, 10^{12} solar masses.
- In the Newtonian approximation, find the tidal force between the feet and head of a human at the event horizon of each black hole. (The human is pointing her feet directly at the black hole.)

Black holes

- A) Using the Schwarzschild metric, find an expression for the coordinate speed of light in the φ direction.
- B) The coordinate speed for an object in circular orbit around a massive object is $v^2 = GM/r$. Use this fact and your result from part A to show that photons can make circular orbits around a black hole at 1.5 R_S.

Accretion Disks

- A) Starting from the virial theorem, derive an expression for the dependence of temperature on radius in an accretion disk.
- B) Derive an expression for the amount of luminosity (dL/dr) radiated in each annulus as a function of radius.

Eddington Luminosity

- η Carinae is a luminous blue variable star (LBV) with a mass of 120 solar masses. Currently, its luminosity is $5\times10^6\ L_{sun}$.
- During the "Great eruption" lasting 20 years, η Car had a luminosity of $2\times10^7\ L_{sun}$ and ejected 3 M_{sun} of material at a speed of 650 km/s.
- Calculate the Eddington luminosity of η Car and compare to its current luminosity and luminosity during the eruption. Is the star's behavior reasonable?
- Estimate the power of the mass outflow during the eruption and compare to the luminosity.

Boltzmann Factor

- Significant numbers of photons are emitted from an excited state only when the occupation number of excited state is reasonably high. Consider two states with an energy difference $\Delta E = h\nu$.
- A) Using the Boltzman factor, find the temperature that produces as occupation number ratio, $n_2/n_1 = 0.1$, for a given energy difference. You may neglect the statistical weights.
- B) How does the temperature in part A compare with the blackbody temperature that would produce a peak at $h\nu$?