General Astronomy - Spring 2011
Home work #8 - Due March 28

1. Calculate the fractional gravitational redshift for light emitted from the surface of a star with a mass of 10 solar masses and a radius of one Earth radius.

The ratio of the observed to original frequency is

$$\frac{\nu'}{\nu} = \sqrt{1 - \frac{2GM}{Rc^2}} = \sqrt{1 - \frac{2(6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}})(2 \times 10^{31}\,\mathrm{kg})}{(6.4 \times 10^6\,\mathrm{m})(3 \times 10^8\,\mathrm{m\,s^{-1}})^2}} = 0.9977$$

The fraction change in frequency is

$$\frac{\Delta\nu}{\nu} = \frac{\nu' - \nu}{\nu} = -2.3 \times 10^{-3}$$

2. What is the Schwarzschild radius of a 3,000,000 solar mass black hole

The Schwarzschild radius scales linearly with black hole mass and is 2.95 km for a one solar mass black hole. So, the Schwarzschild radius of a 3,000,000 solar mass black hole is 3,000,000×2.95 km or about 9,000,000 km. It would fit easily inside the orbit of Mercury.

3. What is the maximum possible amount of energy which could be released by dropping 1 kg of hydrogen gas onto a maximally rotating black hole?

The total energy equivalent of 1 kg of hydrogen gas is $E = mc^2 = (1.0\,\mathrm{kg}) \times (3.0 \times 10^8\,\mathrm{m/s})^2 = 9.0 \times 10^{16}$ J. A maximum of 42% of this could be released by dropping the gas into a maximally rotating black hole, or 3.8×10^{16} J.

To put this in perspective the US annual energy consumption is about 10^{20} J. Therefore, this amount of energy would power the whole US for about 3.3 hours. (If one threw a Chevy Tahoe into the black hole, it could power the US for a whole year.)

4. A black hole is observed at a luminosity of 10^{33} W. What does this tell us about the black hole mass?

This luminosity is equivalent to 2.6×10^6 solar luminosities. Assuming that the black hole does not violate the Eddington luminosity, the black hole mass must be greater than $2.6 \times 10^6 / 3 \times 10^4 = 88$ solar masses.

5. Calculate the peak temperature of the blackbody radiation from a 100 solar mass black hole radiating at the Eddington luminosity.

The Schwarzschild radius of a 100 solar mass black hole is $R = 100 \times 2.95$ km $= 300$ km. The luminosity of a 100 solar mass black hole radiating at the Eddington luminosity is $L = 100 \times 30,000 L_\odot = 1.1 \times 10^{33}$ W. The Stefan-Boltzmann law for a spherical object with radius R, luminosity L, and temperature T is $L = 4\pi R^2 \sigma T^4$. Solve for T

$$T = \left(\frac{L}{4\pi\sigma R^2}\right)^{1/4} = \left(\frac{1.1 \times 10^{33}\,\mathrm{W}}{4\pi(5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}})(3 \times 10^5\,\mathrm{m})^2}\right)^{1/4} = 1.1 \times 10^7\,\mathrm{K}$$

The temperature is about 11 million degrees Kelvin. A 100 solar mass black hole is somewhat cooler than a 1 solar mass black hole.