Spherical Coordinates and Astrometry

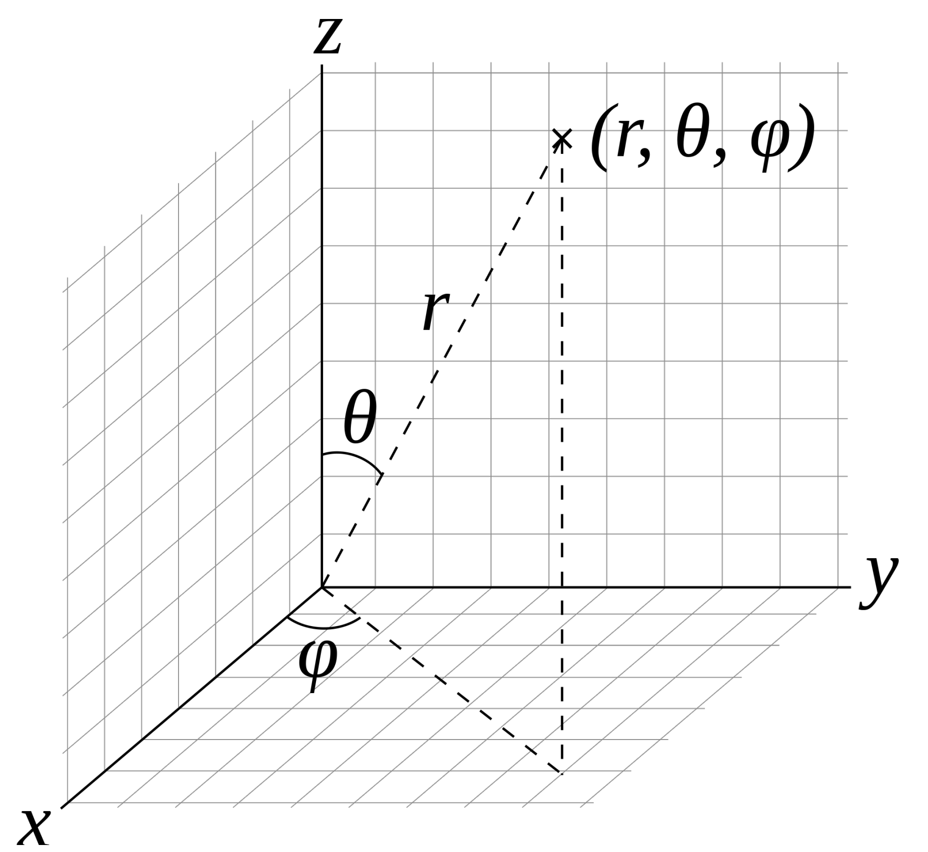
ASTR4850
10/5/2015

Overview

- Definition & Units
- Relation to other coordinate systems
- Using it with Astrometry

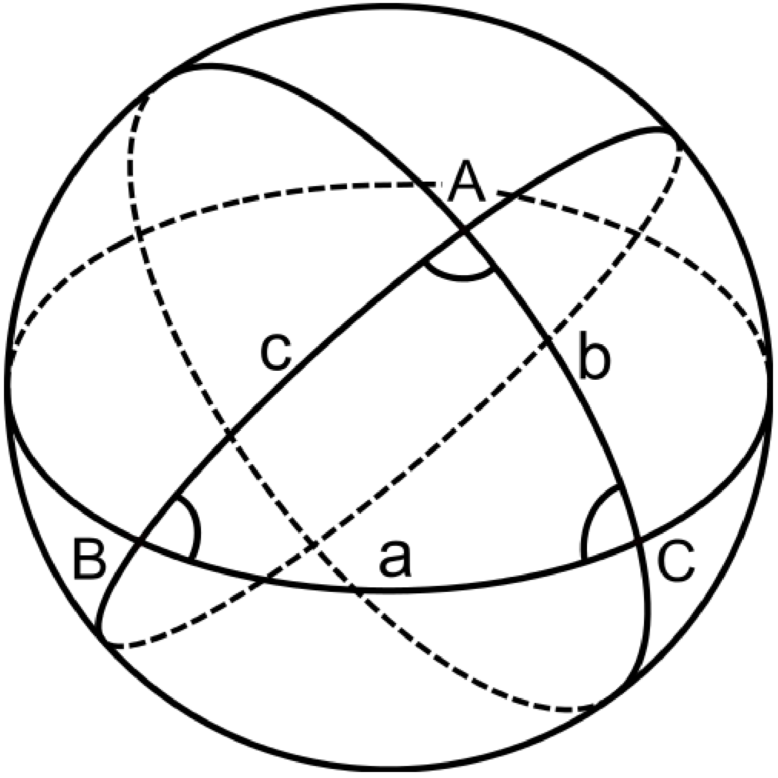
Definition

- Radius = ρ (or r)
- Polar angle or z-inclination angle = θ
- Azimuthal angle = φ
- Why polar angle instead of elevation angle?



Units

- All units in arc (radians, degrees..)
- Lower-case letters just are subtended arcs from the center
- Arcs of least distance between points lie on great circles
- Triangles have a sum of angles >180°



Relation to Terrestrial Coordinates

- Longitude ~ azimuthal angle
- Latitude ~ $\pi/2$ – polar angle
- Radius?

Relation to Celestial Coordinates

- Right ascension (α) = azimuthal angle (φ)
- Declination (δ) = $\pi/2$ – polar angle (θ)
- Radius?

Use with Astrometry

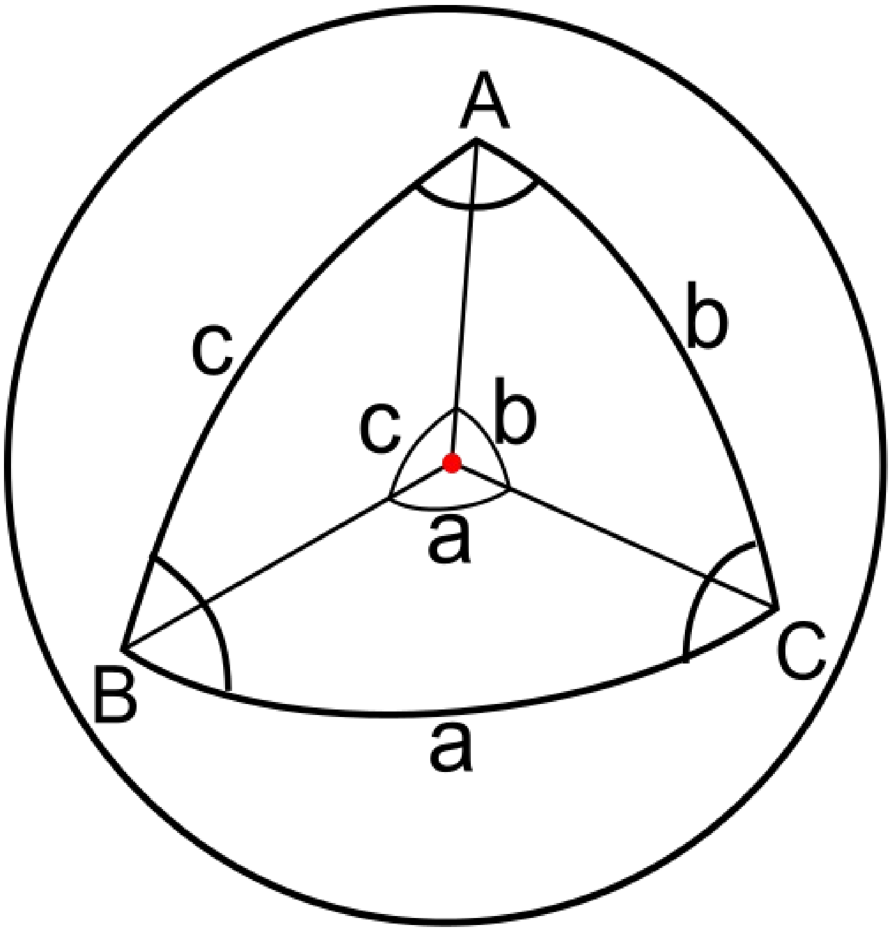
- Knowns:
  - Celestial position of at least two stars (α,δ)
    - Why are at least two star positions necessary, instead of just one?
  - Physical position on a CCD of those two stars (x,y)
  - Relative physical position of a third star on a CCD (x,y)
- Find:
  - Celestial position of the third star (α,δ)

Use with Astrometry, cont.

- One can find spherical angles by using Cartesian conversion (NB: these x,y,z listed aren't physical coordinates of CCD, but Cartesian locations)
  $x=r*\sin(\theta)*\cos(\varphi)$
  $y=r*\sin(\theta)*\sin(\varphi)$
  $z=r*\cos(\theta)$

  Reminder: θ the not the same as declination
- Example: B and C positions are known in RA, Dec, find their angular separation (a)
- Solution: Use dot product with position vectors, in Cartesian coordinates, for B & C.
  |**B**| = |**C**| = r = 1 on the celestial sphere.

  So...
  B●C = |**B**||**C**|*cos(a) = cos(a)
  a = arccos(**B●C**)



Use with Astrometry, cont.

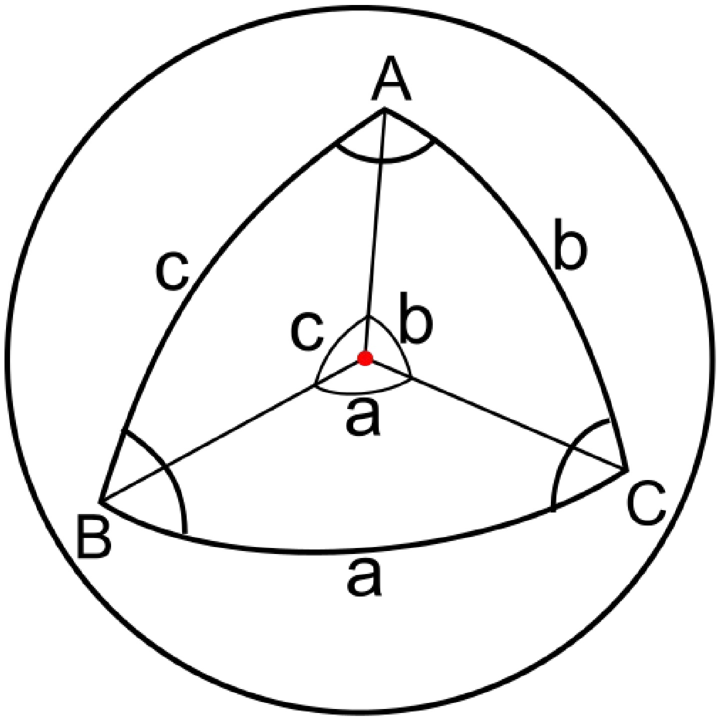
- Example: Find angle B relative to star C and the North Celestial Pole (a useful angle when deciding how your image is rotated compared to celestial coordinates)

  Hint: If we choose A as the NCP, then
  b = 90° - C_Dec
  A = B_RA - C_RA
  a = arccos(**B•C**)

  This only works since we are doing this relative to the NCP. If A was a non-pole, this would not be as straightforward.

- Solution: Use spherical law of sines.
  sin(A)/sin(a) = sin(B)/sin(b) = sin(C)/sin(c)

  B = arcsin{sin(90-C_Dec)*sin(B_RA - C_RA) / sin[arccos(**B•C**)]}



Credits

- Images credits: Peter Mercator (wikimedia)