Outline

• Go over homework
  – What does it mean that the Universe is expanding?
• Equation of state of universe
• Solutions of the Friedmann equations
**Friedmann Equations**

- **First Friedmann equation:** \[ \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G \rho - \frac{k c^2}{R^2} \]

- **Acceleration equation:** \[ \frac{\ddot{R}}{R} = -\frac{4\pi G}{3 c^2} \left( \rho c^2 + 3 P \right) \]

- **Energy conservation:** \[ \dot{\rho} c^2 = -3 \frac{\dot{R}}{R} \left( \rho c^2 + P \right) \]

- **Want to find** \( R(t) \)
  - First need to know dependence of \( \rho \) and \( P \) on \( R \).
  - The equation of state of the universe.
Equation of State of the Universe

• Matter dominated: $\rho c^2 >> P$
  – Use energy conservation to show $\rho \sim R^{-3}$

• Radiation dominated: $P = (1/3)u = (1/3)\rho c^2$
  – Use energy conservation to show $\rho \sim R^{-4}$
Time Evolution of the Universe

- Want to find $R(t)$, use first equation
  \[\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G \rho - \frac{k c^2}{R^2}\]

- $R$ is smaller at earlier times

- Early enough, $\rho \sim R^{-3}$ or $R^{-4}$ will dominate over $R^{-2}$

- Matter dominated: $R \sim t^{2/3}$ (work out on board)

- Radiation dominated: $R \sim t^{1/2}$ (work out on board)

- Radiation dominates at very early times, $R \to 0$, $\rho \to \infty = \text{big bang}$

- Find evolution at later times, keeping $k$ term, on board.
  - Recall $H = \dot{R}/R$
• Find time for transition from radiation to matter dominated and age of universe on board.
Homework

• For next class:
  – Problem 8.1