

# Outline

- Go over homework
  - What does it mean that the Universe is expanding?
- Equation of state of universe
- Solutions of the Friedmann equations

# Friedmann Equations

- First Friedmann equation: 
$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$
- Acceleration equation: 
$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P)$$
- Energy conservation: 
$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)$$
- Want to find  $R(t)$ 
  - First need to know dependence of  $\rho$  and  $P$  on  $R$ .
  - The equation of state of the universe.

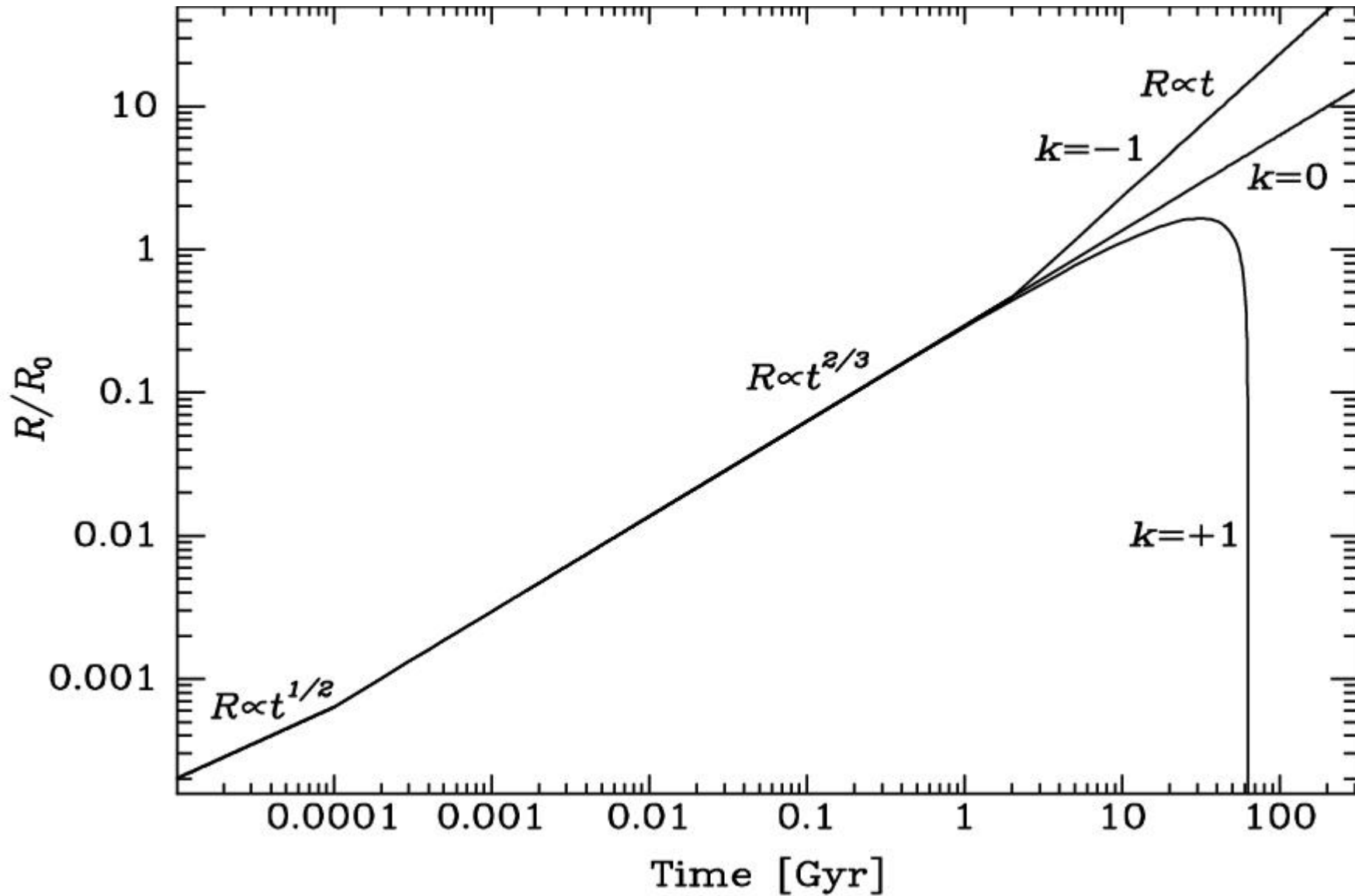
# Equation of State of the Universe

- Matter dominated:  $\rho c^2 \gg P$ 
  - Use energy conservation to show  $\rho \sim R^{-3}$
- Radiation dominated:  $P = (1/3)u = (1/3) \rho c^2$ 
  - Use energy conservation to show  $\rho \sim R^{-4}$

# Time Evolution of the Universe

- Want to find  $R(t)$ , use first equation 
$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$
- $R$  is smaller at earlier times
- Early enough,  $\rho \sim R^{-3}$  or  $R^{-4}$  will dominate over  $R^{-2}$
- Matter dominated:  $R \sim t^{2/3}$  (work out on board)
- Radiation dominated:  $R \sim t^{1/2}$  (work out on board)
- Radiation dominates at very early times,  $R \rightarrow 0$ ,  $\rho \rightarrow \infty$  = big bang
- Find evolution at later times, keeping  $k$  term, on board.
  - Recall  $H = \dot{R}/R$

# Time Evolution of the Universe



- Find time for transition from radiation to matter dominated and age of universe on board.

# Homework

- For next class:
  - Problem 8.1