

Outline

- Go over problem 9.2
- Fluctuations in the Cosmic Microwave Background
Cosmic Microwave Background

- After removing a dipole due to our peculiar motion (600 km/s), the CMB is uniform with root-mean-square fluctuations of $\delta T = 29 \mu K$ or $\delta T/T \sim 10^{-5}$.

- This is actually a problem, but we'll ignore it for now and assume that the universe started off very homogeneous, but with some distribution of small fluctuations in matter/radiation.

- Fluctuations are sound waves: hot/dense versus cool/rarefied.

- How do we describe the fluctuations?
Fourier Transform

- Any periodic function can be described as a sum of sines and cosines (Fourier's theorem).

- The “Power spectrum” shows how strongly the signal fluctuations at a particular frequency or wavelength.

- Perturbations vary at their characteristic frequency.
Describe CMB in terms of correlation function and power spectrum.

Need to do in terms of spherical harmonics, $l = 1$ dipole, $l = 2$ quadrapole, ...

Angular scale $\theta \sim \pi/l = 180^\circ/l$

Specify power at each $l$ as $C_l$
Fluctuations in CMB

- How is the size of a mode related to its frequency?
  - Wavelength $\lambda = c_s \nu$, where $c_s$ is the speed of sound in the medium.
  - Or $\lambda = c_s/\tau$, where $\tau = 1/\nu$ is the period of the mode.

- Early universe was radiation dominated, equation of state: $P = (1/3)\rho c^2$.

- Speed of sound:
  
  $$c_s = \sqrt{\frac{dP}{d\rho}} = \frac{c}{\sqrt{3}}$$

- CMB lets us see the universe at the time of recombination $t_{\text{rec}}$.

- What fluctuations should be largest at that point?
Acoustic Peaks

- Modes with largest amplitude will be those with $\tau = \frac{t_{\text{rec}}}{2}$.
- Hence, $\lambda = \frac{c_s}{\tau} = 2c_s t_{\text{rec}} = \frac{2c t_{\text{rec}}}{\sqrt{3}}$

- This gives us a
  - “ruler” = size of the fluctuations in the CMB
  - at a known distance = distance to when age of universe = $\frac{t_{\text{rec}}}{2}$.

- We can use this to directly measure the geometry of the universe.
Acoustic Peaks

- Fluctuations with wavelengths near \( 2c_s t_{\text{rec}} = 2ct_{\text{rec}}/\sqrt{3} \) should have the largest amplitudes.

- We can use this to directly measure the geometry of the universe by comparing measured angular size of fluctuations to calculated size.

- Calculate expected angular size for \( k=0 \) on board.
Acoustic Peaks

- Peak is at ~0.8° or $l \sim 200$.
- What sets width of peak? Why are there multiple peaks?
- Position of first peak gives $\Omega = 1.02 \pm 0.02$ from WMAP data.
Acoustic Peaks

- All of the wiggles can be modeled using a models of the atomic physics of recombination, the geometry and evolution of the universe, and the spectrum of fluctuations at early times.
- Gives constraints on $\Omega_A$, $\Omega_m$, $\Omega_B$, $t_0$, ...
Homework

For next class: problem 9.3