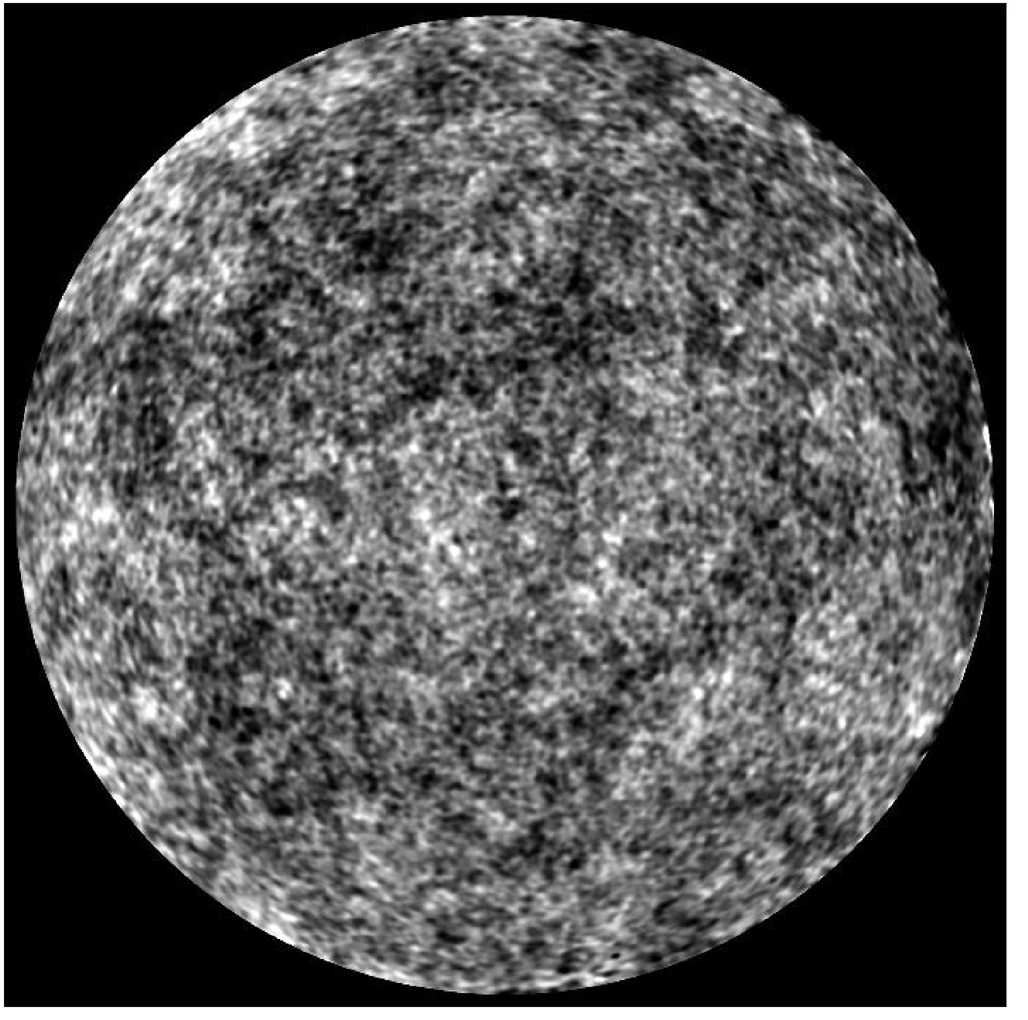
Outline

- Go over problem 9.2
- Fluctuations in the Cosmic Microwave Background

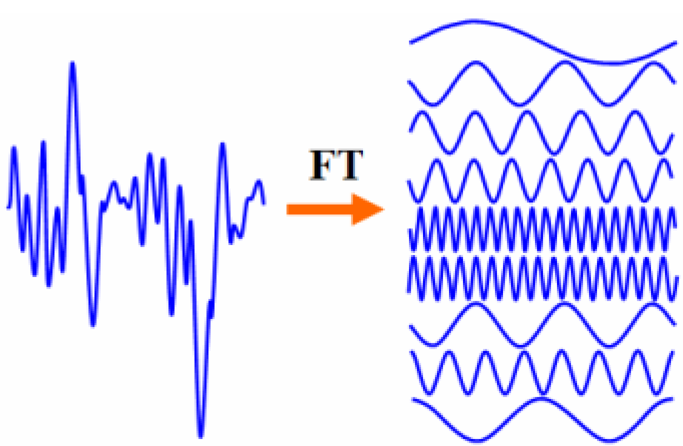
Cosmic Microwave Background

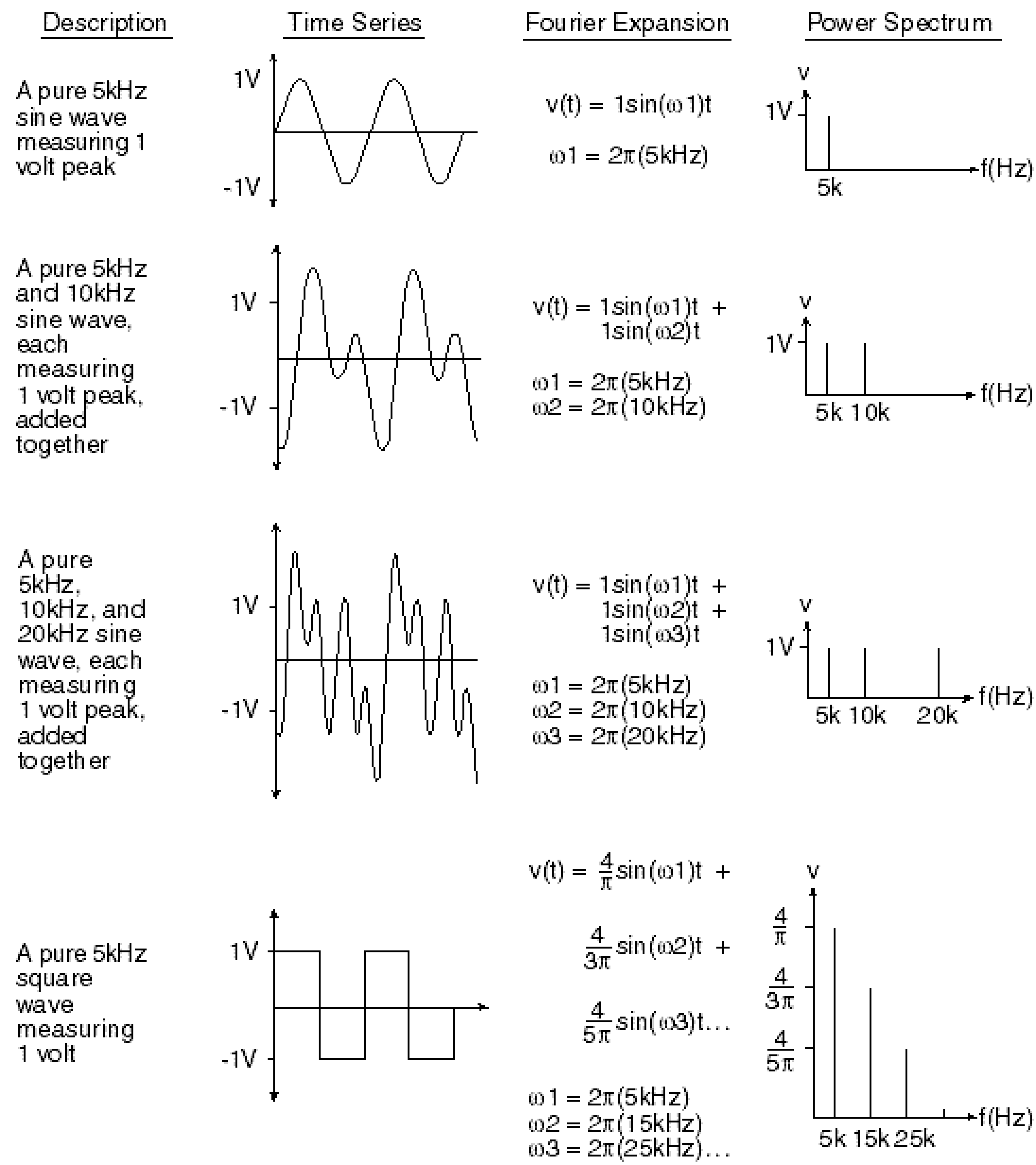
- After removing a dipole due to our peculiar motion (600 km/s), the CMB is uniform with root-mean-square fluctuations of $\delta T = 29\ \mu K$ or $\delta T/T \sim 10^{-5}$.
- This is actually a problem, but we'll ignore it for now and assume that the universe started off very homogeneous, but with some distribution of small fluctuations in matter/radiation.
- Fluctuations are sound waves: hot/dense versus cool/rarefied.
- How do we describe the fluctuations?



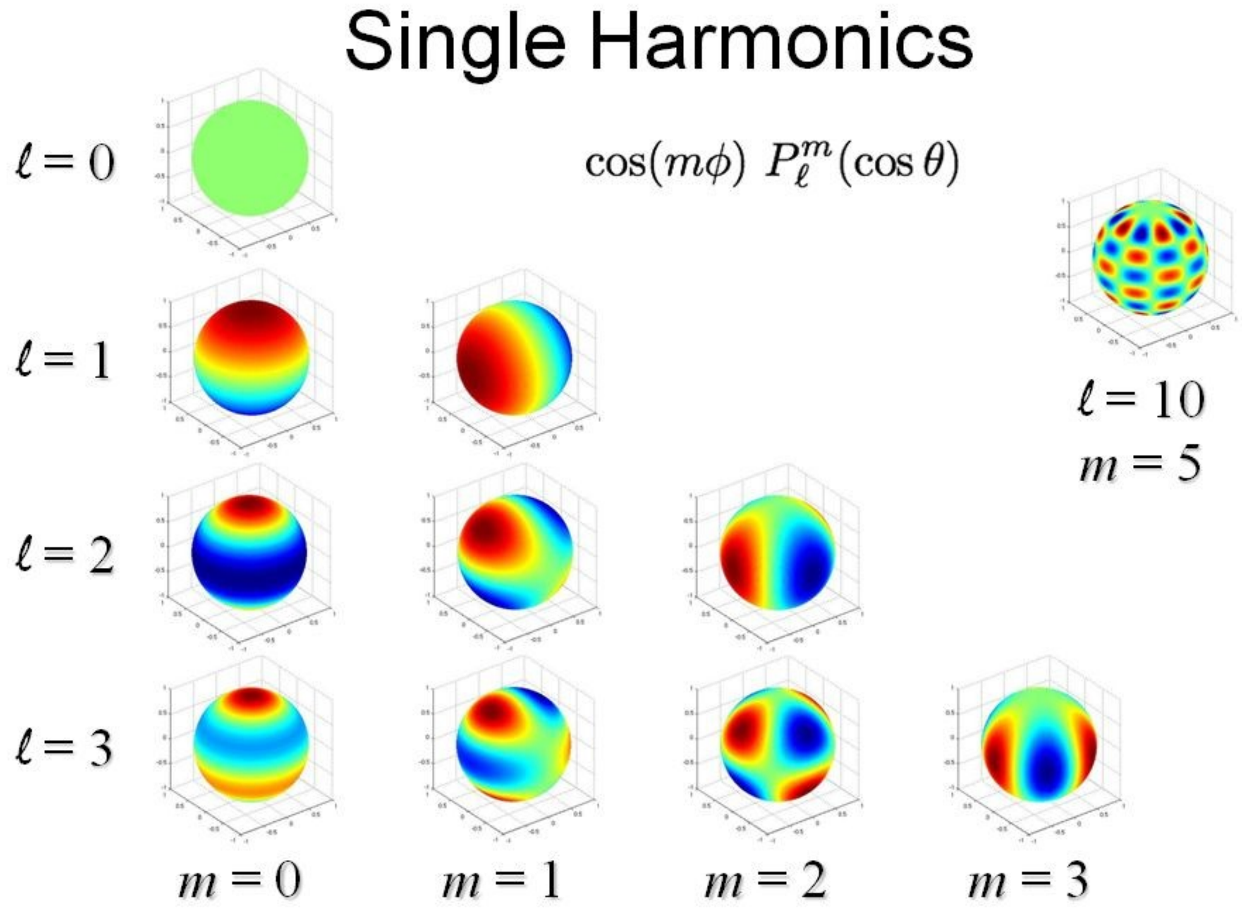
Fourier Transform

- Any periodic function can be described as a sum of sines and cosines (Fourier's theorem).
- The "Power spectrum" shows how strongly the signal fluctuations at a particular frequency or wavelength.
- Perturbations vary at their characteristic frequency.





CMB power spectra



- Describe CMB in terms of correlation function and power spectrum.
- Need to do in terms of spherical harmonics, $l = 1$ dipole, $l = 2$ quadrapole, ...
- Angular scale $\theta \sim \pi/l = 180^\circ/l$
- Specify power at each l as C_l

Fluctuations in CMB

- How is the size of a mode related to its frequency?
  - Wavelength $\lambda = c_s \nu$, where c_s is the speed of sound in the medium.
  - Or $\lambda = c_s/\tau$, where $\tau = 1/\nu$ is the period of the mode.
- Early universe was radiation dominated, equation of state: $P = (1/3)\rho c^2$.
- Speed of sound:

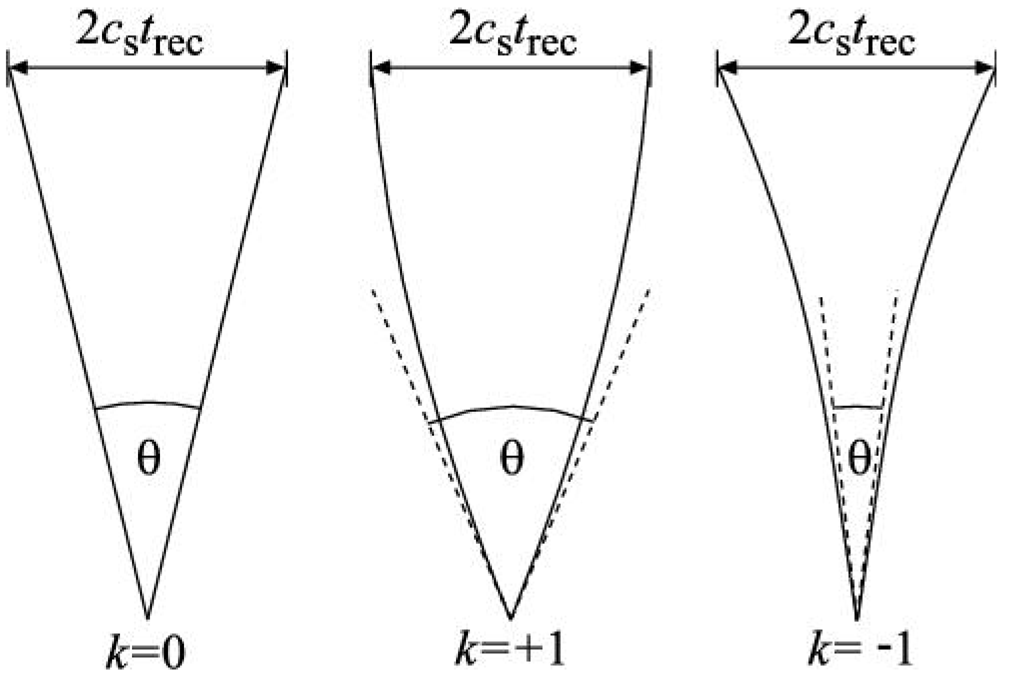
$$c_s = \sqrt{\frac{dP}{d\rho}} = \frac{c}{\sqrt{3}}$$

- CMB lets us see the universe at the time of recombination t_{rec}.
- What fluctuations should be largest at that point?

Acoustic Peaks

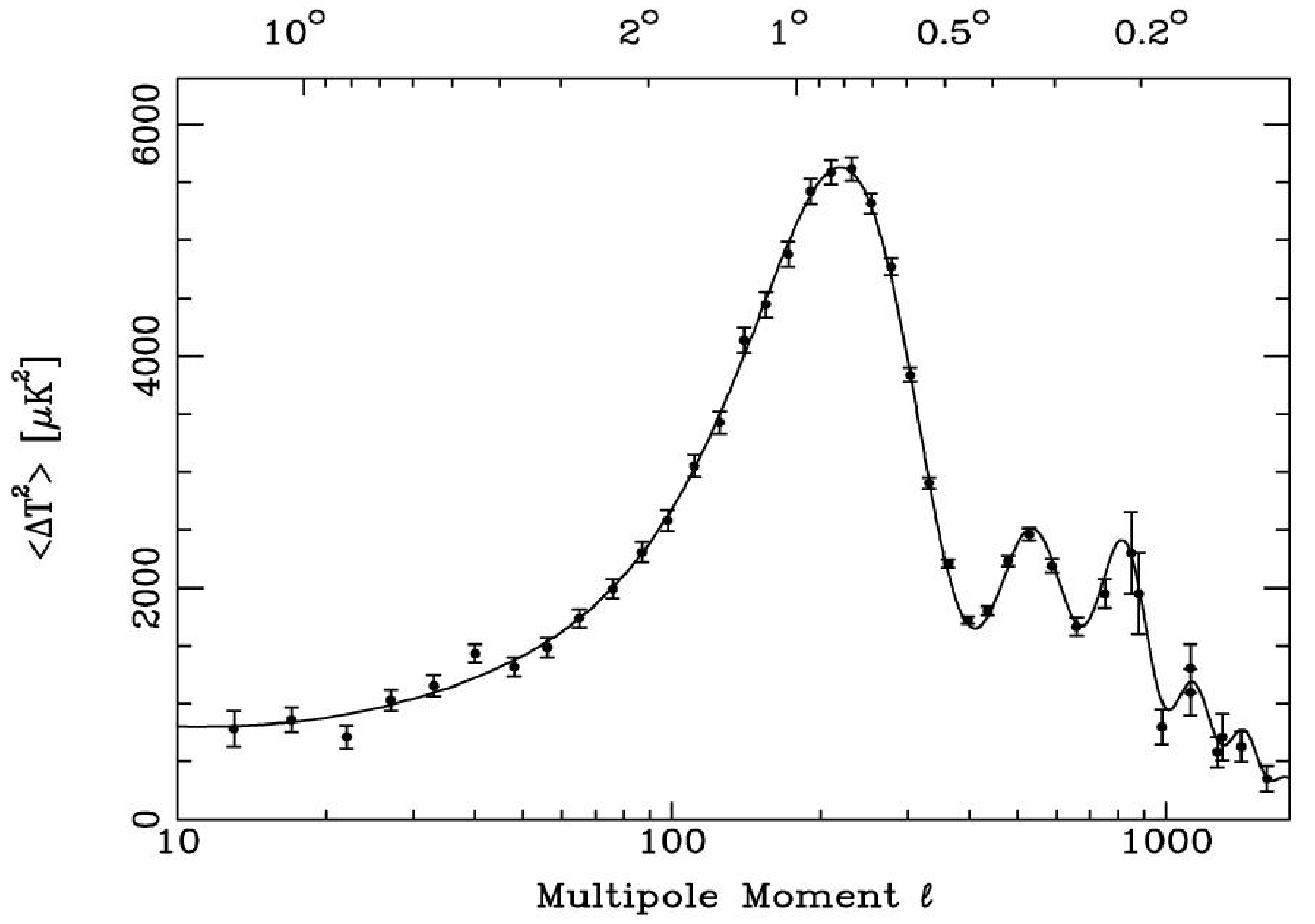
- Modes with largest amplitude will be those with $\tau = t_{\text{rec}}/2$.
- Hence, $\lambda = c_s/\tau = 2c_s t_{\text{rec}} = \dfrac{2ct_{\text{rec}}}{\sqrt{3}}$
- This gives us a
  - "ruler" = size of the fluctuations in the CMB
  - at a known distance = distance to when age of universe = $t_{\text{rec}}/2$.
- We can use this to directly measure the geometry of the universe.

Acoustic Peaks



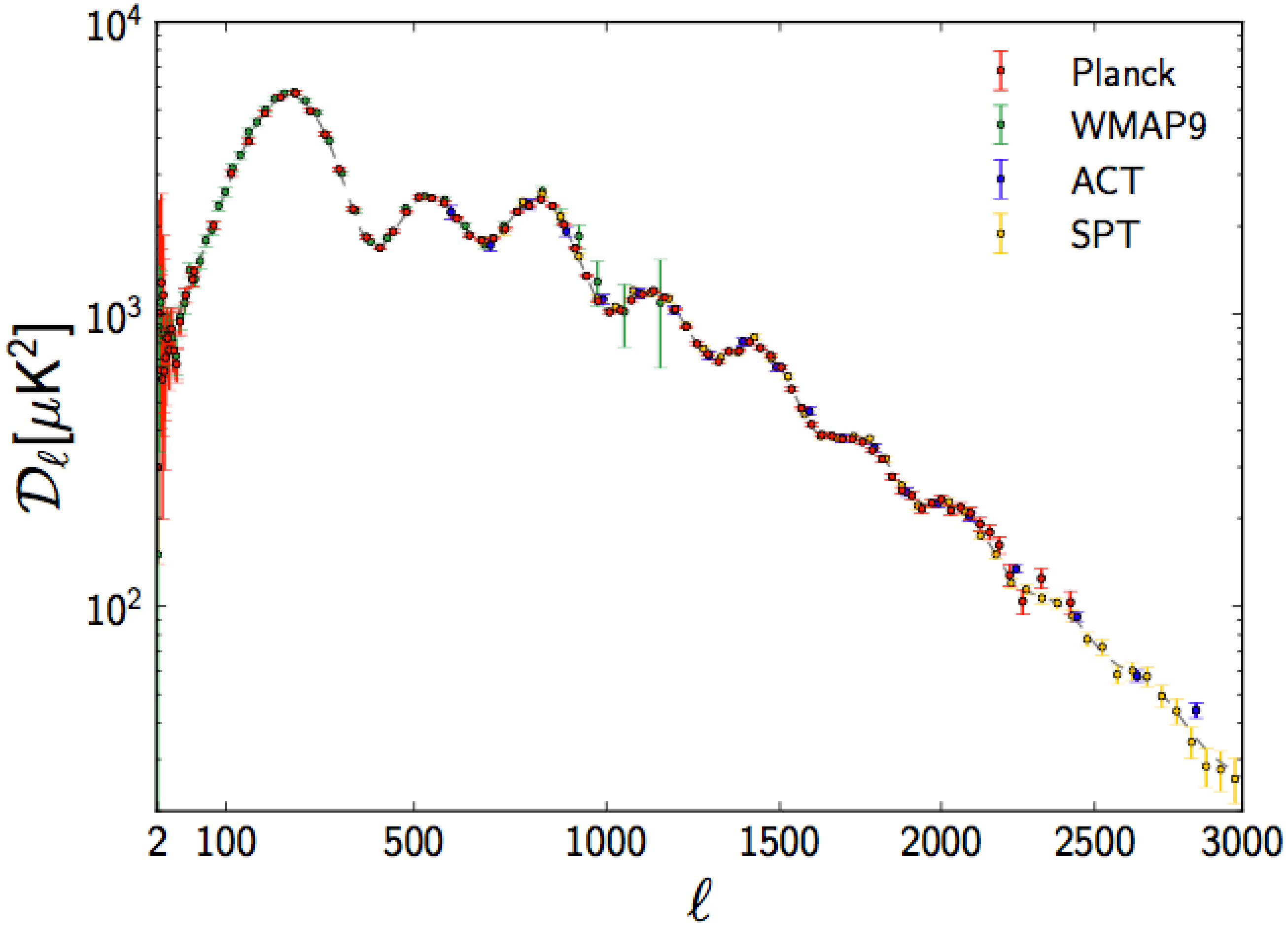
- Fluctuations with wavelengths near $2c_s t_{rec} = 2ct_{rec}/\sqrt{3}$ should have the largest amplitudes.
- We can use this to directly measure the geometry of the universe by comparing measured angular size of fluctuations to calculated size.
- Calculate expected angular size for k=0 on board.

Acoustic Peaks



- Peak is at ~0.8° or *l* ~ 200.
- What sets width of peak? Why are there multiple peaks?
- Position of first peak gives $\Omega = 1.02 \pm 0.02$ from WMAP data.

Acoustic Peaks



- All of the wiggles can be modeled using a models of the atomic physics of recombination, the geometry and evolution of the universe, and the spectrum of fluctuations at early times.
- Gives constraints on Ω_Λ, Ω_m, Ω_B, t_0, ...

Homework

For next class: problem 9.3