Outline

- Go over problem 9.3
- Saha ionization equation
- Cosmic nucleosynthesis

Saha Ionization Equilibrium

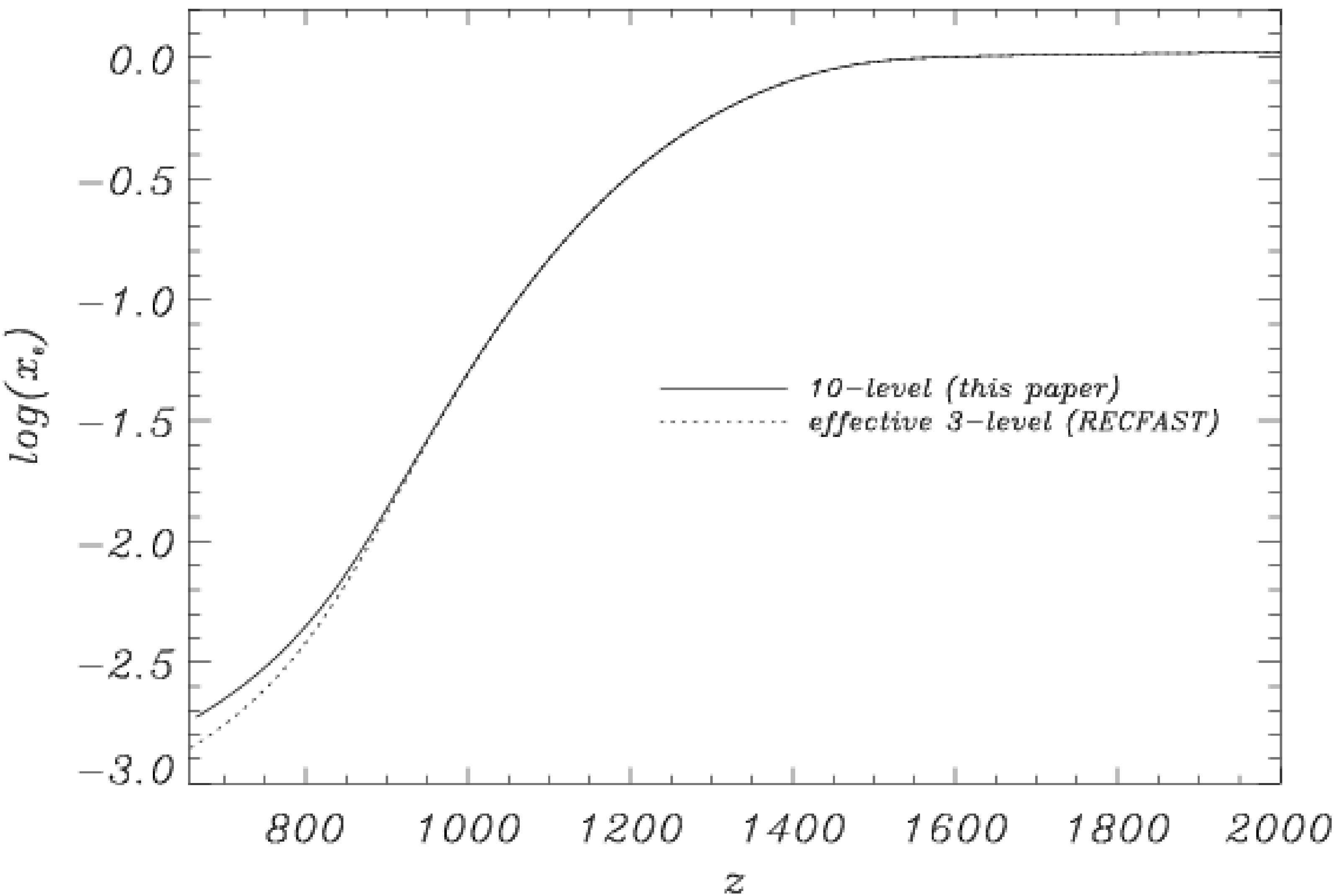
- Hydrogen interacts with radiation: $p + e^- \leftrightarrow H + \gamma$
- At a given temperature T, which sets the typical photon energy kT, what is are the equilibrium number densities?
  - At low T, expect $n_\mathrm{H} >> n_\mathrm{p} = n_\mathrm{e}$
  - At high T, expect $n_\mathrm{e} >> n_\mathrm{H}$
- In general, use Saha equation:

$$\frac{n_p n_e}{n_H} = \left(\frac{m_e k T}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{\Delta E}{kT}\right)$$

- This must be modified because H has many energy levels.
  - "Effective three-level atom" adds only n=2 (why is that useful?)
  - Can add more levels

Ionization Equilibrium

- Define free electron fraction, $x = n_e/(n_p + n_H)$



Nucleosynthesis

- At higher redshifts, the temperature gets higher, $T \sim (z+1)\times 2.7$ K.
- For a system with two energy levels separated by energy ΔE, the excited state will be occupied when $kT \sim \Delta E$.
- Consider protons and neutrons:
  - $p + e^{-} + 0.8\ \text{MeV} \leftrightarrow n + \nu_e$
  - $p + \bar{\nu}_e + 1.8\ \text{MeV} \leftrightarrow n + e^{+}$
- Equilibrium is described by Saha equation, $\Delta E = m_n c^2 - m_p c^2 = 1.3$ MeV

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{\Delta E}{kT}\right)$$

- When $kT < 0.8$ MeV, neutrons can no longer be created, they "freeze out".
  - $n_n/n_p \sim \exp(-1.3/0.8) = 0.2$
- Neutrinos also decouple, creating cosmic neutrino background.

Nucleosynthesis

- Neutrons are unstable (life time ~ 15 minutes), and can survive only if bound into nuclei.
- Most of the neutrons get bound into helium nuclei
  - $n + p \rightarrow d + \gamma$
  - $p + d \rightarrow {}^3\mathrm{He} + \gamma$
  - $d + d \rightarrow {}^3\mathrm{He} + n$
  - $n + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + \gamma$
  - $d + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + p$
- Some neutrons decay into protons, some go into heavier elements ${}^7\mathrm{Li}$, ${}^7\mathrm{Be}$
- By calculating the time evolution of these reactions, one finds the ratio of neutrons in ${}^4\mathrm{He}$ to protons is ~ 0.15 ~ 1/7.

Nucleosynthesis

- By calculating the time evolution of these reactions, one finds the ratio of neutrons in ^{4}He to protons is ~ 0.15 ~ 1/7.
- This means there are 14 protons for every 2 neutrons
  - Or one ^{4}He (=2n+2p) for every 12 free protons
  - Ratio of helium to hydrogen is 1/12
  - Mass fraction of helium is 0.24
- This calculation depends only on relatively low-energy (few MeV) nuclear physics which is very well studied in the laboratory.
- Measurements of intergalactic gas clouds with no evidence of star formation (why pick those?) show the ratio of helium to hydrogen is ~1/12 and there are no gas clouds with helium ratios significantly below 1/12.

Nucleosynthesis

- Deuterium is also produced cosmologically
  - $n + p \rightarrow d + \gamma$
  - Binding energy is 2.2 MeV.
- Recall that the baryon to photon ratio today is small, $\eta \sim 5\times10^{-10}$
- Number densities of photons and baryons both scale as R^{-3}, so ratio was the same in the past.
- Large number of photons means that deuterium can be destroyed by photons way out in the exponential tail of the blackbody.
- Net deuterium productions doesn't begin until kT ~ -2.2 MeV/ln(η)
- This is at about 2 minutes and is in a race with neutron decay.
- Measurement of primordial deuterium abundance constrains the baryon to photon ratio, thus the baryon density.
- Better measurement comes from acoustic peaks in CMB.

Time	Redshift z	Temperature T(K)	Event
$\sim 10^{-34}$ s	$\sim 10^{27}$	$\sim 10^{27}$	Inflation ends, $\Omega_m + \Omega_\Lambda \to 1$, causally connected regions have expanded exponentially, initial fluctuation spectrum determined.
2 s	4×10^9	10^{10}	Neutron freezeout, no more neutrons formed.
3 min	4×10^8	10^9	Primordial nucleosynthesis over—light element abundances set.
65,000 yr	3500	10^4	Radiation domination $\to$ mass domination, $R \sim t^{1/2} \to R \sim t^{2/3}$, dark-matter structures start growing at a significant rate.
400,000 yr	1100	3000	Hydrogen atoms recombine, matter and radiation decouple, Universe becomes transparent to radiation of wavelengths longer than Lyα, CMB fluctuation pattern frozen in space, baryon perturbations start growing.
$\sim 10^8$–10^9 yr	$\sim$6–20	$\sim$20–60	First stars form and reionize the Universe, ending the Dark Ages. The Universe becomes transparent also to radiation with wavelengths shorter than Lyα.
$\sim$6 Gyr	$\sim$1	$\sim$5	Transition from deceleration to acceleration under the influence of dark energy.
14 Gyr	0	2.725 ± 0.002	Today.

Neutrino Background

At some point back in cosmic time, the Universe was dense enough to be opaque to neutrinos. Then, as the Universe expanded, the density decreased until neutrinos could stream freely. A cosmic neutrino background (which is undetected to date) must have formed when this decoupling between neutrinos and normal matter occurred, in analogy to the CMB that results from the electron–photon decoupling at the time of hydrogen recombination. Find the temperature at which neutrino decoupling occurred.

Assume in your calculation that decoupling occurs during the radiation-dominated era, photons pose the main targets for the neutrinos, neutrino interactions have an cross section $\sigma = 10^{-43}\ \mathrm{cm}^2\ (E/1\ \mathrm{MeV})^2$ and the neutrinos are relativistic. Assume a flat universe with no cosmological constant.

Neutrinos actually couple to electrons and positrons, e.g. $\bar{\nu}_e + \nu_e \leftrightarrow e^+ + e^-$, which are well coupled to radiation at these energies.

Friedmann Equations

- First Friedmann equation: $$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{k c^2}{R^2} + \frac{\Lambda}{3}$$
- Acceleration equation: $$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}\left(\rho c^2 + 3P\right) + \frac{\Lambda}{3}$$
- Energy conservation: $$\dot{\rho} c^2 = -3\frac{\dot{R}}{R}\left(\rho c^2 + P\right)$$

Homework

For next class: problem 9.4