Equation Sheet - Astrophysics II - Spring 2015

$$\Delta\theta = 1.22\lambda/D \qquad \mathrm{SNR} = N_{\mathrm{star}}/\sqrt{N_{\mathrm{total}}} \qquad N_{\mathrm{total}} = N_{\mathrm{star}} + p \times n_{\mathrm{sky}}$$

$$d = 1/p \qquad D = \theta d \qquad \lambda\nu = c \qquad E = h\nu \qquad \lambda = h/p \qquad \Delta x \Delta p_x > h$$

$$L = 4\pi r_*^2 \sigma T^4 = f 4\pi d^2 \qquad \lambda_{max} T = 0.29\,\mathrm{cm\,K} \qquad \mathrm{h}\nu_{\mathrm{max}} = 2.82\mathrm{kT}$$

$$P_{gas} = nkT = \frac{\rho}{\bar{m}} kT \qquad E_K = \frac{f}{2} kT \qquad P_{rad} = \frac{1}{3} aT^4$$

$$l = 1/n\sigma \qquad \tau = l/v_{\mathrm{ran}} \qquad \sigma_e = \sigma_{geom}\left(1 + \frac{v_e^2}{v_{ran}^2}\right)$$

$$\rho(r,z) = \rho_0 \left[\exp\left(-\frac{r}{r_d}\right)\right]\left[\exp\left(-\frac{|z|}{h_d}\right)\right]$$

$$M(r) = v^2 r/G \qquad \tau \sim r/\sigma \qquad M \sim \sigma^2 r_{cl}/G$$

$$\alpha = \frac{4GM}{c^2 b} = \frac{2r_s}{b} \qquad r_s = \frac{2GM}{c^2} = (3\,\mathrm{km})\frac{M}{M_\odot}$$

$$\theta_E = \left(\frac{4GM}{c^2}\frac{D_{ls}}{D_{ol}D_{os}}\right)^{1/2} \qquad \theta_\pm = \frac{1}{2}\left[\beta \pm (\beta^2 + 4\theta_E^2)^{1/2}\right]$$

$$a_{tot} = \frac{u^2+2}{u(u^2+4)^{1/2}} \quad u = \beta/\theta_E \quad u(t) = \left[u_0^2 + \frac{(t-t_0)^2}{\tau^2}\right]^{1/2} \quad u_0 = \beta_0/\theta_E \quad \tau = \frac{\theta_E D_{ol}}{v}$$

$$\phi(L)dL = \phi(L*)\left(\frac{L*}{L}\right)\exp\left(\frac{L}{L*}\right)dL$$

$$T(r) = \left(\frac{GM\dot{M}}{8\pi\sigma}\right)^{1/4} r^{-3/4} \quad L \approx \frac{GM\dot{M}}{2r_{in}} \quad L_E = \frac{4\pi c G M m_p}{\sigma_T} = (1.3{\times}10^{38}\,\mathrm{erg/s})\frac{M}{M_\odot}$$

$$L = \eta \dot{M}_{disk} c^2 \qquad \dot{M}_{BH} = \frac{1-\eta}{\eta c^2} L$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$ds^2 = c^2dt^2 - R^2\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right) \qquad l = \int_{r=0}^{r} dl = R(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}}$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2} + \frac{\Lambda}{3} \qquad \frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P) + \frac{\Lambda}{3} \qquad \dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)$$

$$H = \frac{\dot{R}}{R} \qquad \frac{R}{R_0} = \frac{1}{1+z} \qquad \rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

$$\Omega_m = \frac{\rho_m}{\rho_c} \qquad \Omega_\Lambda = \frac{\Lambda}{3H_0^2} = \frac{\rho_\Lambda}{\rho_c}$$

for flat universe: $D_{PM} = rR_0, D_A = rR_0/(1+z), D_L = rR_0(1+z)$

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2}\exp\left(-\frac{\Delta E}{kT}\right) \qquad \frac{n_p n_e}{n_H} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}\exp\left(-\frac{\Delta E}{kT}\right)$$

$G = 6.67\times10^{-8}\,\mathrm{erg\,cm\,g^{-2}}$ $\quad h = 6.63\times10^{-27}\,\mathrm{erg\,s}$ $\quad c = 3.0\times10^{10}\,\mathrm{cm/s}$
$k = 1.38\times10^{-16}\,\mathrm{erg/K} = 8.62\times10^{-5}\,\mathrm{eV/K}$
$\sigma = 5.67\times10^{-5}\,\mathrm{erg\,cm^{-2}\,K^{-4}}$ $\quad a = 4\sigma/c = 7.6\times10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}}$
$m_p = 1.67\times10^{-24}\,\mathrm{g}$ $\quad m_e = 9.1\times10^{-28}\,\mathrm{g}$
$e = 4.8\times10^{-10}$ statcoulombs $\quad 1$ statcoulomb $= 1$ erg$^{1/2}$ cm$^{1/2}$
$\sigma_T = 6.7\times10^{-25}\,\mathrm{cm^2}$
CGS units: erg = g cm^2 s^{-2} = dyne cm
$1\,\mathrm{pc} = 3.086\times10^{18}\,\mathrm{cm}$
Sun:
$M_\odot = 2.0\times10^{33}\,\mathrm{g}$ $\quad R_\odot = 7.0\times10^{10}\,\mathrm{cm}$ $\quad L_\odot = 3.8\times10^{33}\,\mathrm{erg/s}$
$T_{E\odot} = 5800\,\mathrm{K}$ $\quad \bar{\rho}_\odot = 1.4$ g cm^{-3} $\quad T_{C\odot} = 15\times10^6\,\mathrm{K}$ $\quad \rho_{C\odot} = 150\,\mathrm{g\,cm^{-3}}$
Sun to Galactic center = 8.0 kpc $\quad v_\odot = 220$ km/s
Radius of Milky Way = 50 kpc
$H_0 = 70$ km/s/MPC
Age of universe = 13.8 Gyr
$\rho_c = 9.2\times10^{-30}\,\mathrm{g\,cm^{-3}}$
$\rho_{m,0}c^2 = 2.5\times10^{-9}\,\mathrm{erg\,cm^{-3}}$
$u_{rad,0} = 4.2\times10^{-13}\,\mathrm{erg\,cm^{-3}}$
$\Omega_\Lambda \approx 0.7$ $\quad \Omega_{m,0} \approx 0.3$ $\quad \Omega_{B,0} = 0.044\pm0.004$ $\quad \Omega_m + \Omega_\Lambda = 1.02\pm0.02$
$T_{CMB} = 2.725\pm0.002$ K
Lyman limit = 912 Å