Outline

- Exam will be draw from material in Chapters 8 and 9.

- It may be useful to study these problems and the homework problems. Also read over the textbook and class notes for short answer questions.

Hubble Constant

- Write down an expression for the distance from us, located at r = 0 in co-moving coordinates, to a galaxy at a position r in co-moving coordinates in terms of the scale factor of the universe, $R(t)$, and the curvature parameter of the FRW metric.

- Find the Hubble law, i.e. the relation between distance and recession velocity of the galaxy. Express the Hubble 'constant' in terms of $R(t)$ and its derivatives.

Light ray

- A light ray is emitted from $r = 0$ at time $t = 0$. Find an expression for $r(t)$ in a positively curved universe ($k = 1$).

Energy-Momentum Tensor

- The energy-momentum tensor has 16 components, but is symmetric. How many components are independent?

- Define the following components of the tensor in words: T_{00}, T_{ii}, T_{i0}, T_{0i}, T_{ij}, where i > 0 and i ≠ j.

- If the universe is homogenous and istropic, what is true of the energy-momentum tensor?

Einstein Tensor

- Fill in the blanks. The Einstein tensor is a function of the _____ and provides a measure of the _____ of spacetime as measured by _____ _____.

Expansion

- Put the following in chronological order
    - Matter dominated era
    - Cosmological constant dominated era
    - Radiation dominated era

- Use the Friedman equations to find the dependence of R on t in the radiation dominated era.

- Use the energy conservation equation to find the dependence of the energy density of radiation on R.

Expansion

- Find an expression for when the universe made the transition from being radiation to matter dominated in terms of the current densities of matter and radiation and the current age of the universe. Remember the factor of 1.7 for the energy density of neutrinos.

Curvature

- What is the critical density?

- Using the Friedmann equations, find an expression for the critical density in terms of the Hubble constant.

Acceleration vs Decceleration

- Use the acceleration equation, with the cosmological constant, to show that the acceleration of the universe changed sign when the scale factor was

$$\frac{R_a}{R_0} = \left(\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}} \right)^{1/3}$$

- Find the value of z when $R = R_a$.

Acceleration

- Find an expression for the dependence of the scale factor on time, $R(t)$, for a universe in which the cosmological constant, Λ, dominates.

Redshift

- Starting from the FRW metric, derive an expression for the redshift of light emitted from a galaxy at a time when the scale factor of the universe was R. Use R_0 for the current scale factor.

Distances

- Write down expressions for and explain the following:

  - Proper motion distance

  - Angular diameter distance

  - Luminosity distance

Cosmological Constant

- Derive the equation of state of the cosmological constant. What's weird about it?

Cosmic Microwave Background

- How and when was the cosmic microwave background produced?

- What is the spectrum of the CMB?

- What was the spectrum of the CMB when it was produced?

- What is the number density of CMB photons?

- How does one measure the power spectrum of the CMB?

- What is the relation between angular scale and spherical harmonic order?

Cosmic Microwave Background

- The early universe was radiation dominated, with an equation of state $P = (1/3)\rho c^2$ and the speed of sound was $c_s = c/\sqrt{3}$.

- The CMB was produced at the time of recombination t_{rec}.

- Calculate the physical size of the largest fluctuations in the density/temperature of matter at the time of recombination. Explain your derivation.

- Find an expression for t_{rec} in terms of z_{rec} and the current age of the universe.

- How can one measure z_{rec}?

- Calculate the angle subtended on the sky of the largest fluctuations in the CMB.

Nucleosynthesis

- Use the Saha equation to show that the neutron freeze-out, the ratio of neutrons to proton is approximately 0.2.

- At the time of nucleosythesis, the ratio of neutrons bound in ^{4}He nuclei to the total number of protons (in any type of nuclei) is 0.15. Estimate the mass fraction in helium.

Cosmological Probes

- At what redshift did the space density of AGN peak?

- How can high redshift AGN be used to probe the intergalactic medium?

- The spectra of star forming galaxies tends to cutoff at what wavelength. Explain.

- Explain the idea of photometric redshifts. What filters should you use to find galaxies at a redshift of 4.5? (You may specific the filter band edge rather than the name.)

- Fill in the blanks: most of the energy in starburst galaxies is produced at _____ wavelengths, but emitted at _____ wavelengths. Explain.

Background radiation

- In what band is most of the energy density in the cosmic background radiation?

- The CMB is different from the cosmic background radiation at other wavelengths in what way?

- The cosmic X-ray background is primarily produced by what sort of objects?