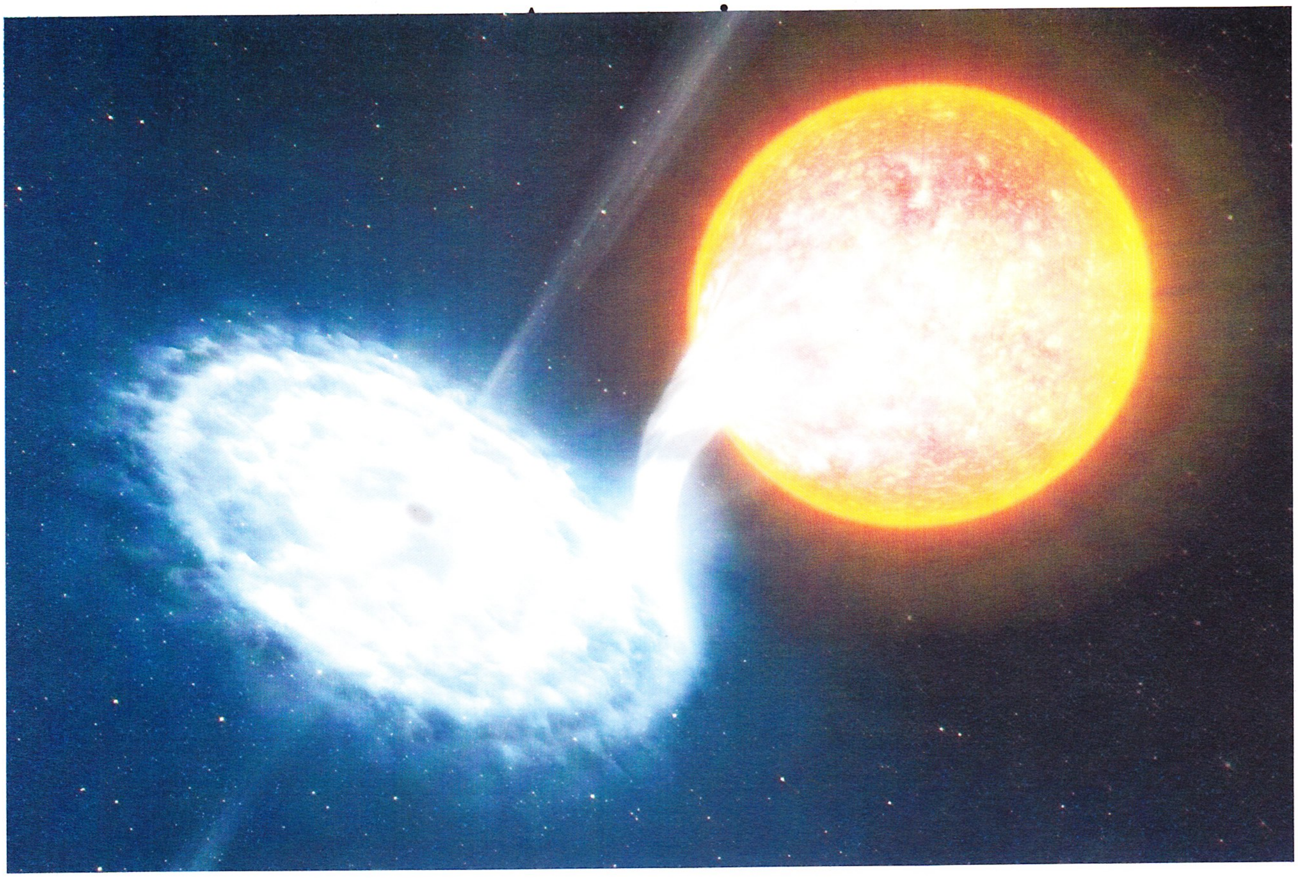
Accretion

- Efficiency of accretion onto surface
- Efficiency of accretion through disk
- Inner edge of disk around a black hole
- Eddington luminosity
- Neutron stars emit X-rays



Accretion

- Efficiency of accretion onto surface
  - Do on board

Efficiency of accretion onto surface

Particle falling in from ∞ to surface of object

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

at $r = \infty$ $\quad E = 0$

at $r = R$ $\quad \frac{1}{2} m v^2 = \frac{GMm}{R}$

particle stops at surface
$K = \frac{1}{2} m v^2$ is released

If matter falls in at rate $\dot{m}$ = accretion rate, then luminosity

$$L = \frac{1}{2} \dot{m} v^2 = \frac{GM\dot{m}}{R}$$

Introduce Schwarzschild radius

$$r_g = \frac{2GM}{c^2} \qquad r_g \simeq \underset{2.95}{3\,km} \frac{M}{M_\odot}$$

$$L = \frac{1}{2} \frac{r_g}{R} \dot{m} c^2 = \xi \dot{m} c^2$$

ξ = efficiency

Nuclear burning $\xi = 7\times10^{-3}$

White dwarf $M \sim M_\odot$ $R \sim 5\times10^{3}$ km

$$\xi = \frac{1}{2}\,\frac{3\text{ km}}{5\times10^{3}\text{ km}} \sim 3\times10^{-4}$$

Neutron star $M \sim 1.4\,M_\odot$ $R \sim 12$ km

$$\xi = \frac{1}{2}\,\frac{1.4\,(3\text{ km})}{12\text{ km}} \sim 0.2$$

Accretion

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Efficiency of accretion through disk

Now look at black hole

direct in-fall from ∞ releases
no energy,
mass falls
into event horizon

But matter usually has angular
momentum, forms disk to
dissipate L

Consider circular orbit

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r} = K + U$$

Virial theorem $\langle K \rangle = -\frac{1}{2} \langle U \rangle$

$$\langle E \rangle = \langle K \rangle + \langle U \rangle = -\frac{1}{2}\langle U \rangle + \langle U \rangle = \frac{1}{2}\langle U \rangle$$

$$E = -\frac{1}{2}\frac{GMm}{r}$$

$$\Delta E = E(r \to \infty) - E(r) = \frac{1}{2}\frac{GMm}{r}$$

Orbital speed

$$\frac{1}{2}mv^2 = K = -\frac{1}{2}U = +\frac{1}{2}\frac{GMm}{r}$$

$$v_{orb} = \sqrt{\frac{GM}{r}}$$

Free fall speed

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \qquad v_{ff} = \sqrt{\frac{2GM}{r}}$$

Difference in energy must be dissipated in order to make disk

$$\Delta E = \frac{1}{2}mv_{ff}^2 - \frac{1}{2}mv_{orb}^2$$

$$= \frac{1}{2}m\frac{2GM}{r} - \frac{1}{2}m\frac{GM}{r} = \frac{1}{2}\frac{GMm}{r}$$

Black hole disk efficiency

as before assume mass flow $\dot{m}$

$$L = \frac{1}{2} \frac{GM\dot{m}}{r}$$

$$= \frac{1}{4} \frac{r_g}{r} \dot{m} c^2$$

efficiency $\xi = \frac{1}{4} \frac{r_g}{r}$

depends on radius of innermost orbit in disk

For "Schwarzchild" or non-rotating BH

$r_{ms} = 3 r_g$ $\quad \xi \sim \frac{1}{4 \cdot 3} = 0.08$

GR effects make $\xi = 0.057$

For maximally rotating black hole

$r_{ms} = \frac{1}{2} r_g$ $\quad \xi = 0.422$

Accretion

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Inner edge of disk around BH

Equation of motion in Schwarzschild metric is

$$\left(\frac{dr}{d\tau}\right)^2 = \tilde{E}^2 - \left(1 - \frac{2GM}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right)$$

define effective potential

$$\left(\frac{dr}{d\tau}\right)^2 = \tilde{E}^2 - \hat{V}(\tilde{L}, r)^2$$

need a local minimum in $\tilde{V}$ to have a stable orbit

$$\tilde{V} = \left(1 - \frac{2GM}{r}\right)\left(1 + \frac{L^2}{r^2}\right)$$ ← show plot on slide

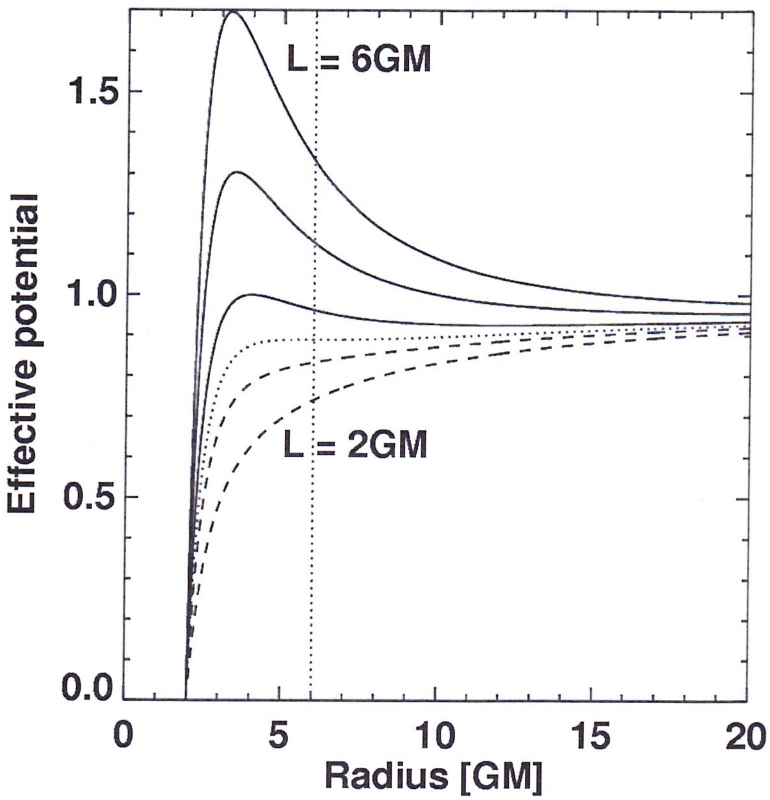
$$= 1 - \frac{2GM}{r} + \frac{L^2}{r^2} - \frac{2GML^2}{r^3}$$

$$\frac{d\tilde{V}}{dr} = \frac{2GM}{r^2} - \frac{2L^2}{r^3} + \frac{6GML^2}{r^4} = 0$$

$$r^2 - \frac{L^2}{GM} r + 3L^2 = 0$$

$$r = \frac{1}{2}\frac{L^2}{GM} \pm \frac{1}{2}\sqrt{\frac{L^4}{G^2M^2} - 4 \cdot 3L^2}$$

Orbits around Black Holes



real roots if

$$\frac{L^4}{G^2M^2} - 12L^2 \geq 0$$

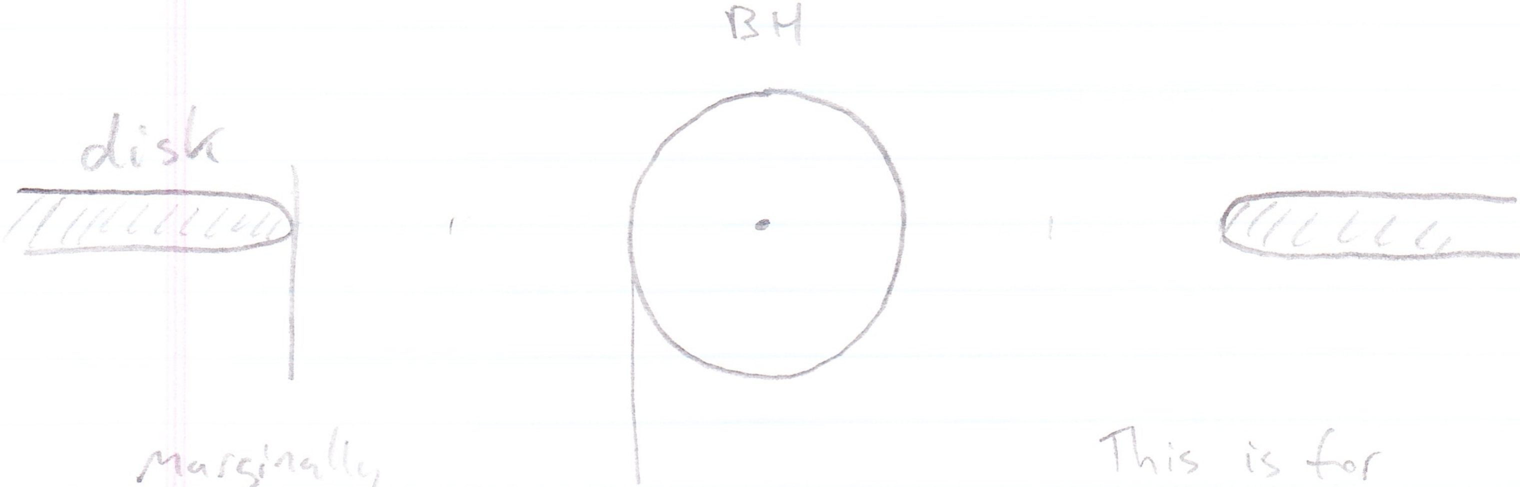
$$L^2 \geq 12G^2M^2$$

$$|L| \geq \sqrt{12}\,GM$$

critical value $L = \sqrt{12}\,GM$

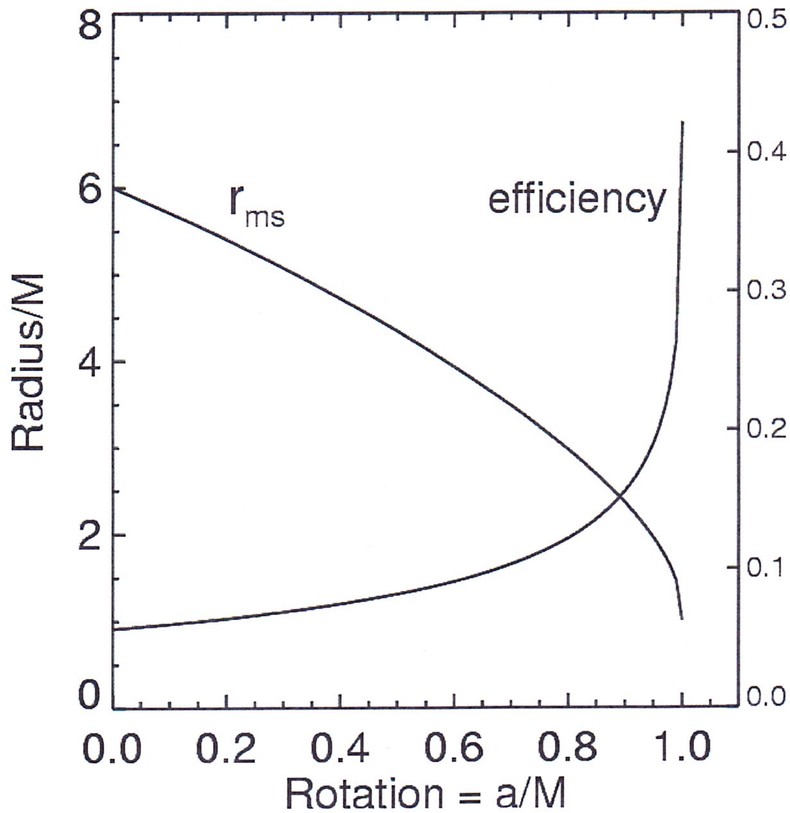
$$r = \frac{1}{2}\,\frac{12G^2M^2}{GM} \pm 0$$

$$r_{ms} = 6GM$$



This is for non-rotating BH. Show slide for rotating BH.

Orbits around Black Holes



Accretion

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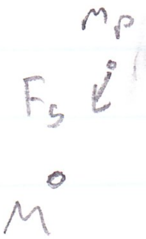
Eddington Luminosity

atom $p + e^-$ falling in toward luminous object



Gravitational force inwards $\quad F_G = \dfrac{G M m_p}{r^2}$

Radiation force

photon momentum $\quad p = \dfrac{E}{c}$

absorption of photon imparts $\Delta p = \dfrac{E_\gamma}{c}$ to atom

need rate of interactions

Photon flux $\quad N = \# \gamma / cm^2 \cdot s$

Luminosity $\quad L = N \cdot E_\gamma \cdot 4\pi r^2$

rate of interactions $= \sigma_T N$

σ_T = Thomson cross section

Radiation force $\quad F_R$ = rate of momentum transfer

$$F_R = \sigma_T N p = \sigma_T \frac{L}{E_\gamma 4\pi r^2} \frac{E_\gamma}{c} = \frac{\sigma_T L}{4\pi c r^2}$$

If source is powered by accretion, must have $F_G \geq F_R$

need inflowing matter to power source

Eddington limit at $F_G = F_R$

$$\frac{G M m_p}{r^2} = \frac{\sigma_T L}{4\pi c r^2}$$

$$L = \frac{4\pi c G m_p}{\sigma_T} M$$

$$= 1.3 \times 10^{38} \text{ erg/s } \frac{M}{M_\odot}$$

Neutron stars make X-rays

take NS accreting at Eddington

$M = 1.4\, M_\odot \qquad R = 12$ km

$L = 1.8 \times 10^{38}$ erg/s

$L = 4\pi R^2 \sigma T^4$

$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}

$$T = \left(\frac{L}{4\pi R^2 \sigma} \right)^{1/4}$$

$= 2.0 \times 10^7$ K

$kT = 1.7$ keV

Variability time scale of NS

$$t \sim \frac{R}{c} \sim 10^{-4} \text{ s}$$

Variability time scale of $10\, M_\odot$ BH

$$t \sim \frac{6 r_g}{c} \sim 10^{-3} \text{ s}$$