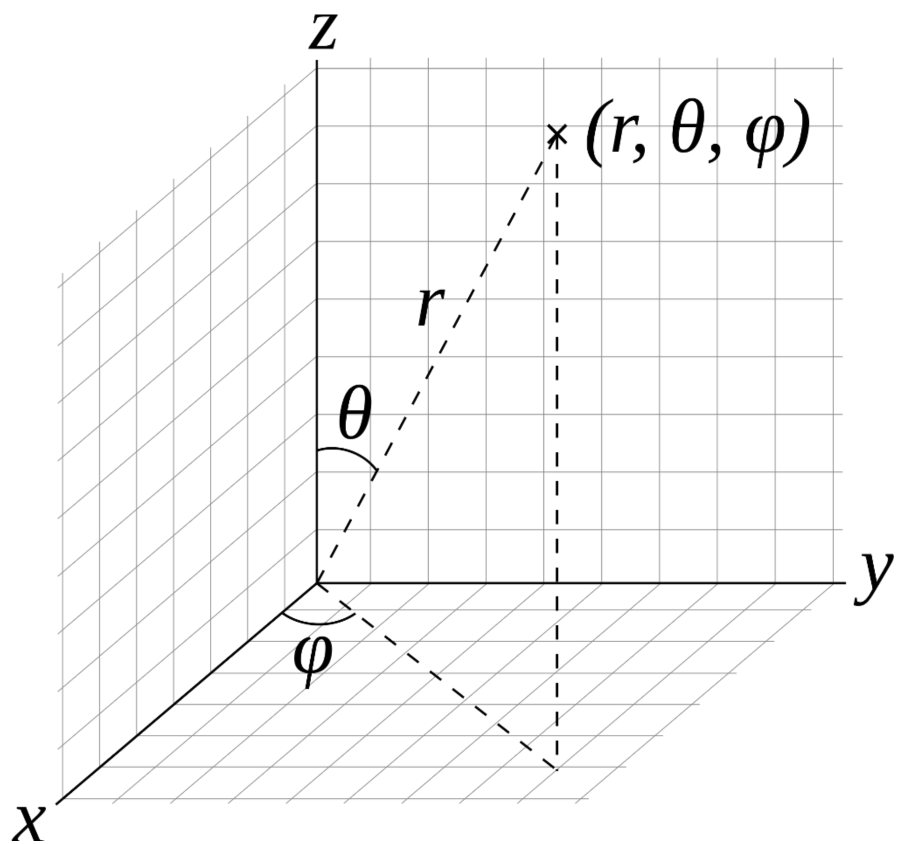
Spherical Coordinates and Astrometry

Polar Coordinates

- Radius = ρ (or *r*)
- Polar angle or z-inclination angle = θ
- Azimuthal angle = φ

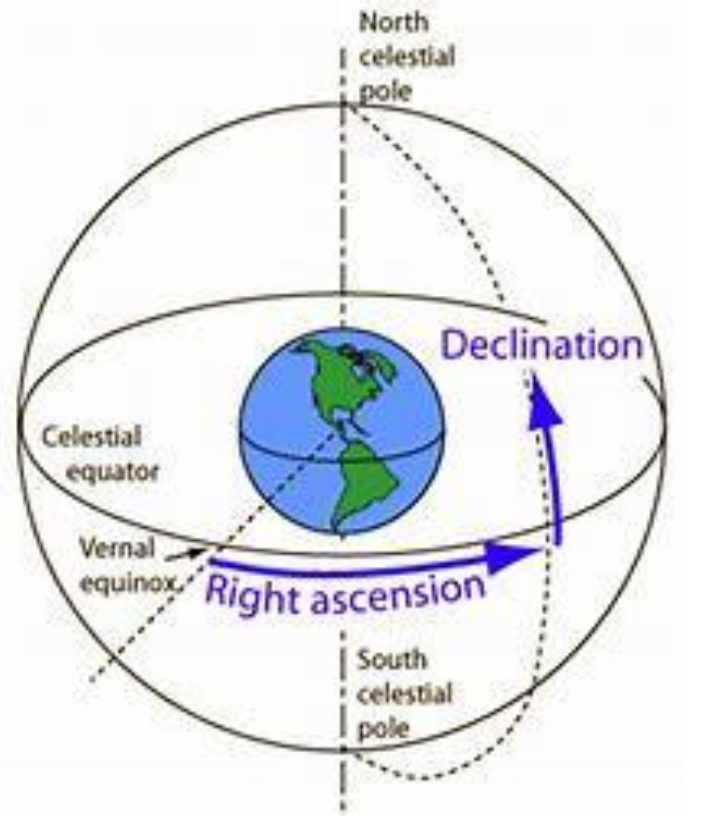


Cartesian coordinates

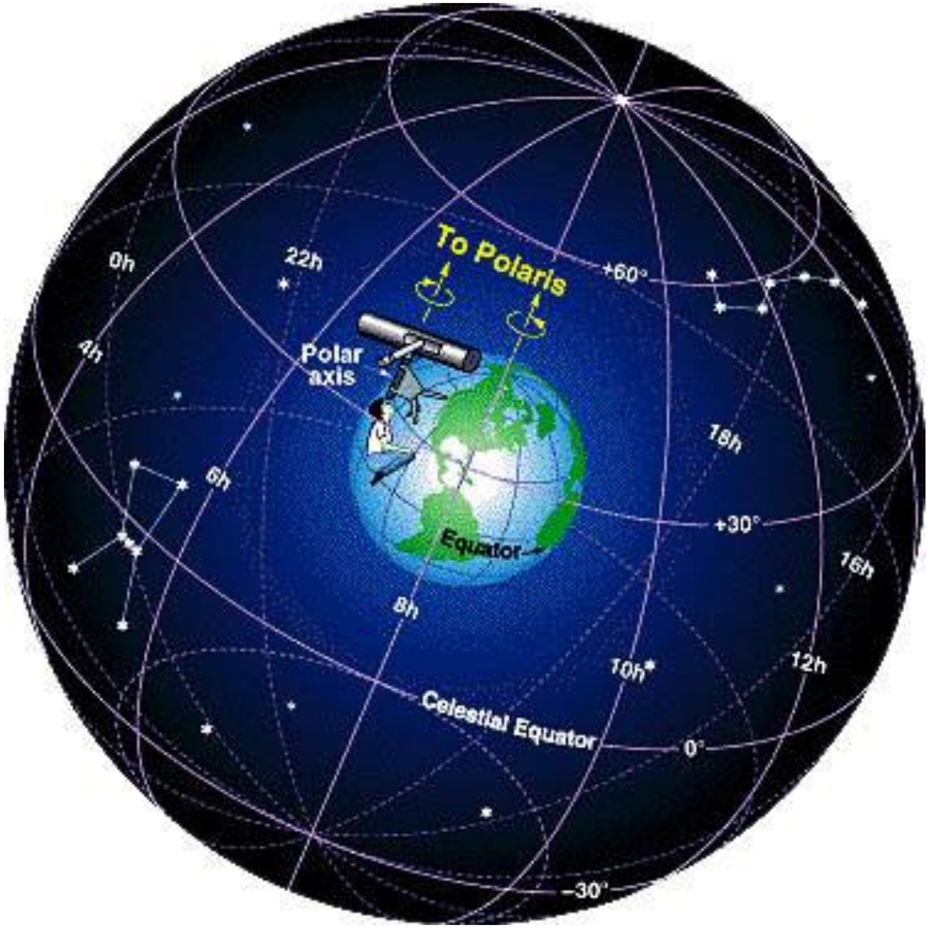
$x = r \sin(\theta) \cos(\varphi)$
$y = r \sin(\theta) \sin(\varphi)$
$z = r \cos(\theta)$

Relation to Celestial Coordinates

- Right ascension (α) = azimuthal angle (φ)
- Declination (δ) = $\pi/2$ – polar angle (θ)
- Radius?



How to find angular distance between two stars?



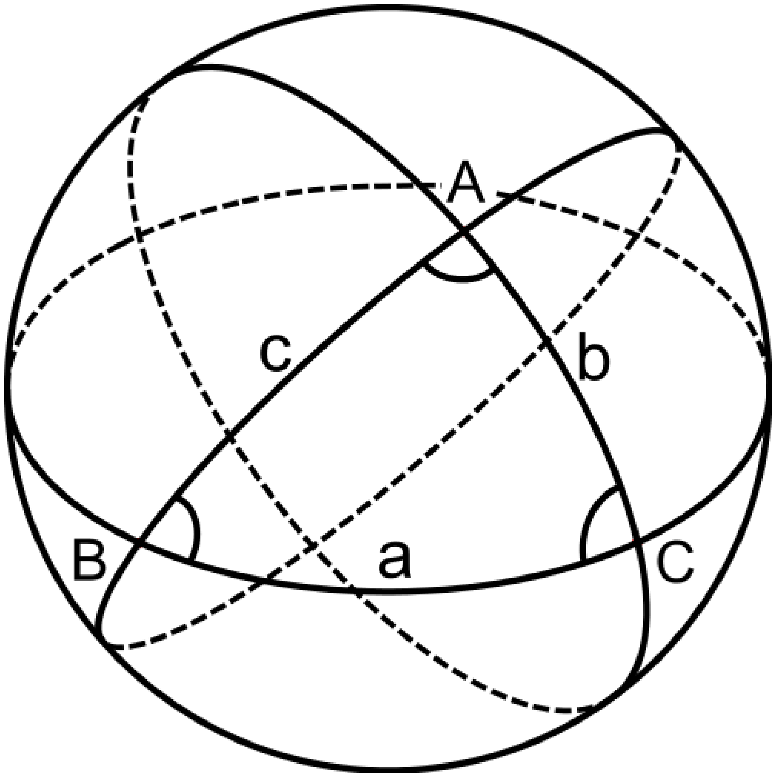
How to find angular distance between two stars?

- Dot product: $\vec{a} \cdot \vec{b} = ab\cos\gamma$

- Write the position of each star as a unit vector. Reminder: θ the not the same as declination
- Take the dot product of the Cartesian coordinates,

$$\cos\gamma = a_x b_x + a_y b_y + a_z b_z$$

Spherical Geometry

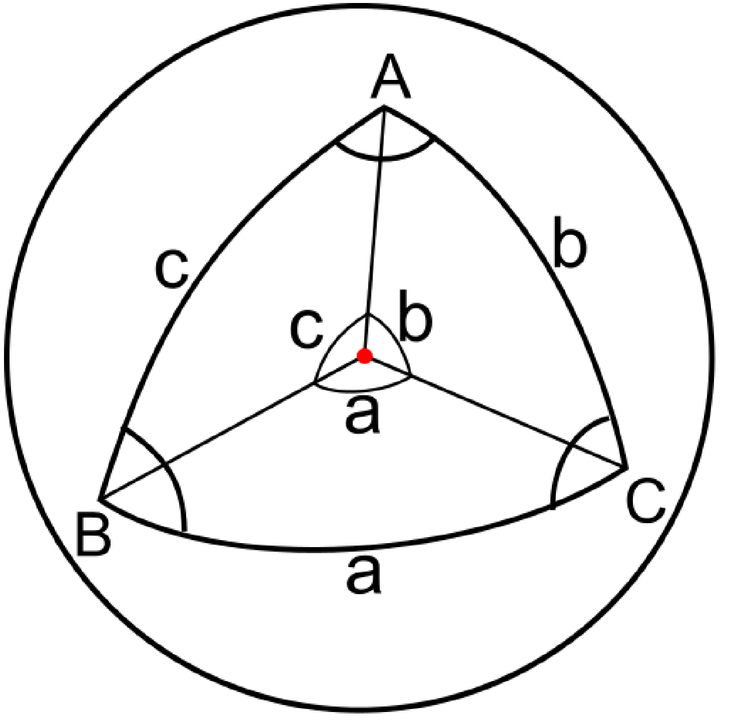


- All units in arc (radians, degrees..)
- Lower-case letters are subtended arcs from the center.
- Upper-case letters are angles or points on the sphere.
- Triangles have a sum of angles >180°.
- Spherical law of sines:
  sin(A)/sin(a) = sin(B)/sin(b) = sin(C)/sin(c)

For use in Astrometry Lab

- Find angle B between star C and the North Celestial Pole (a useful angle when deciding how your image is rotated compared to celestial coordinates).

- If we choose A as the NCP, then
  b = 90˚ - C_Dec
  A = B_RA - C_RA
  $\cos \boldsymbol{a} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{C}}$



- Use spherical law of sines $\frac{\sin B}{\sin b} = \frac{\sin A}{\sin a}$

$$\sin B = \sin(90 - C_{\text{Dec}}) \frac{\sin(B_{\text{RA}} - C_{\text{RA}})}{\sin\left(\cos^{-1}\left(\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{C}}\right)\right)}$$