Cosmology

- Scale factor
- Cosmology à la Newton
- Cosmology à la Einstein
- Cosmological constant
- SN and dark energy
- Evolution of the Universe

Scale Factor

- Assume expansion of Universe is homogeneous and isotropic
- Then expansion can be described by a scale factor $a(t)$, such that

$$r(t) = a(t)\, r_0$$

where $r_0 = r(\text{now})$ and a is dimensionless

Hubble Parameter

- Scale factor $a(t)$, such that $r(t) = a(t)\, r_0$
- Hubble law $v = Hr$
- Becomes

$$v = \frac{dr}{dt} = \dot{r} = \dot{a} r_0 = Hr = Har_0$$

$$H = \frac{\dot{a}}{a}$$

Cosmology à la Newton

- Model universe as homogeneous sphere with mass *M* and radius *r*, consider test mass *m* at surface. Then energy is:

$$E = km = K - U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

- Rewrite with scale factor

$$r = ar_0 \quad v = \dot{a}r_0 \qquad M = \frac{4}{3}\pi\, r^3 \rho$$

$$\frac{1}{2}v^2 = \frac{1}{2}r_0^2\dot{a}^2 = \frac{GM}{r} + k = G\frac{4}{3}\pi a^2 r_0^2 \rho + k$$

Cosmology à la Einstein

$$\frac{1}{2} r_0^2 \dot{a}^2 = \frac{4}{3} \pi a^2 r_0^2 \rho + k \quad \Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho + \frac{2k}{r_0^2 a^2}$$

$k < 0$: universe is bound, $k > 0$: universe is unbound

Change to relativistic version with parameters:

u = energy density

r_c = curvature of universe (always positive)

κ = curvature parameter +1=positive, 0=flat, -1=negative

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} u - \frac{\kappa c^2}{r_c^2 a^2}$$

Friedmann/Lemaitre Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}u - \frac{\kappa c^2}{r_c^2 a^2} + \frac{\Lambda}{3}$$

Extra term with Λ = "cosmological constant" was added by Einstein.

Equivalent to adding a component to the Universe that has a constant energy density as a function of time, perhaps the energy of quantum fluctuations in a vacuum.

$$u_\Lambda = \frac{c^2\Lambda}{8\pi G}$$

Energy densities

Rewrite Friedmann/Lemaitre equation in terms of energy densities.

u_{r} = radiation energy density

u_{m} = energy density of matter

u_{Λ} = energy density of cosmological constant or dark energy

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(u_r + u_m + u_\Lambda\right) - \frac{\kappa c^2}{r_c^2 a^2}$$

Evolution of energy densities

- Energy density of Λ is constant in time.
- Energy density of matter (normal or dark)
  - Assume non-relativistic particles, then energy is dominated by rest mass
  - Rest mass is not red-shifted, so energy density varies like number density of particles, decreases as volume of universe increases

$$u_{\mathrm{m}}(t) = n(t)\varepsilon = n(t)mc^2 = mc^2\, N/V \propto a(t)^{-3}$$

Evolution of energy densities

- Energy density of radiation
  - Number density of photons as volume of universe increases

    $n(t) = N/V \propto a(t)^{-3}$
  - Wavelength of photons increases as size of universe increases

    $\lambda(\mathrm{t}) \propto a(t)$ so $\varepsilon(\mathrm{t}) = \mathrm{hc}/\lambda(\mathrm{t}) \propto a(t)^{-1}$
  - Combine both factors

    $u_{\mathrm{r}}(t) = n(t)\varepsilon \propto a(t)^{-3}\, a(t)^{-1} \propto a(t)^{-4}$

Friedmann/Lemaitre Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(u_r + u_m + u_\Lambda\right) - \frac{\kappa c^2}{r_c^2 a^2}$$

Previous equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(\frac{u_{r,0}}{a^4} + \frac{u_{m,0}}{a^3} + u_\Lambda\right)$$

Know how u's scale

Take κ=0

$$\dot{a}^2 = H_0^2\left(\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_\Lambda a^2\right)$$

$$u_c = \frac{3H_0^2 c^2}{8\pi G} \quad \Omega_m = \frac{u_m}{u_c}$$

$$\dot{a} = H_0\left[\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2\right]^{1/2}$$

Energy densities

Critical density

$$u_c = \rho_c c^2 = \frac{3H_0^2 c^2}{8\pi G} = 5200\,\text{MeV m}^{-3}$$

Express densities in terms of density parameters:

$$\Omega_m = \frac{u_m}{u_c} \quad ,\ldots$$

From CMB curvature measurement:

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1.02 \pm 0.02$$

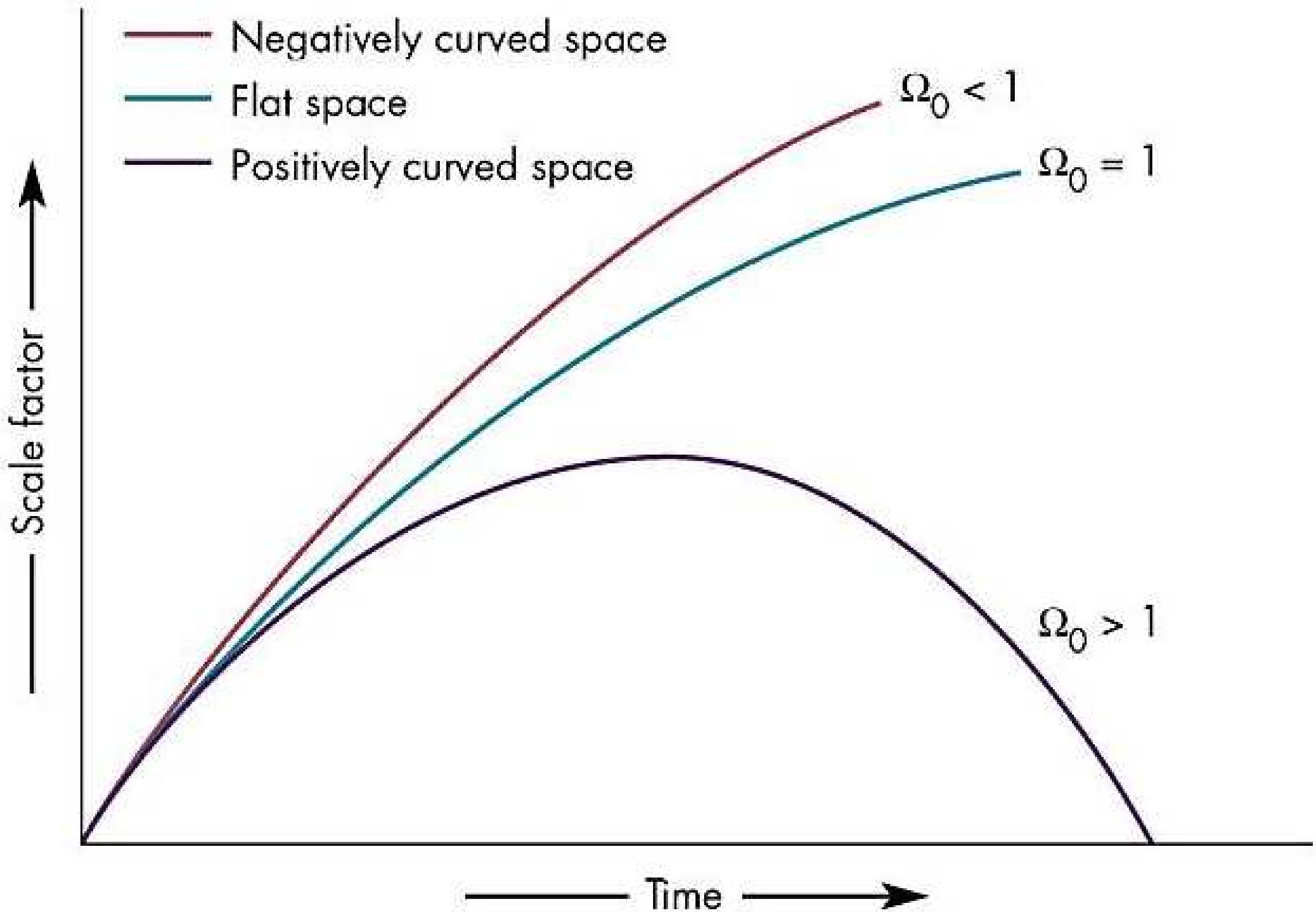
Friedmann/Lemaitre Equation

$$\dot{a} = H_0 \left[\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 \right]^{1/2}$$

$$\ddot{a} = H_0^2 \left[-\frac{\Omega_{r,0}}{a^3} - \frac{\Omega_{m,0}}{2a^2} + \Omega_{\Lambda,0} a \right]$$

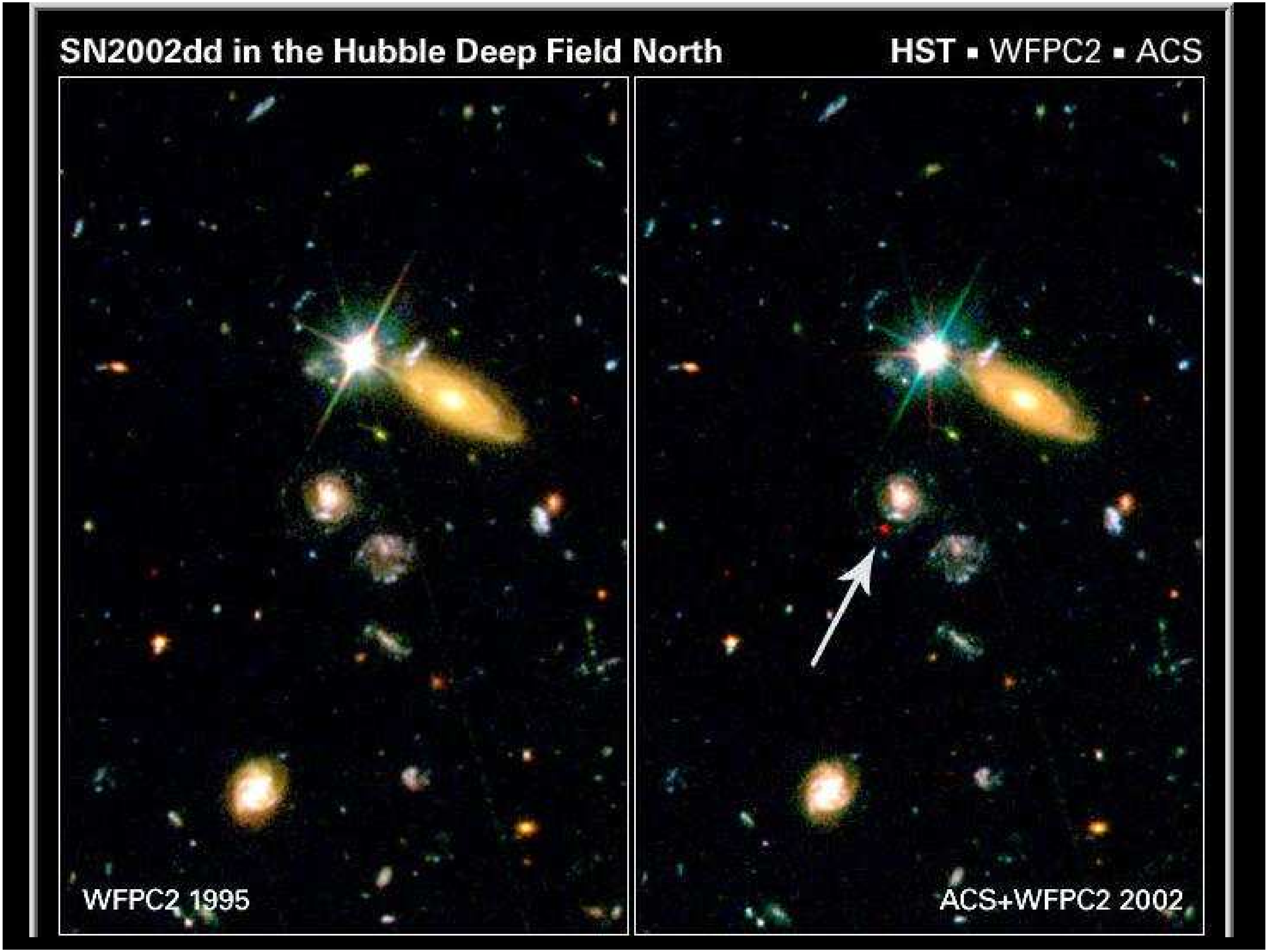
- Radiation and matter slow down expansion
- CC speeds up expansion
- Impossible to get static universe without CC

Matter slows down expansion

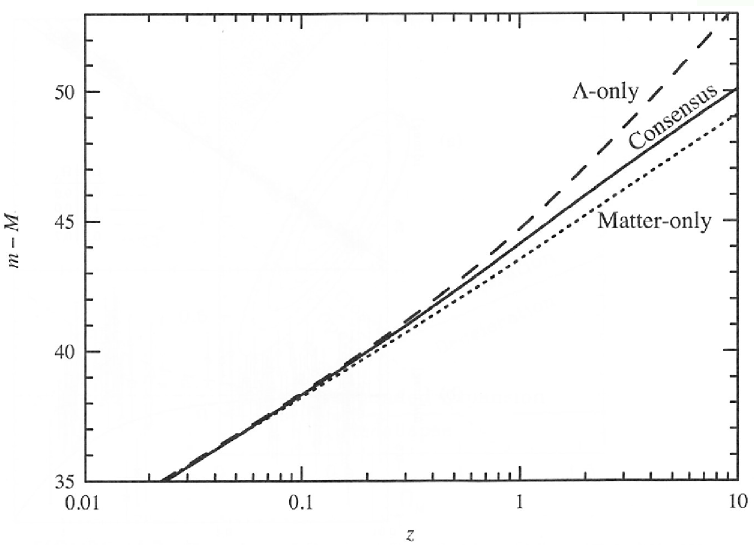


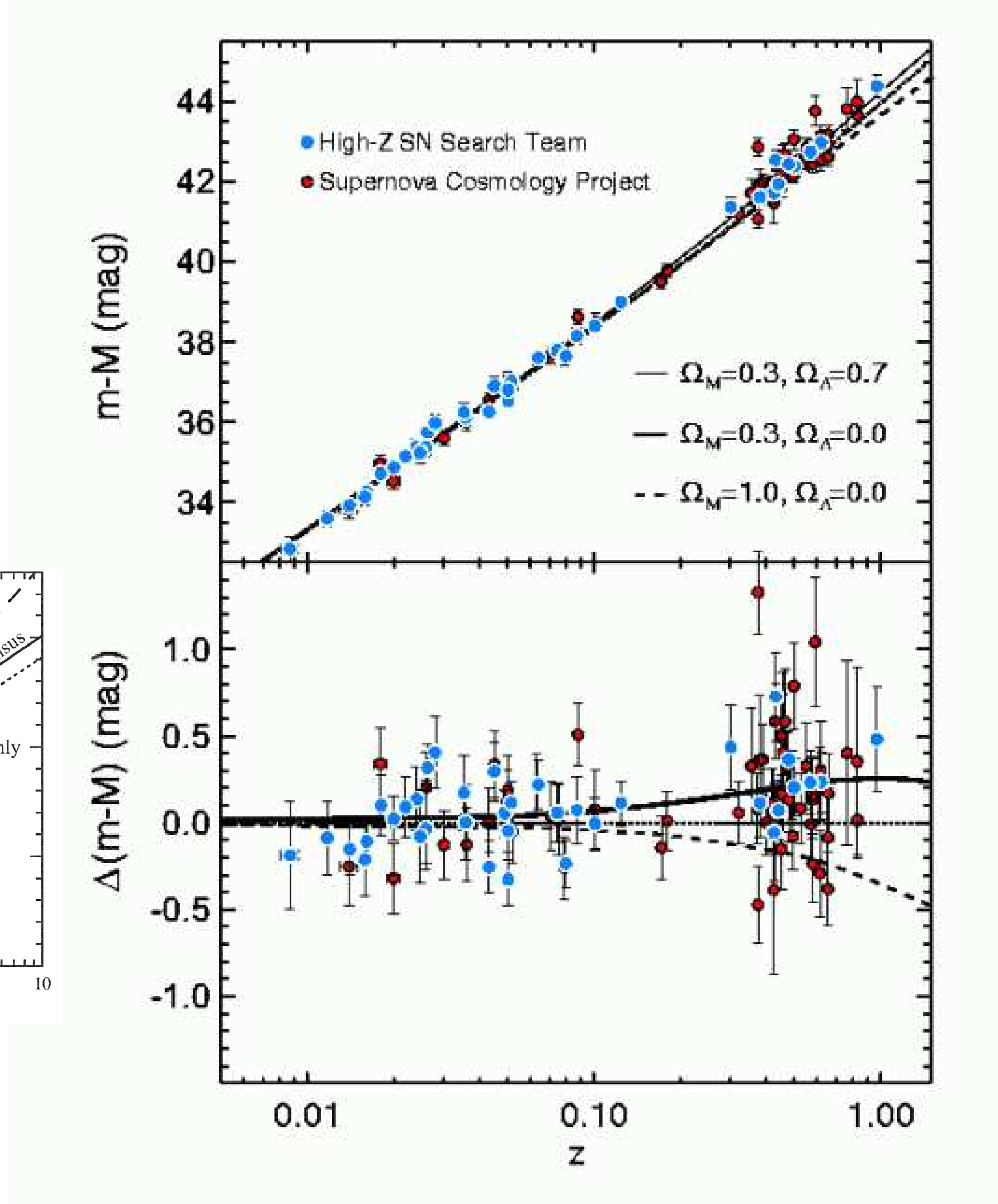
Einstein and Cosmology

- After Einstein wrote down the equations for General Relativity, he made a model of the Universe and found that the Universe had to be either expanding or contracting.
- He introduced a new term, the cosmological constant or Λ, in his equations representing a energy field which could create antigravity to allow a static model.
- After Hubble found the expansion of the Universe, Einstein called Λ his greatest blunder.
- Quantum physics predicts some energy fields that act like Λ.


SN2002dd in the Hubble Deep Field North
HST ▪ WFPC2 ▪ ACS
WFPC2 1995
ACS+WFPC2 2002

Accelerating Universe



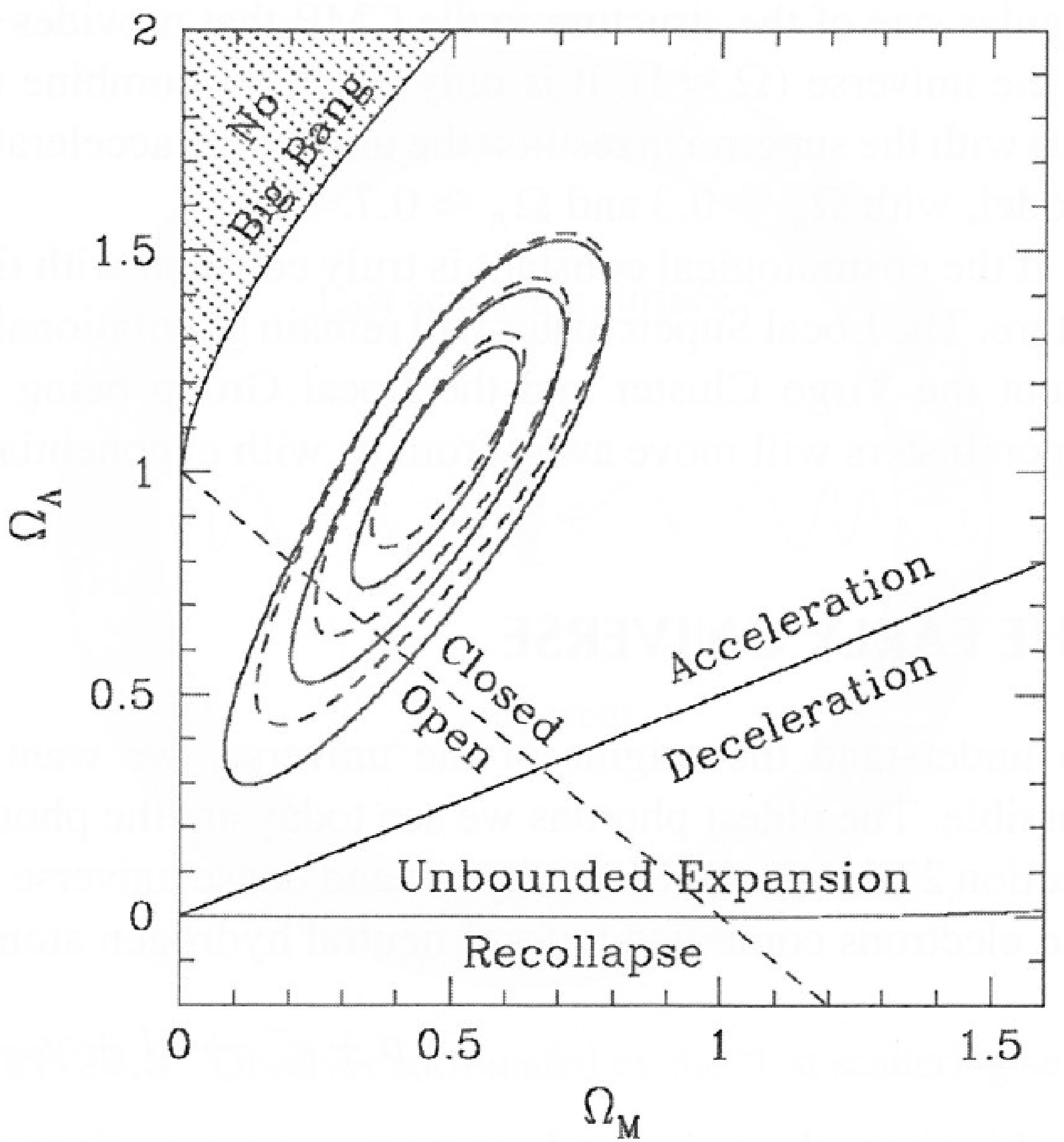


Accelerating Universe

- Hubble expansion appears to be accelerating

- Normal matter cannot cause acceleration, only deceleration of expansion
- Dark energy is required
  - may be cosmological constant
  - may be something else
  - major current problem in astronomy

Supernova constraints on Ωs



- Dashed vs solid are different SN samples
- Use curvature constraint Ω=1.02±0.02 to narrow range

Radiation Energy Density

Main component is CMB, star light is < 10%

$u_{\text{CMB}} = 0.260 \text{ MeV m}^{-3}$

$$\Omega_{CMB} = \frac{u_{CMB}}{u_c} = \frac{0.260 \text{ MeV m}^{-3}}{5200 \text{ MeV m}^{-3}} = 5.0 \times 10^{-5}$$

There are also likely neutrinos left over from the big bang, produced when nucleons froze out

$u_{\text{nu}} = 0.177 \text{ MeV m}^{-3}$

$$\Omega_{CMB} = \frac{u_{CMB}}{u_c} = \frac{0.177 \text{ MeV m}^{-3}}{5200 \text{ MeV m}^{-3}} = 3.4 \times 10^{-5}$$

Total for radiation: $\Omega_{r,0} = 8.4 \times 10^{-5}$

Matter Energy Density

- Matter in baryons (protons, neutrons, electrons): $\Omega_{bary} = 0.04$
- Matter in clusters (part dark): $\Omega_{cluster} = 0.2$
- Best estimate of all matter (baryons+dark): $\Omega_{m,0} = 0.3$

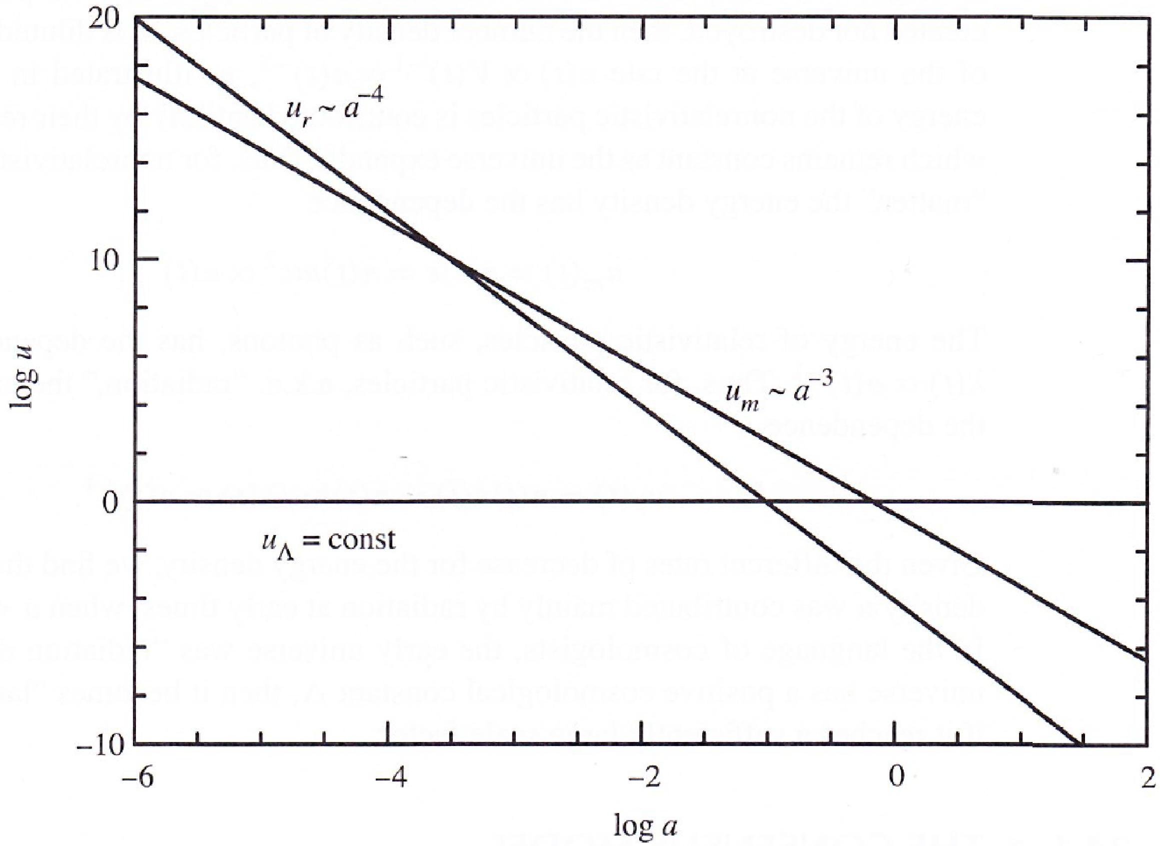
- Ratio of photons to baryons $\sim 2\times10^9$

Consensus Model

Component	Ω
Photons	5.0×10^{-5}
Neutrinos	5.0×10^{-5}
Total radiation	5.0×10^{-5}
Baryons	0.04
Dark matter	0.26
Total matter	0.30
Cosmological constant	~0.7
Curvature	1.02 ± 0.02

- Hubble constant = 70 ± 5 km s^{-1} Mpc^{-1}

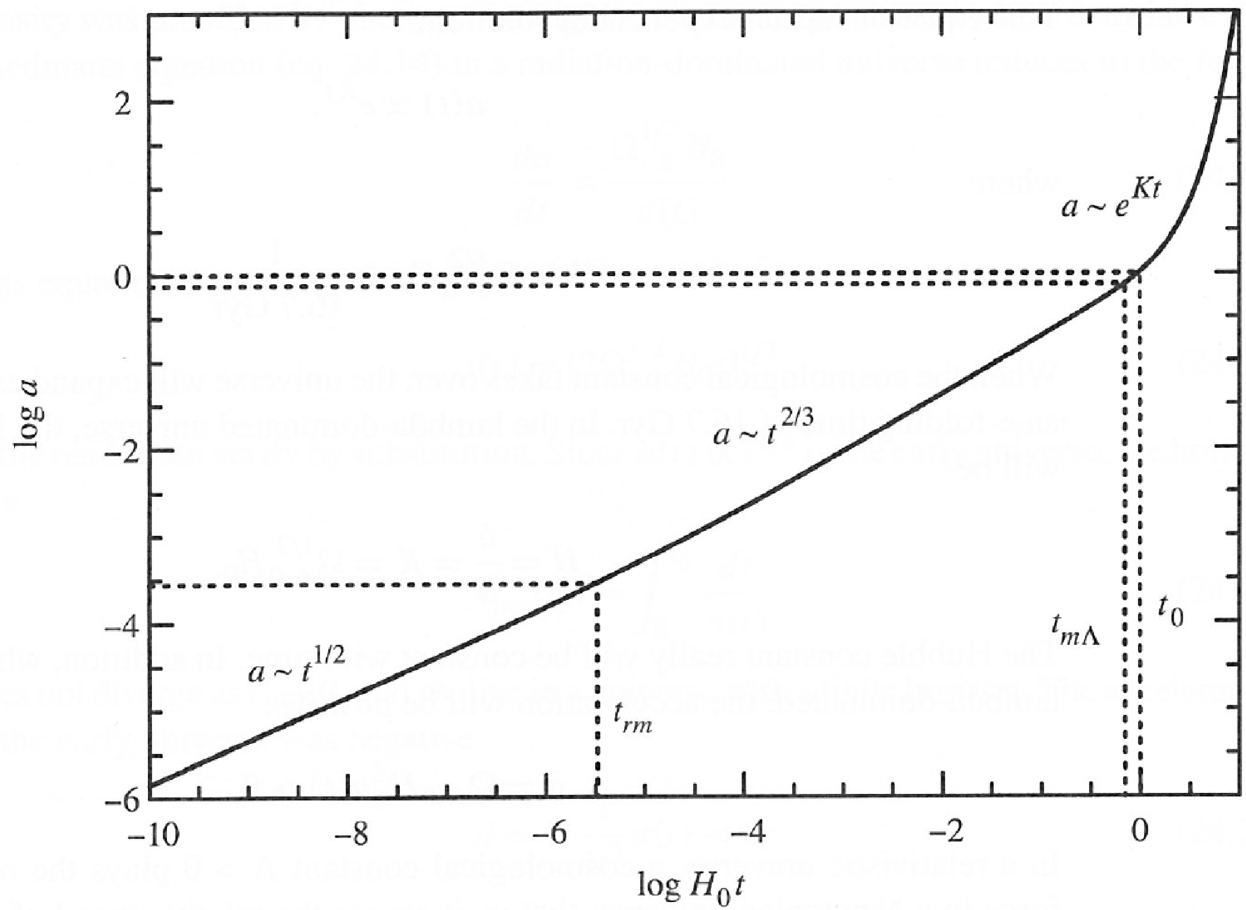
Energy density versus scale factor



$z=1/a-1$

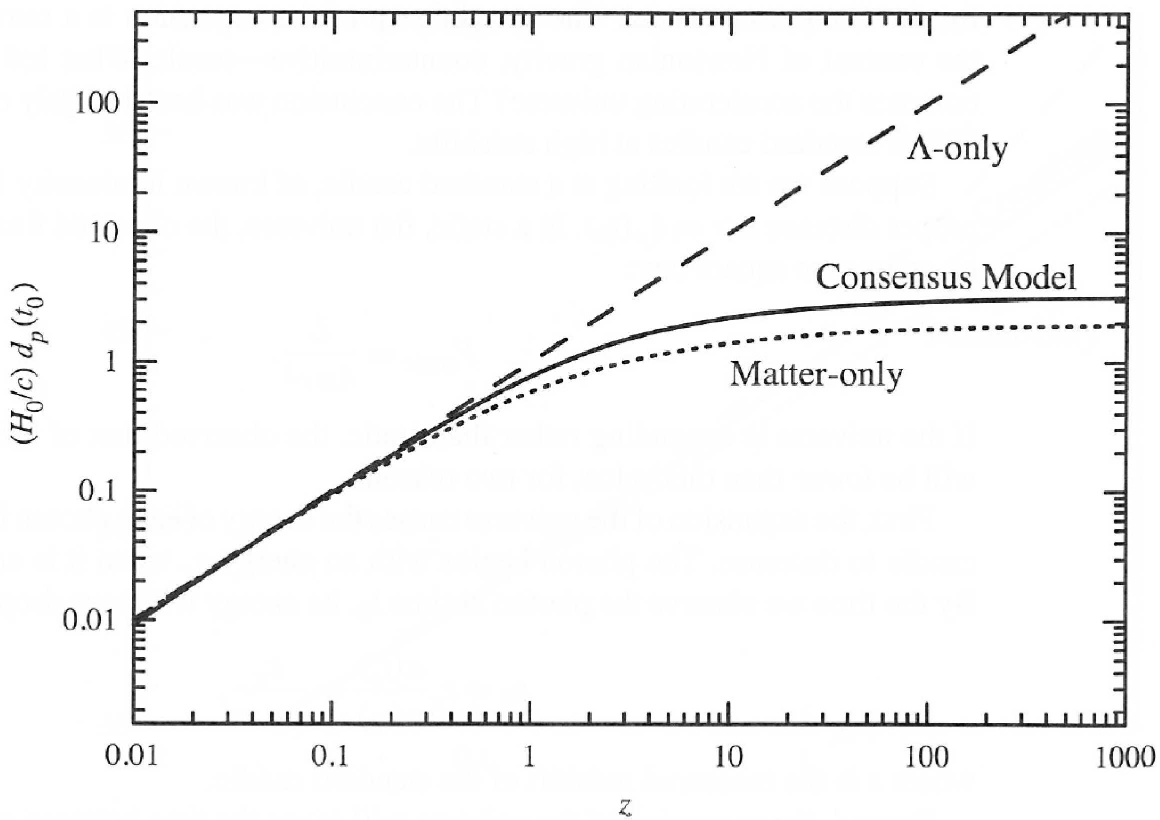
- Early times, z > 3600 or age < 47 kyr, were radiation dominated
- Matter dominated until 9.8 Gyr
- Current age 13.5 Gyr

Scale factor versus time



- Different slopes of expansion in radiation vs matter dominated epochs
- Exponential expansion in Λ dominated epoch (if like cosmological constant)

Proper distance versus redshift



- Proper distance reaches a limiting value of 14 Gpc
- Different distances are needed for different meaurements: distance, angular size, luminosity

Review Questions

- As fractions of the critical density, what are the current energy densities of radiation, baryonic matter, dark matter, and dark energy?
- Derive the equation for the critical density
- How do radiation, matter, and the cosmological constant affect the rate of expansion of the Universe?
- When was the universe dominated by radiation, matter, and dark energy?