

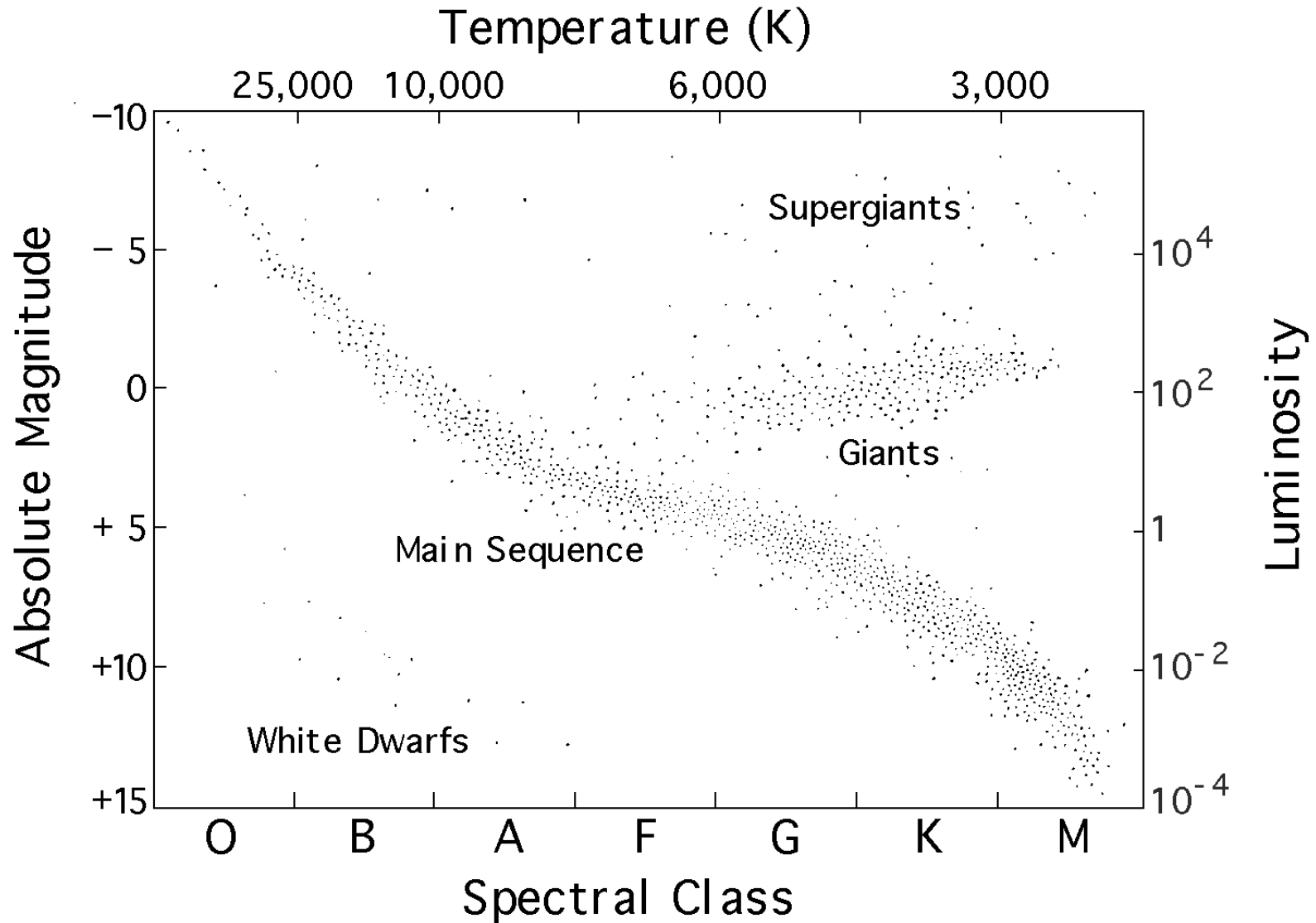
Stars

- Classifying stars: HR diagram
- Luminosity, radius, and temperature
- “Vogt-Russell” theorem
- Main sequence
- Evolution on the HR diagram

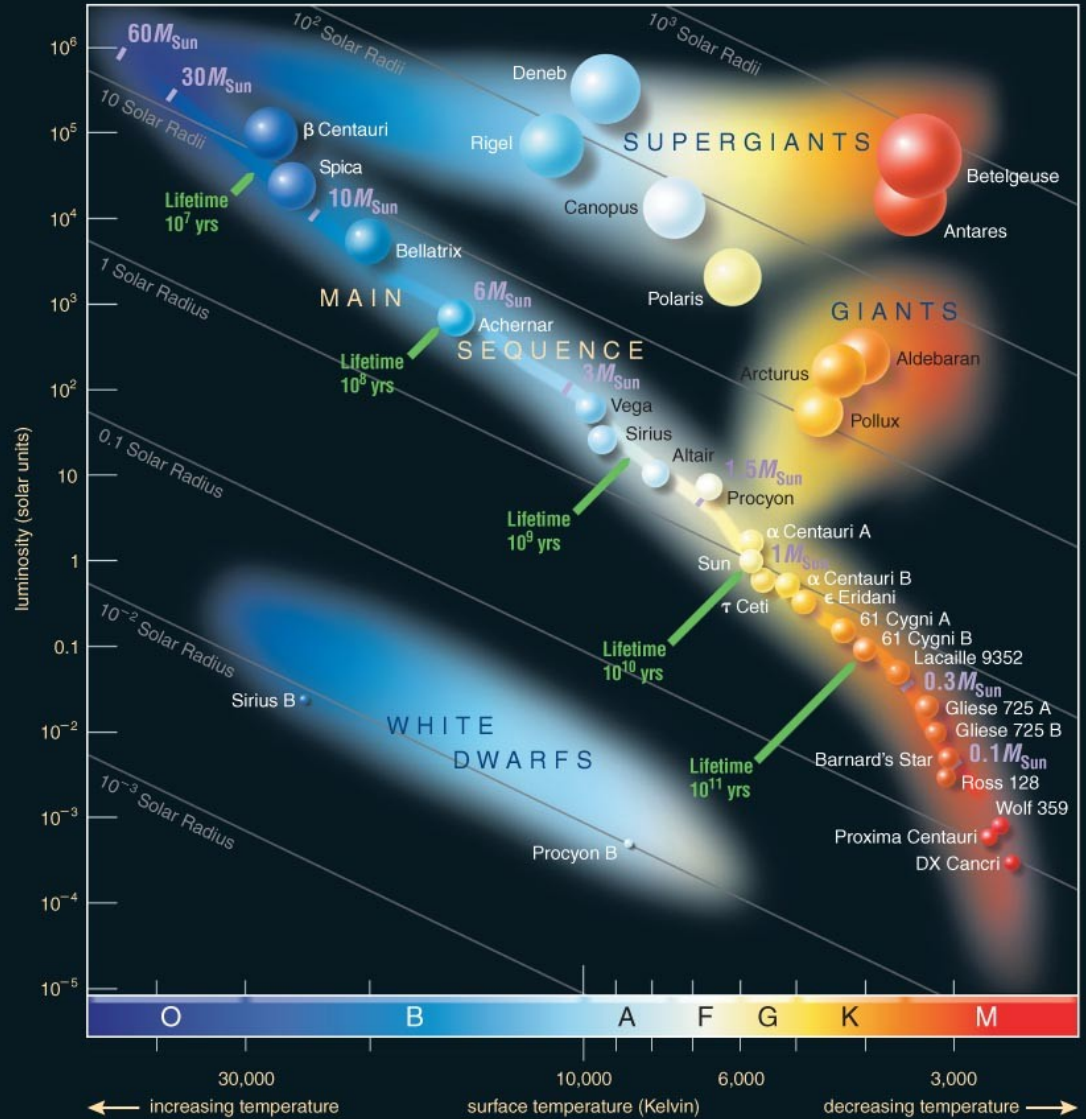
Classifying stars

- We now have two properties of stars that we can measure:
 - Luminosity
 - Color/surface temperature
- Using these two characteristics has proved extraordinarily effective in understanding the properties of stars – the Hertzsprung-Russell (HR) diagram

If we plot lots of stars on the HR diagram, they fall into groups



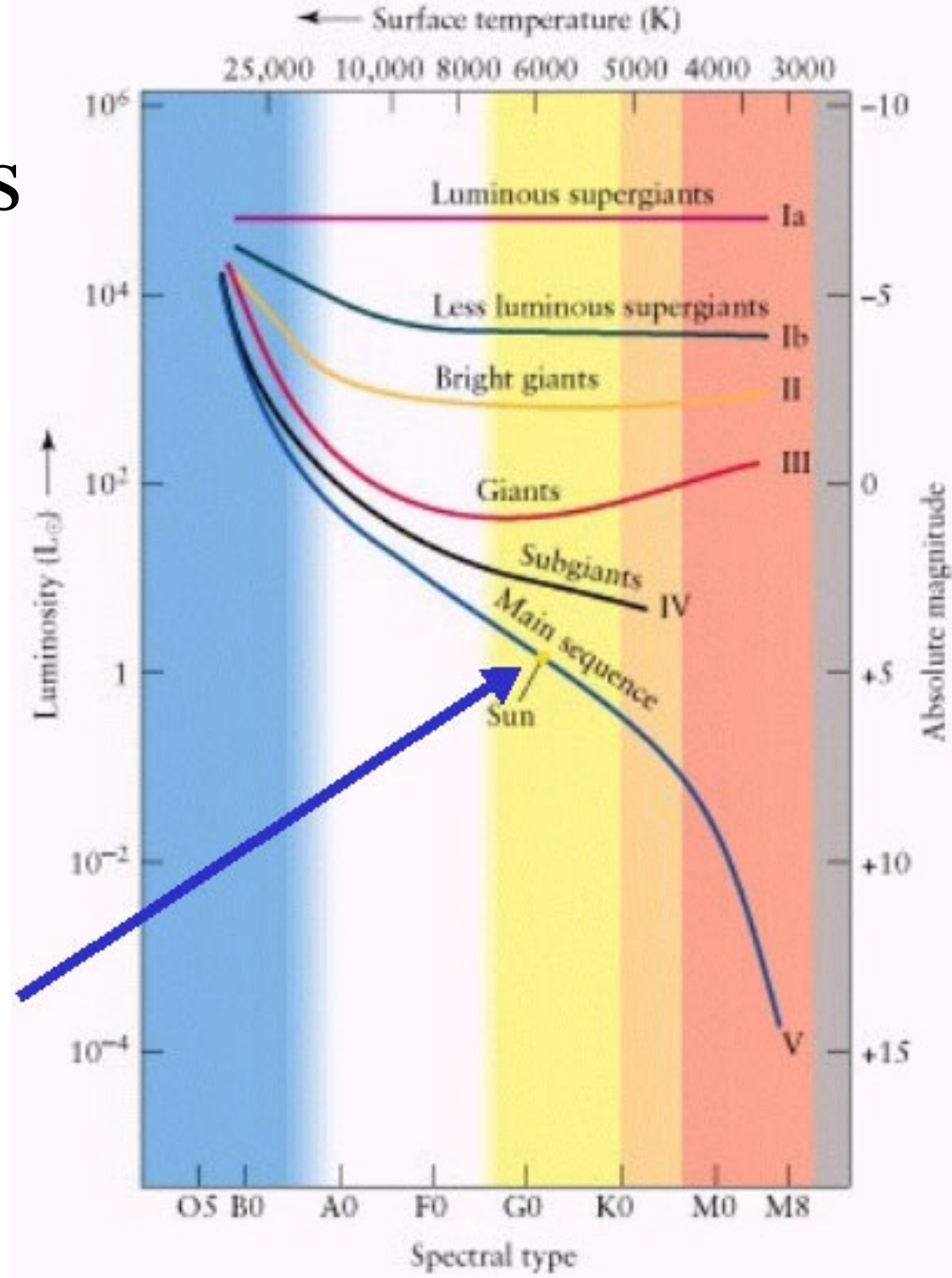
These groups indicate types of stars, or stages in the evolution of stars



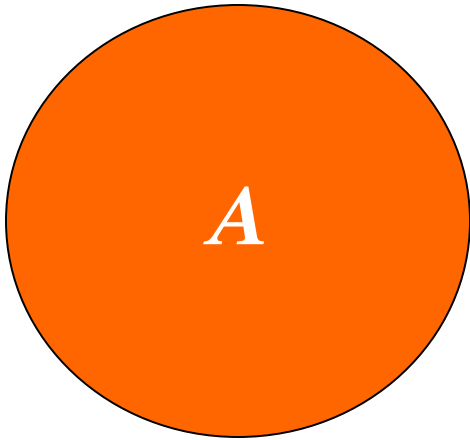
Luminosity classes

- Class Ia,b : Supergiant
- Class II: Bright giant
- Class III: Giant
- Class IV: Sub-giant
- Class V: Dwarf

The Sun is a G2 V star

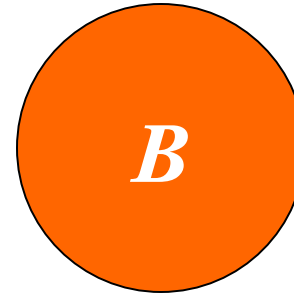


Luminosity versus radius and temperature



$$R = 2 R_{\text{Sun}}$$

$$T = T_{\text{Sun}}$$

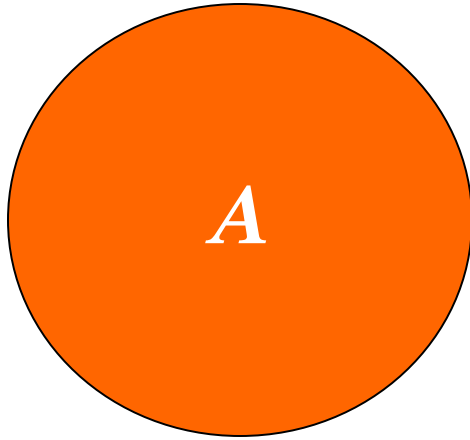


$$R = R_{\text{Sun}}$$

$$T = T_{\text{Sun}}$$

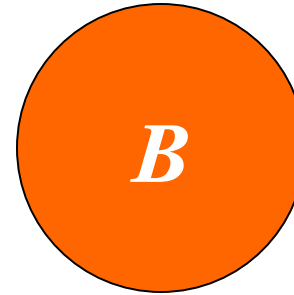
Which star is more luminous?

Luminosity versus radius and temperature



$$R = 2 R_{\text{Sun}}$$

$$T = T_{\text{Sun}}$$

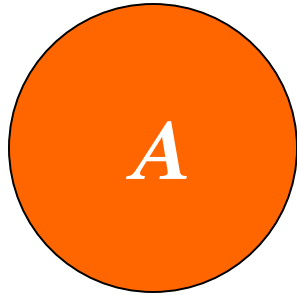


$$R = R_{\text{Sun}}$$

$$T = T_{\text{Sun}}$$

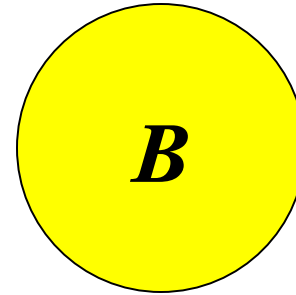
- Each cm^2 of each surface emits the same amount of radiation.
- The larger star emits more radiation because it has a larger surface. It emits 4 times as much radiation.

Luminosity versus radius and temperature



$$R = R_{\text{Sun}}$$

$$T = T_{\text{Sun}}$$



$$R = R_{\text{Sun}}$$

$$T = 2T_{\text{Sun}}$$

Which star is more luminous?

The hotter star is more luminous.

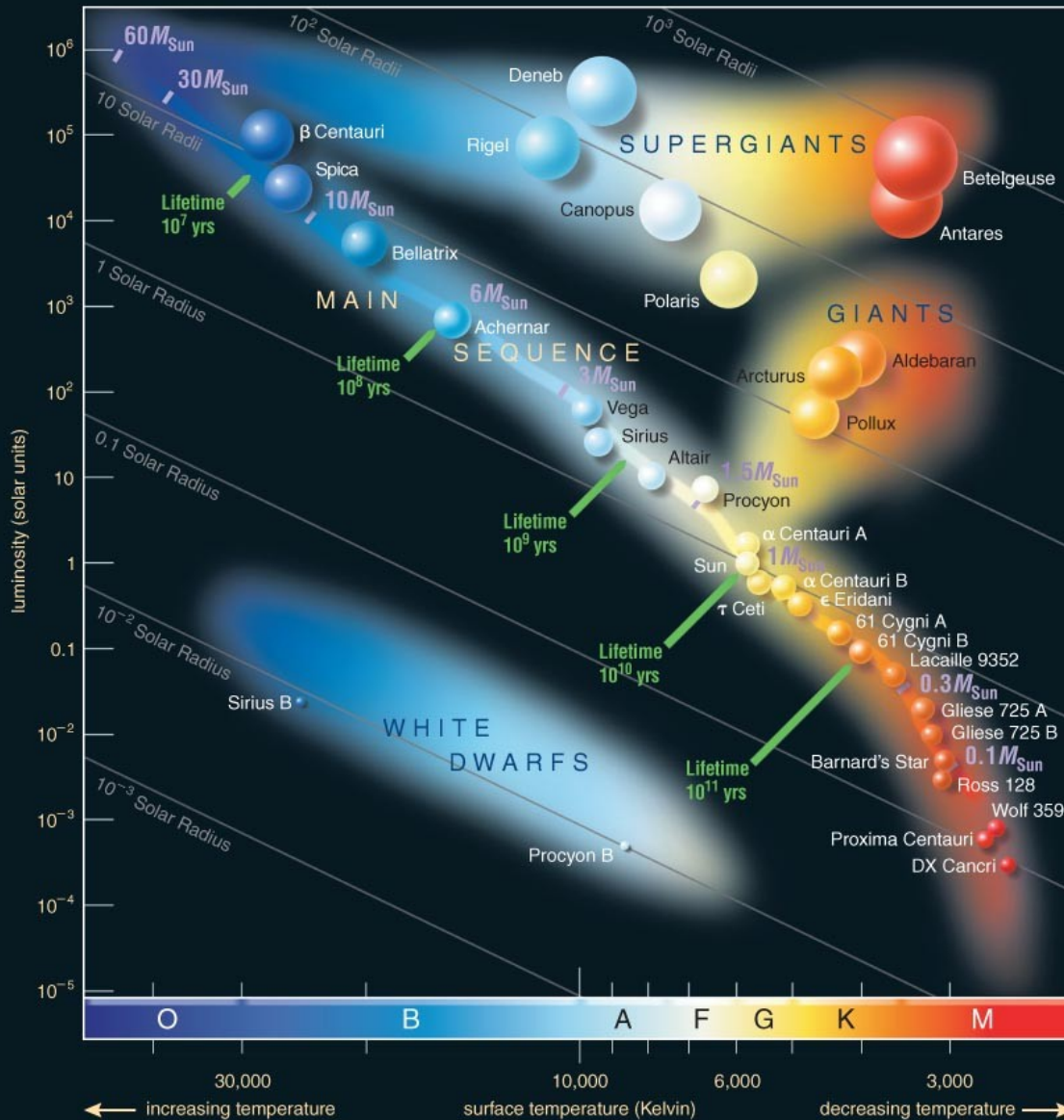
Luminosity varies as T^4

(Stefan-Boltzmann Law)

Luminosity Law

$$\frac{L_A}{L_B} = \frac{R_A^2 T_A^4}{R_B^2 T_B^4}$$

If star A is 2 times as hot as star B, and the same radius, then it will be $2^4 = 16$ times as luminous.



From a star's luminosity and temperature, we can calculate the radius.

Properties of Stars

- Stars have many different properties: mass, luminosity, radius, chemical composition, surface temperature, core temperature, core density, ...
- However, the entire history of how an isolated star will evolve – meaning how the properties of the star will change with time – is determined by just two properties: **mass** and **chemical composition**.
- This is the “Vogt-Russell” theorem.

“Vogt-Russell” theorem for spheres of water

- Spheres of water have several properties: mass, volume, radius, surface area ...
- We can make a “Vogt-Russell” theorem for balls of water that says that all of the other properties of a ball of water are determined by just the mass and even write down equations, i.e.
volume = mass/(density of water).
- The basic idea is that there is only one way to make a sphere of water with a given mass.

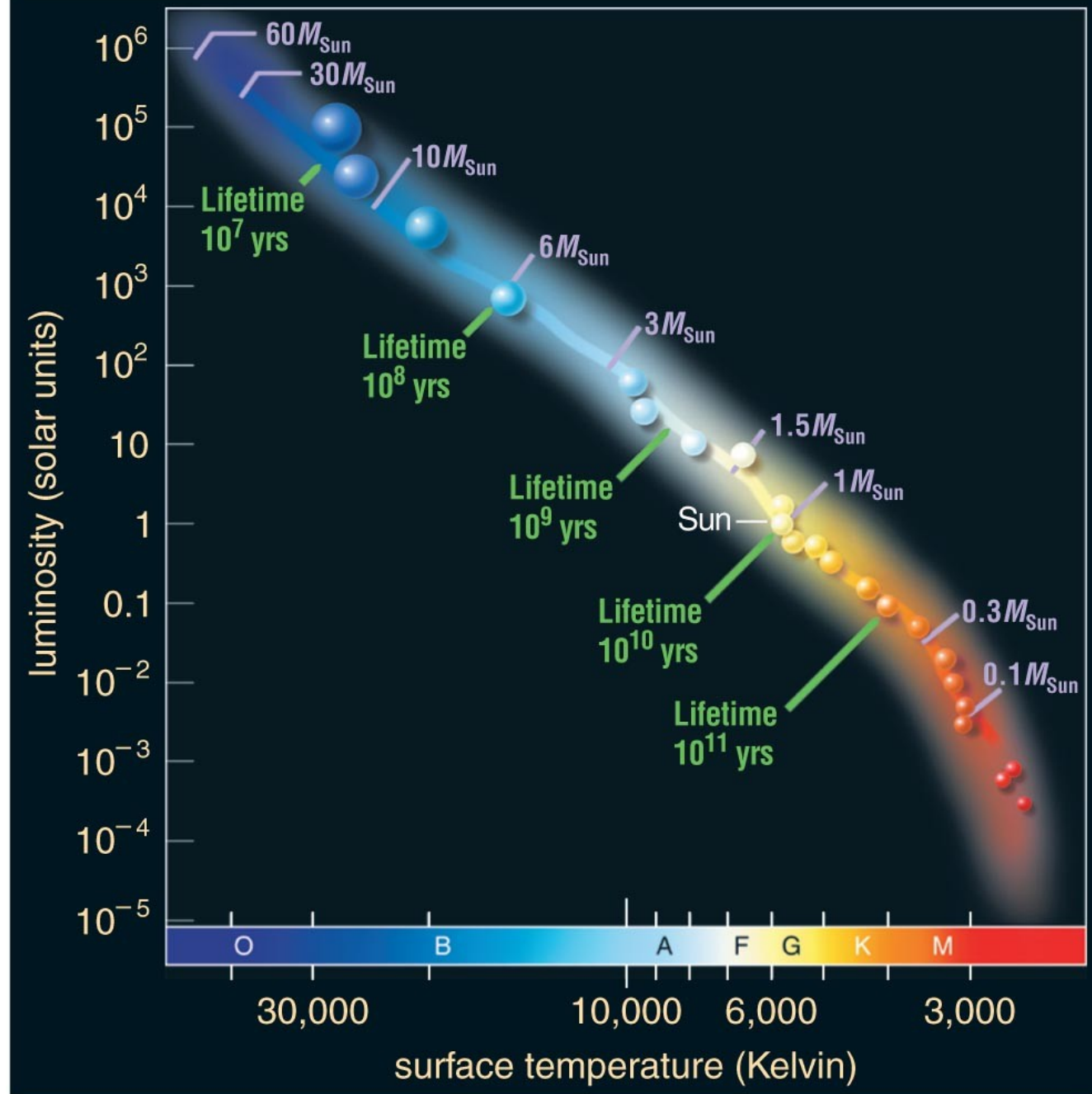
“Vogt-Russell” theorem

- The idea of the “Vogt-Russell” theorem for stars is that there is only one way to make a star with a given mass and chemical composition – if we start with a just formed protostar of a given mass and chemical composition, we can calculate how that star will evolve over its entire life.
- This is extremely useful because it greatly simplifies the study of stars and is the basic reason why the HR diagram is useful.

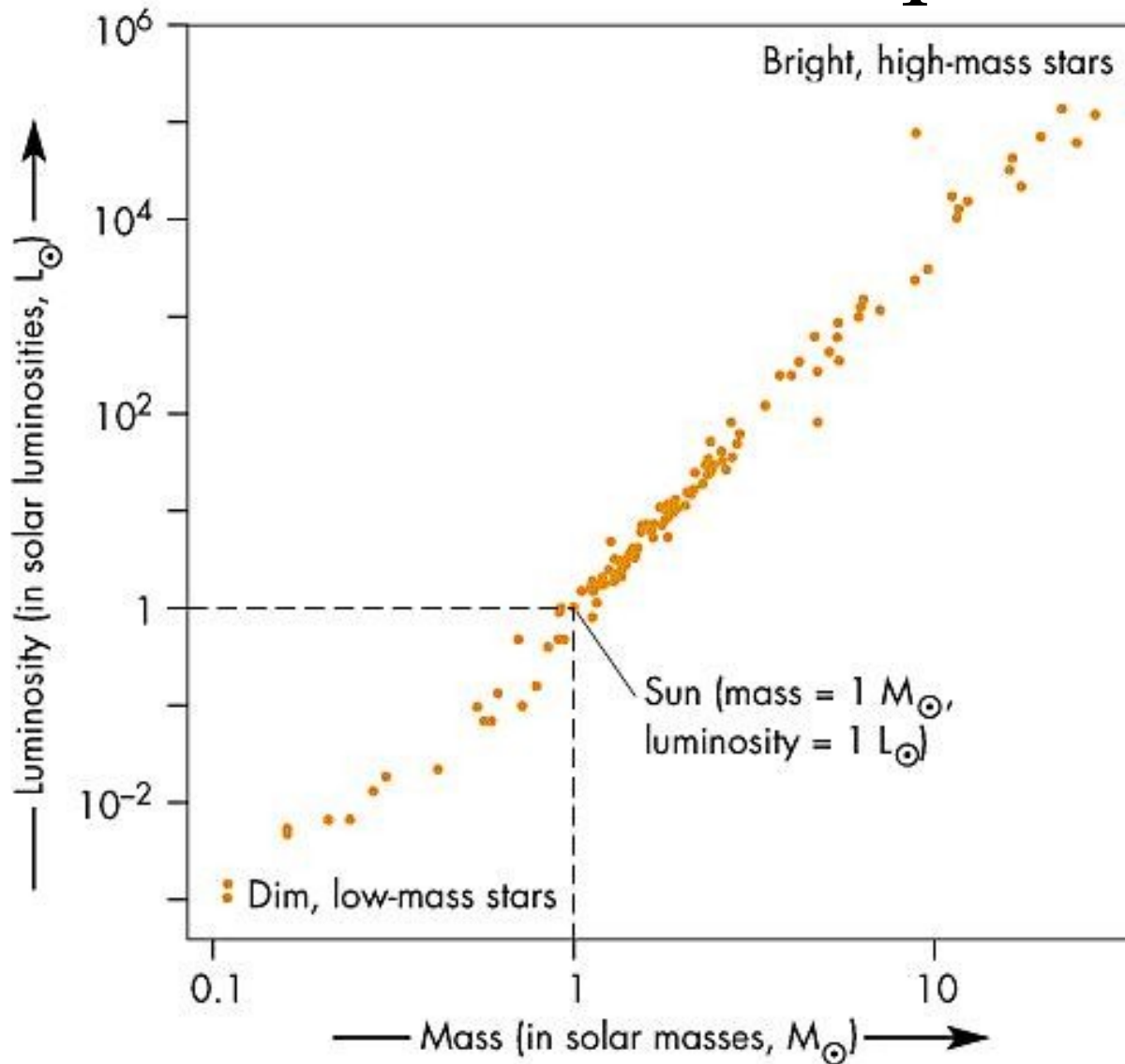
HR diagram

Main sequence is
when a star is
burning hydrogen in
its core.

The luminosity and
temperature of a
main-sequence star
are set by its mass.
More massive
means brighter and
hotter.



Mass-Luminosity relation on the main sequence



$$\frac{L}{L_{\odot}} = \left(\frac{M}{M_{\odot}} \right)^{3.5}$$

Mass-Lifetime relation

- The lifetime of a star (on the main sequence) is longer if more fuel is available and shorter if that fuel is burned more rapidly
- The available fuel is (roughly) proportional to the mass of the star
- From the previous, we know that luminosity is much higher for higher masses
- We conclude that higher mass stars live shorter lives

$$\frac{t_A}{t_B} = \frac{M_A L_B}{M_B L_A} = \frac{M_A M_B^{3.5}}{M_B M_A^{3.5}} = \frac{M_B^{2.5}}{M_A^{2.5}}$$

A ten solar mass star has about ten times the sun's supply of nuclear energy. Its luminosity is 3000 times that of the sun. How does the lifetime of the star compare with that of the sun?

- A) 10 times as long
- B) the same
- C) 1/300 as long
- D) 1/3000 as long

$$\frac{t_A}{t_B} = \frac{M_A L_B}{M_B L_A} = \frac{10}{1} \frac{1}{3000} = \frac{1}{300}$$

Mass-Lifetime relation

Mass/mass of Sun	Lifetime (years)
60	400,000
10	30,000,000
3	600,000,000
1	10,000,000,000
0.3	200,000,000,000
0.1	3,000,000,000,000

Choose the best match:

_____ stars have _____ than _____ stars.

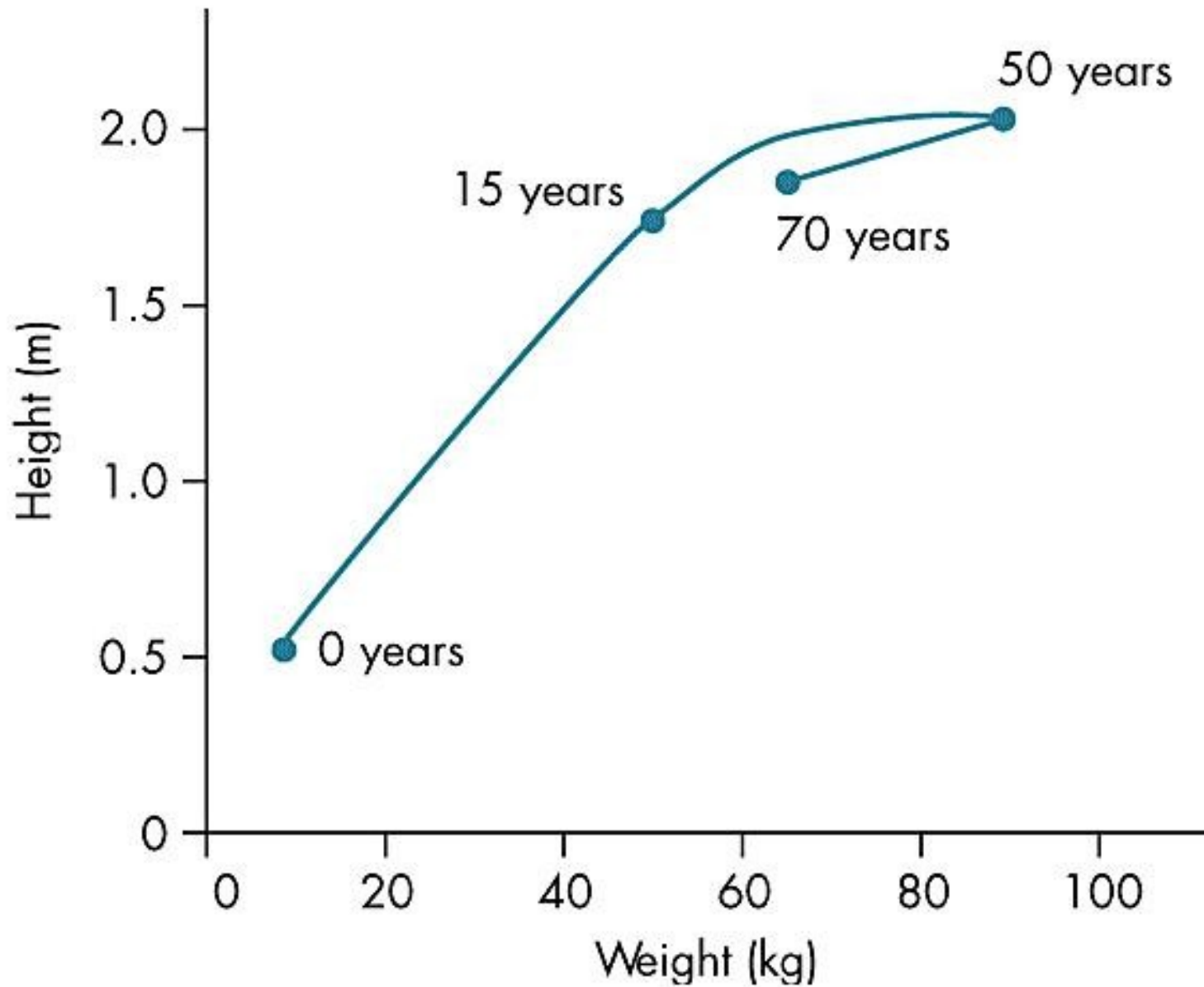
- A) Main-sequence : more mass : pre-main sequence
- B) More massive : shorter lifetimes : less massive
- C) More luminous : larger radii : less luminous
- D) Hollywood : worse tans : Broadway

$$t \propto \frac{M}{L} \propto \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}}$$

Evolution of stars

- Stars change over their lifetimes (from formation to death).
- We can track these changes via motion of the star in the HR diagram.

HW diagram for people



- The Height-Weight diagram was for one person who we followed over their entire life.
- How could we study the height-weight evolution of people if we had to acquire all of the data from people living right now (no questions about the past)?

- We could fill in a single HW diagram using lots of different people. We should see a similar path.
- We can also estimate how long people spend on particular parts of the path by how many people we find on each part of the path.

Review Questions

- What tool do astronomers use to understand the evolution of stars?
- What is the relation between luminosity, radius, and temperature?
- What parameters determine the evolution of a star?
- How does a star's mass influence its lifetime?