The geometry of the Universe

- How many degrees in the angles in a triangle?
- Positively/negatively curved spaces
- Measuring curvature using the cosmic microwave background

Why can't we see radiation produced during the first 1,000 years after the Big Bang?

A) It was absorbed soon after it was emitted.
B) It hasn't reached us yet.
C) It has been deflected by black holes.
D) It passed by our part of the universe a few billion years ago.

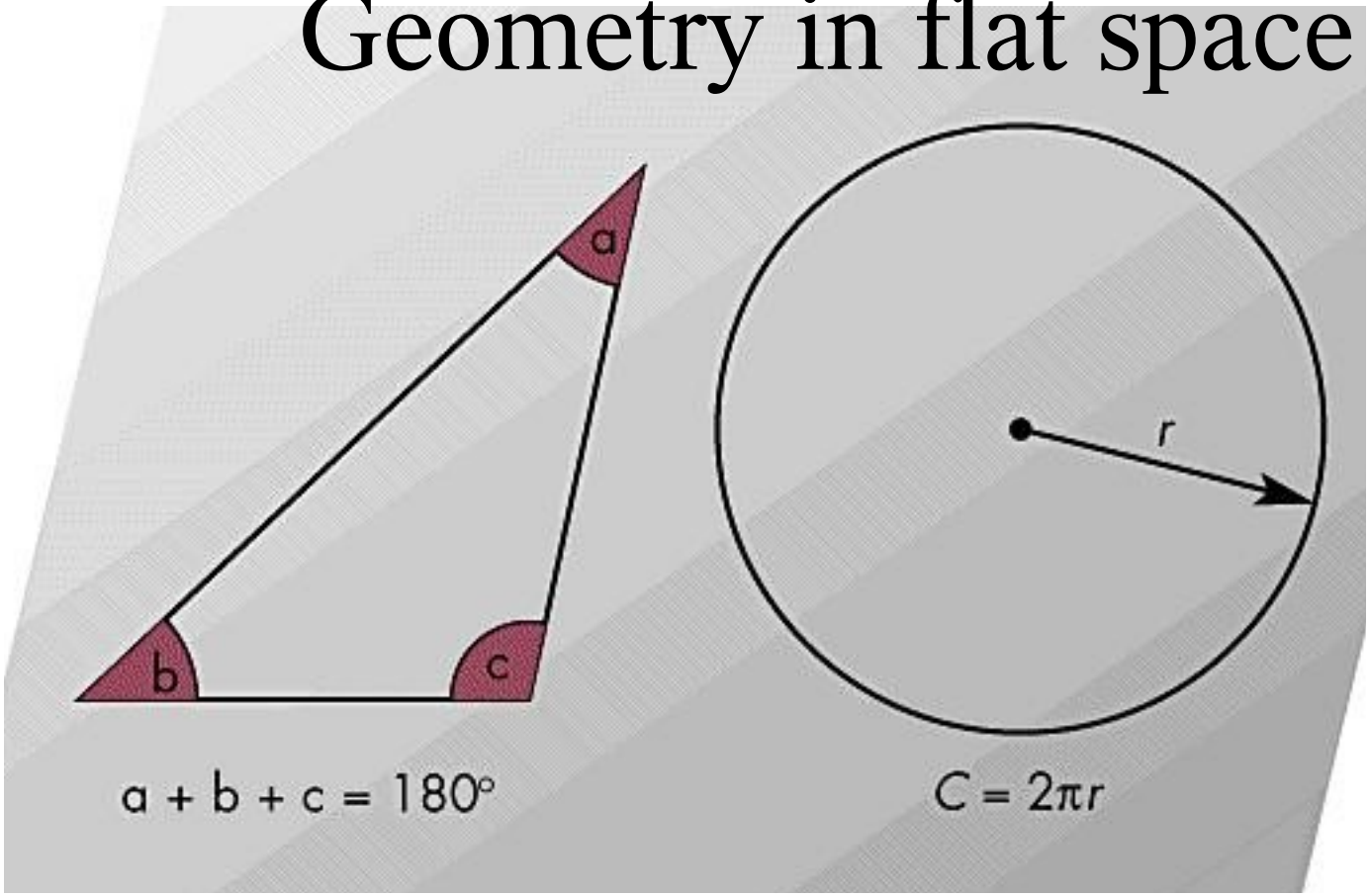
Two dimensional geometry

- Only two directions: up/down and left/right
  : north/south and east/west
- All motion of particles, light confined to two dimensions
- Examples: black board, piece of paper, surface of sphere, surface of donut, surface of saddle

Geometry

- How are the diameter and circumference of a circle related?
- What is the sum of all of the angles in a triangle?

Geometry in flat space



- Circumference = $2\pi \times$ radius

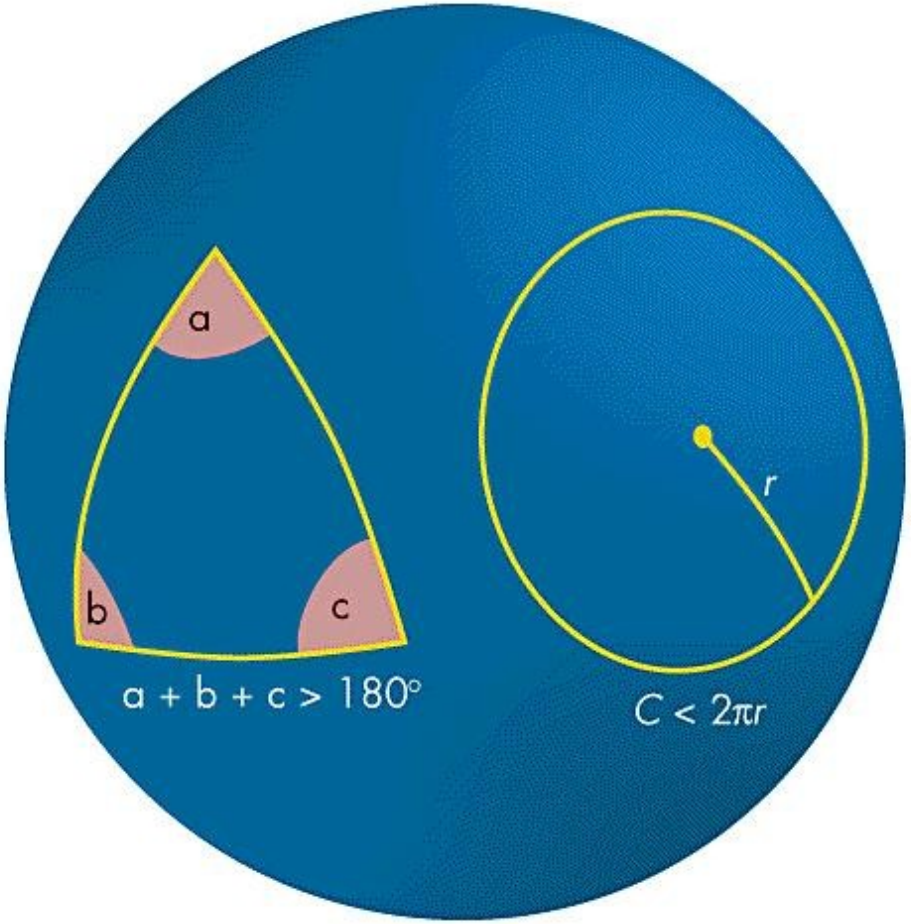
  $\pi = 3.1415926\ldots$ $2\pi = 6.28\ldots$

- The sum of the angles in a triangle is 180º

Non-Euclidean Geometry

- How about in a curved space?
- Do demo 8C10.50

Geometry in positively curved space



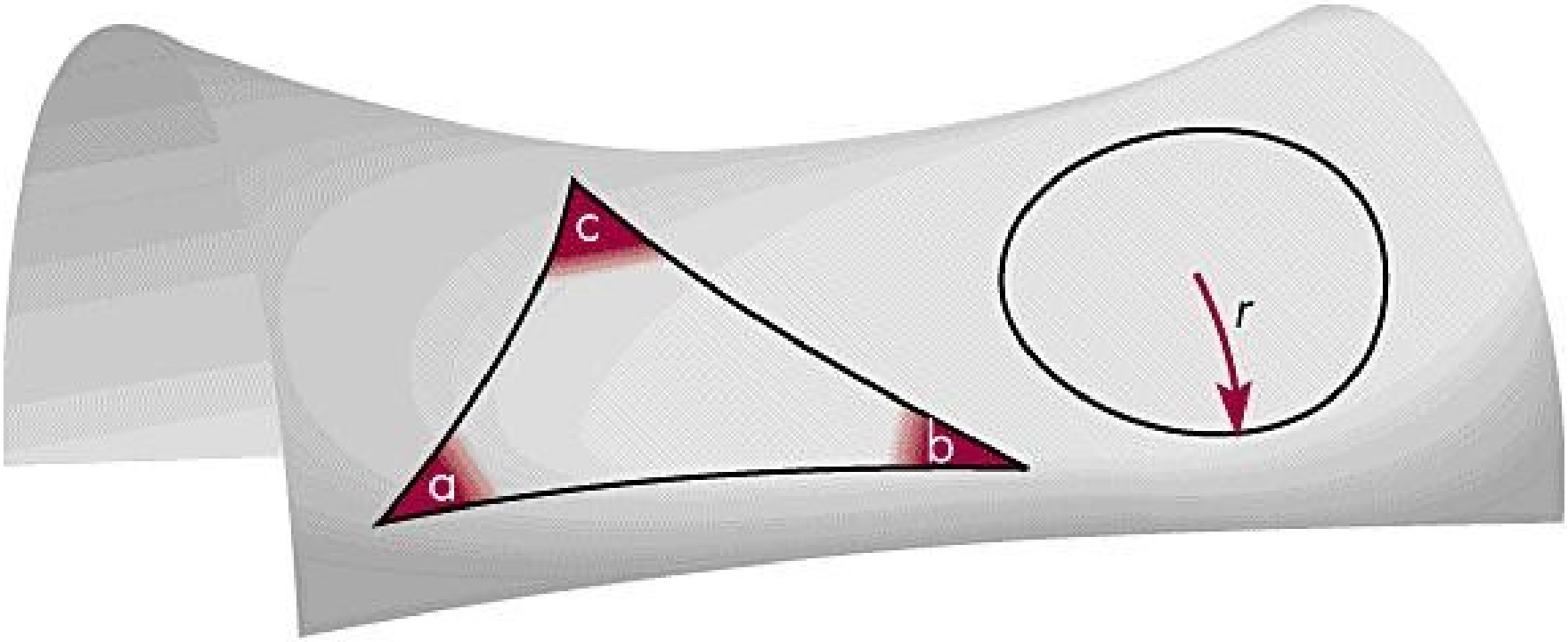
- Circumference $< 2\pi \times$ radius
- The sum of the angles in a triangle $> 180^{o}$

Properties of positively curved space

- Finite
- Unbounded
- No center

How do the properties of positively curved space differ from flat space?

Geometry in negatively curved space



- Circumference > $2\pi \times$ radius
- The sum of the angles in a triangle < 180º

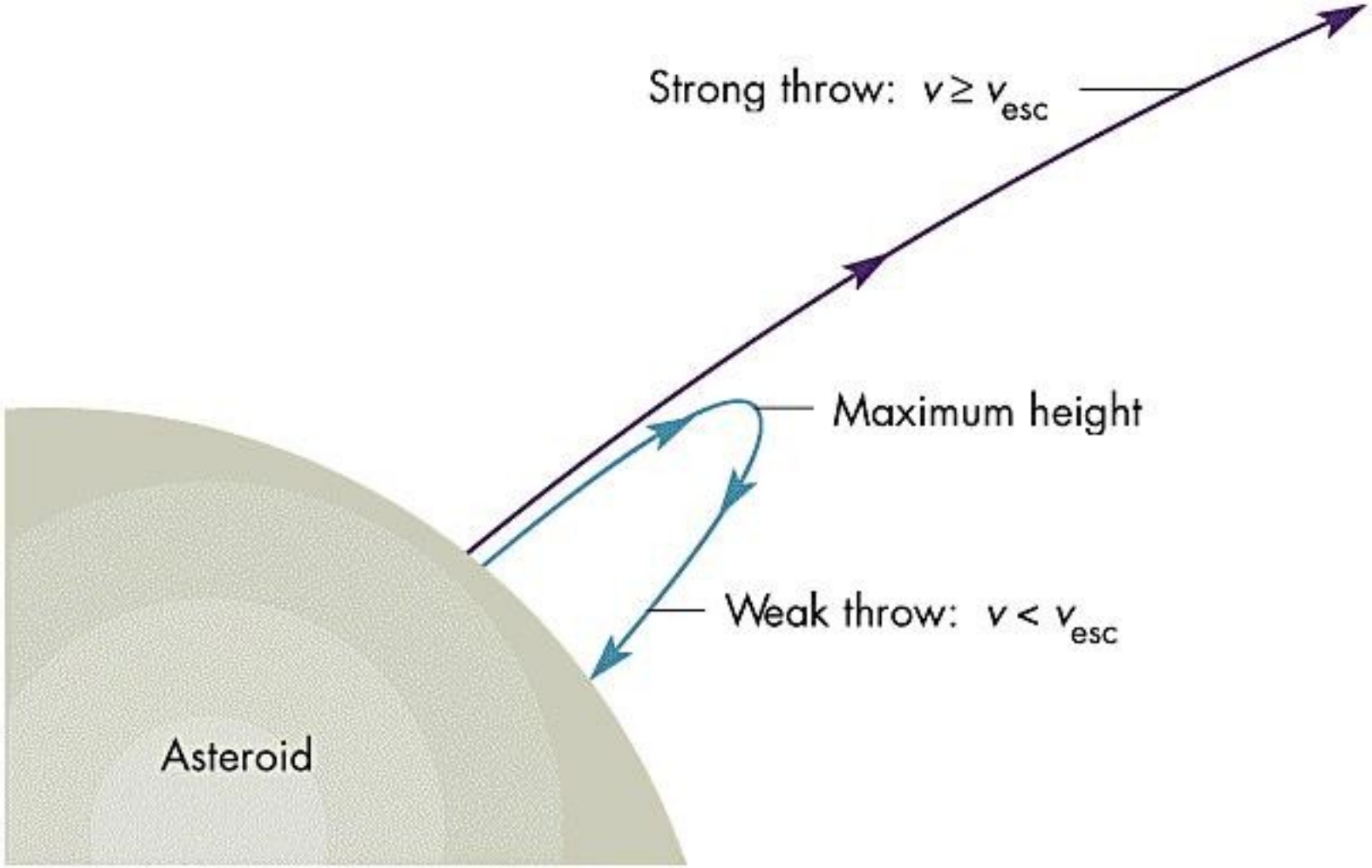
Curvature of the Universe

The curvature of the Universe is determined by:

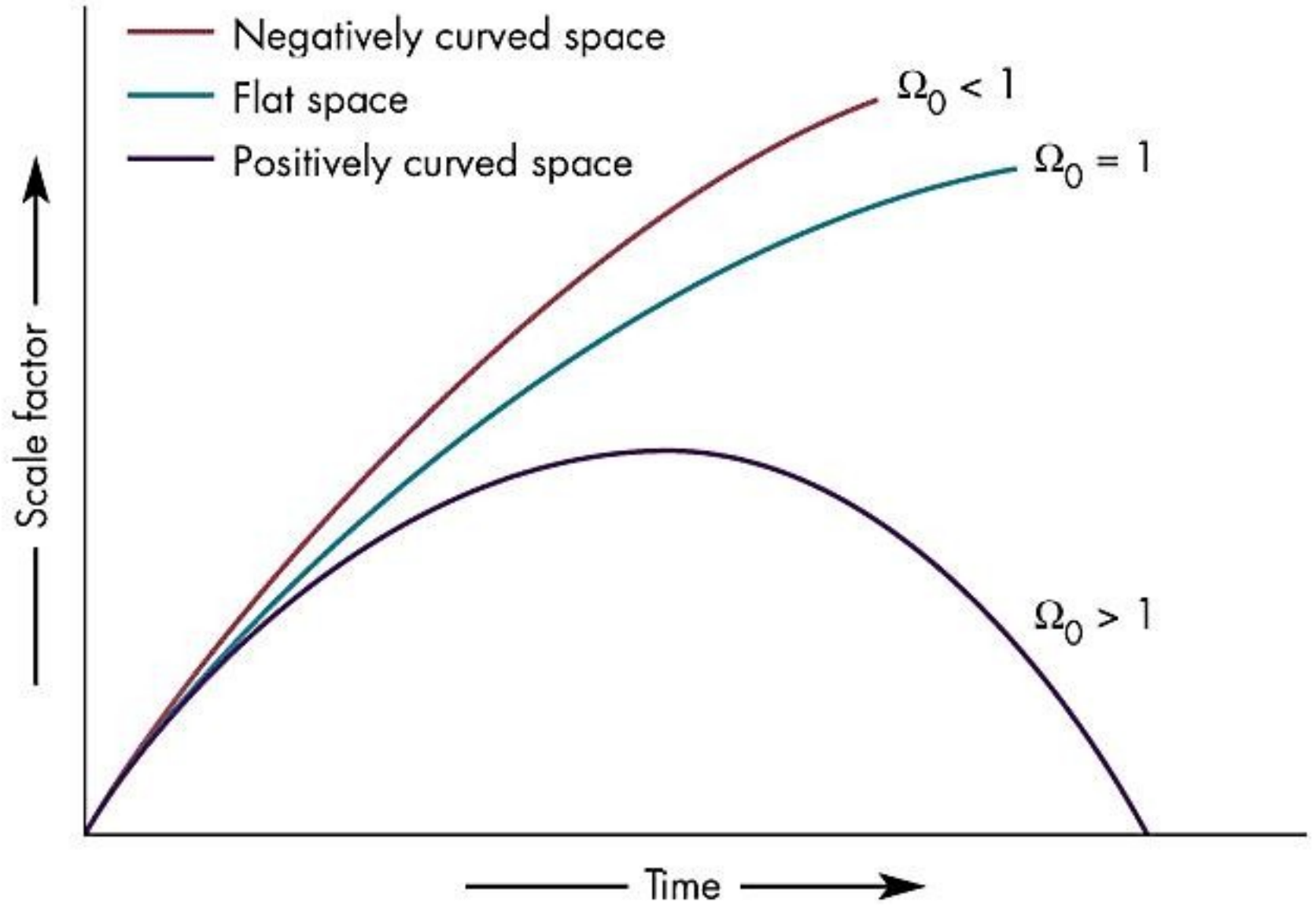
- the density of matter and energy
  - higher density produces positive curvature
  - gravity from matter always makes positive curvature
- the expansion of the Universe
  - more rapid expansion produces negative curvature

At the “critical density”, expansion exactly balances gravity – universe is flat

Fate of the Universe



Fate of the Universe



Curvature of the Universe

The curvature of the Universe is determined by the density parameter Ω_0

$$\Omega_0 = \frac{\rho}{\rho_C}$$

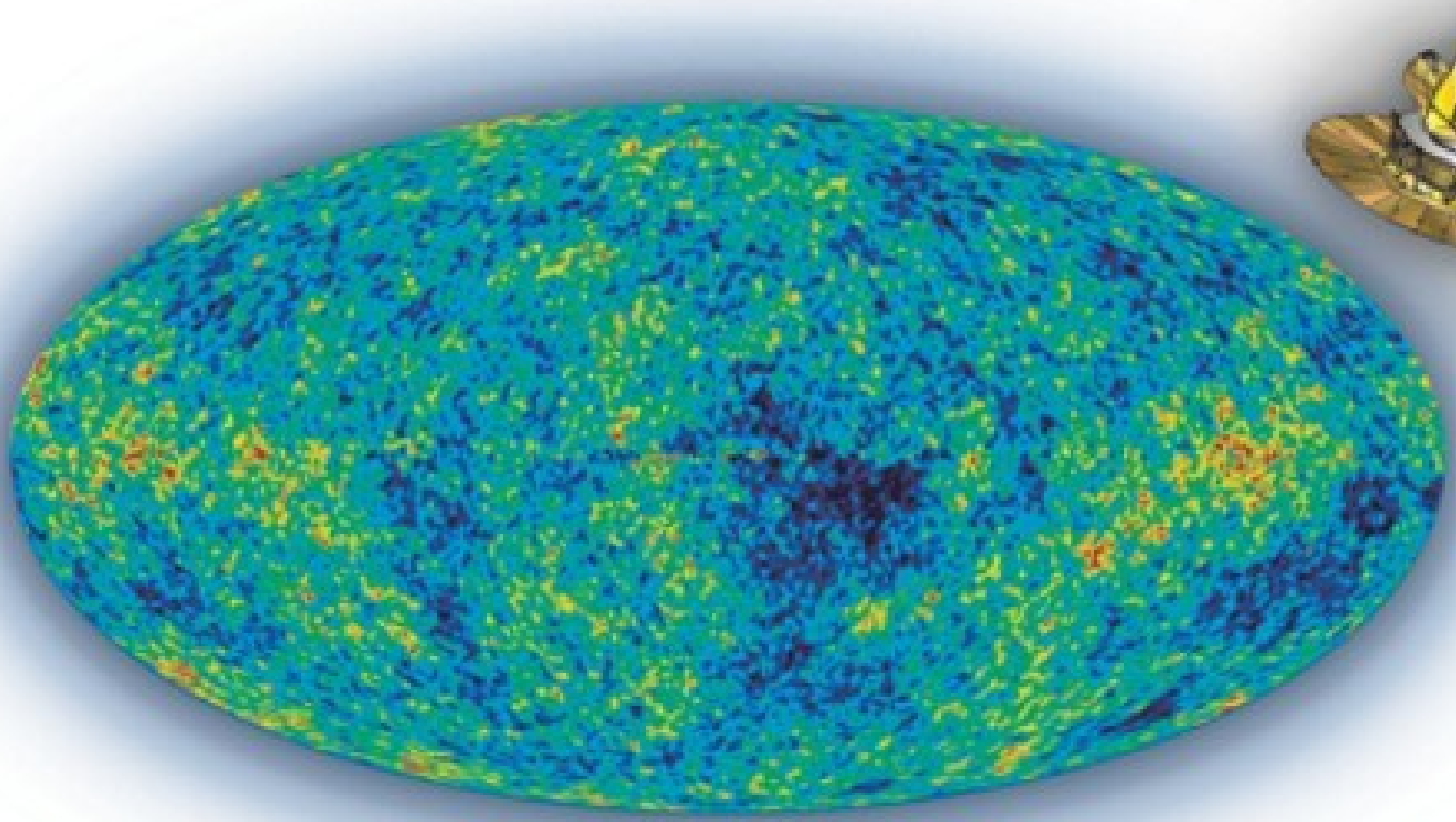
ρ = density of matter/energy

Critical density $\rho_c = 10^{-26}$ kg/m^3

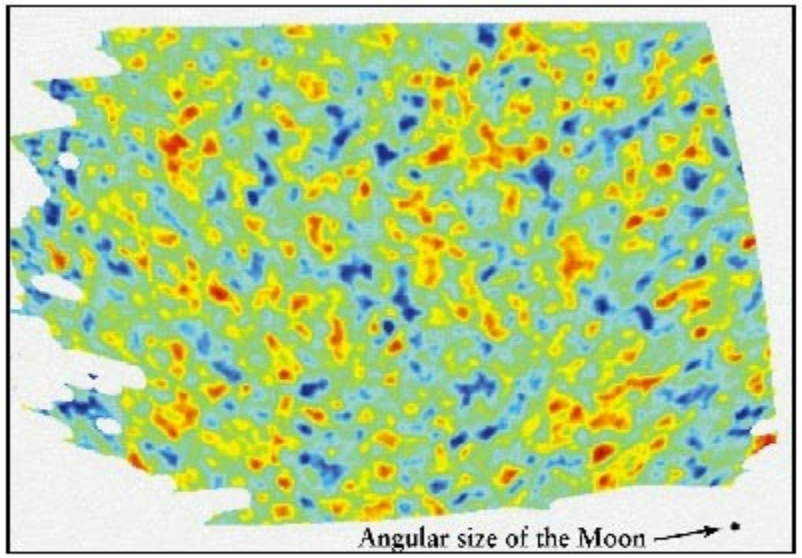
$\Omega_0 < 1 \Rightarrow$ negative curvature

$\Omega_0 > 1 \Rightarrow$ positive curvature

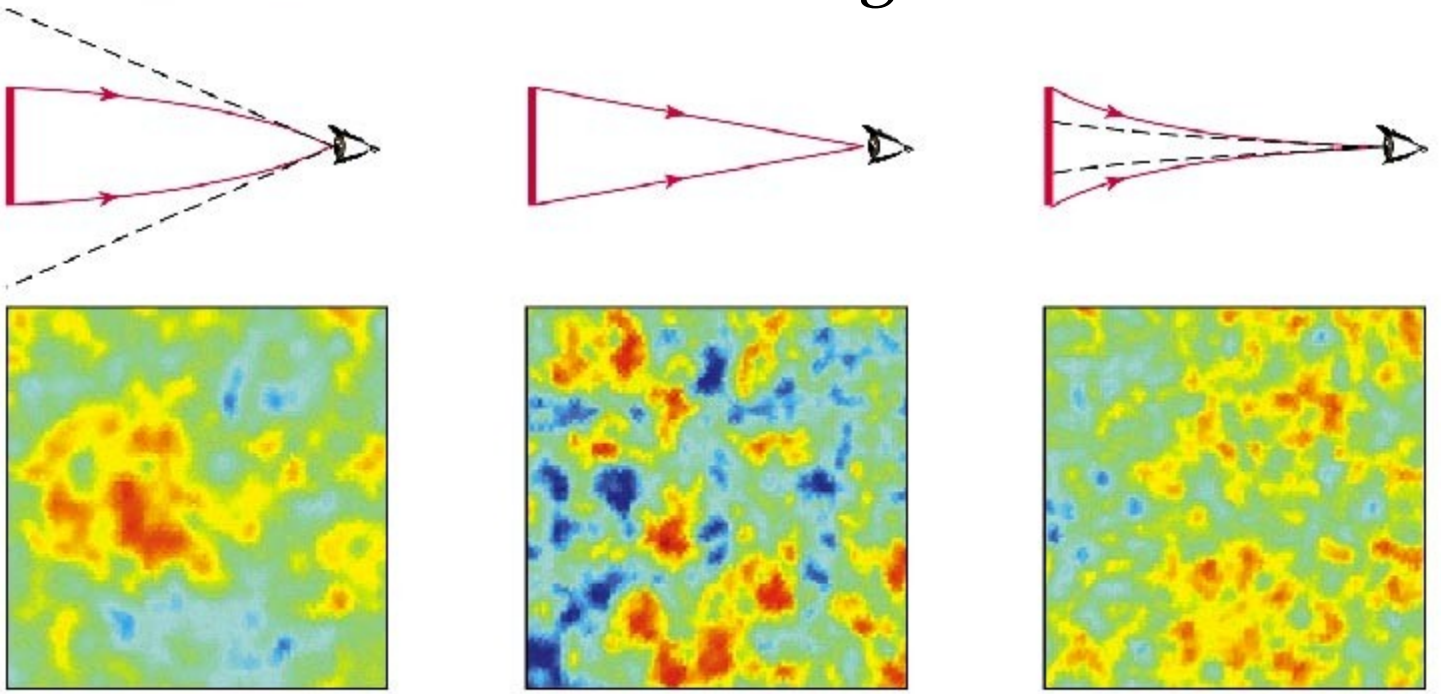
Cosmic Microwave Background



Small fluctuations are due to sound waves at recombination.



Temperature variations in the Cosmic Microwave Background (CMB) are observed to be about 0.0003 K. The expected physical size of the hot/cold regions can be calculated.



a If universe is closed, "hot spots" appear larger than actual size

b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size

Curvature of the Universe

The curvature of the Universe is determined by the density parameter Ω_0

$$\Omega_0 = \frac{\rho}{\rho_C}$$

$\Omega_0 < 1 \Rightarrow$ negative curvature

$\Omega_0 > 1 \Rightarrow$ positive curvature

Measurement of CMB fluctuations gives

$$\Omega_0 = 1.02 \pm 0.02$$