(L-4) Free fall, review

- If we neglect air resistance, all objects, regardless of their mass, fall to earth with the same acceleration $g \approx 10 \text{ m/s}^2$.
- This means that if they start at the same height, they will both hit the ground at the same time.

Free fall – velocity and distance

<table>
<thead>
<tr>
<th>time (s)</th>
<th>speed (m/s)</th>
<th>distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.45</td>
<td>4.5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>125</td>
</tr>
</tbody>
</table>

Motion with constant acceleration

- A ball falling under the influence of gravity is an example of what we call motion with constant acceleration.
- The nice thing about this is that if we know where the ball starts and how fast it is moving at the beginning we can figure out where the ball will be and how fast it is going at any later time!

Simplest case is acceleration = 0

- If the acceleration = 0 then the velocity is constant. [remember that acceleration is the rate of change of velocity]
- In this case the distance an object will travel in a certain amount of time is given by distance = velocity x time
- For example, if you drive at 60 mph for one hour you go 60 mph x 1 hr = 60 mi.

Example – running the 100 m dash

- Justin Gatlin won the 100 m dash in just under 10 s. Did he run with constant velocity, or was his motion accelerated?
- He started from rests and accelerated, so his velocity was not constant.
- Although his average speed was about 100 m/10 s = 10 m/s, he did not maintain this speed all through the race.

The velocity of a falling ball

- Suppose that at the moment you start watching the ball it has an initial velocity equal to $v_0$.
- Then its present velocity is related to the initial velocity and acceleration by present velocity = initial velocity + acceleration x time
- Or in symbols: $v = v_0 + a \cdot t$
Ball dropped from rest
• If the ball is dropped from rest then that means that its initial velocity is zero, \( v_0 = 0 \)
• Then its present velocity = \( a \cdot t \), where \( a \) is the acceleration of gravity \( g = 10 \text{ m/s}^2 \) or 32 ft/s\(^2\), for example:
• What is the velocity of a ball 5 seconds after it is dropped from rest from the Sears Tower? \( v = 32 \text{ ft/s}^2 \cdot 5 \text{ s} = 160 \text{ ft/s} \)

The position of a falling ball
• Suppose we would like to know where a ball would be at a certain time after it was dropped
• Or, for example, how long would it take a ball to fall to the ground from the top of the Sears Tower (1450 ft).
• Since the acceleration is constant (g) we can figure this out!

Falling distance
• Suppose the ball falls from rest so its initial velocity is zero
• After a time \( t \) the ball will have fallen a distance
  \[ \text{distance} = \frac{1}{2} \cdot \text{acceleration} \cdot t^2 \]

Falling from the Sears Tower
• After 5 seconds, the ball falling from the Sears Tower will have fallen
  \[ \text{distance} = \frac{1}{2} \cdot 32 \text{ ft/s}^2 \cdot (5 \text{ s})^2 = 16 \cdot 25 \]
  \[ = 400 \text{ feet}. \]
• We can turn the formula around to figure out how long it would take the ball to fall all the way to the ground (1450 ft)
  \( \text{time} = \sqrt{2 \times \text{distance}/g} \)

Look at below!
• Or \( \text{time} = \sqrt{\frac{2 \times \text{distance}}{g}} \)
• \( \text{time} = \sqrt{\frac{2 \cdot 1450 \text{ ft}}{32 \text{ ft/s}^2}} = \sqrt{\frac{2900}{32}} = \sqrt{90.6} = 9.5 \text{ s} \)
• when it hit the ground it would be moving at \( v = g \cdot t = 32 \text{ ft/s}^2 \cdot 9.5 \text{ sec} = 305 \text{ ft/s} \)
or about 208 mph (watch out!)

How high will it go?
• Let’s consider the problem of throwing a ball straight up with a speed \( v \). How high will it go?
• As it goes up, it slows down because gravity is pulling on it.
• At the very top its speed is zero.
• It takes the same amount of time to come down as go it

\[ v = 0 \text{ for an instant} \]
### An amazing thing!

- When the ball comes back down to ground level it has exactly the same speed as when it was thrown up, but its velocity is reversed.
- This is an example of the law of conservation of energy.
- We give the ball some kinetic energy when we toss it up, but it gets it all back on the way down.

### So how high will it go?

- If the ball is tossed up with a speed $v$, it will reach a maximum height $h$ given by
  \[ h = \frac{v^2}{2g} \quad \Rightarrow \quad v = \sqrt{2gh} \]
- Notice that if $h = 1\text{m}$,
  \[ v = \sqrt{2 \times 10 \times 1} = \sqrt{20} \approx 4.5 \text{ m/s} \]
- This is the same velocity that a ball will have after falling 1 meter.

### Example

- Randy Johnson can throw a baseball at 100 mph. If he could throw one straight up, how high would it go?
  - 1 mph = 0.45 m/s → 100 mph = 45 m/s
  - \[ h = \frac{v^2}{2g} = \frac{(45)^2}{2 \times 10} = 2025 \div 20 = 101 \text{ meters} \]
  - About 100 yards or the length of a football field!

### Example – comparing masses

- If you have 2 cubes of the same material, one with side 1 cubic centimeter and the other with side 2 cubic centimeters, how do the masses compare?
  - The mass is proportional to the volume which is given by $s^3$ where $s$ is the length of the side.
  - Thus the 2 cm cube has 8 times the volume and 8 times the mass.

### Escape from planet earth

- To escape from the gravitational pull of the earth an object must be given a velocity at least as great as the so-called escape velocity.
- For earth the escape velocity is 7 mi/sec or 11,000 m/s, 11 kilometers/sec or about 25,000 mph.
- An object given this velocity on the earth’s surface will not return.