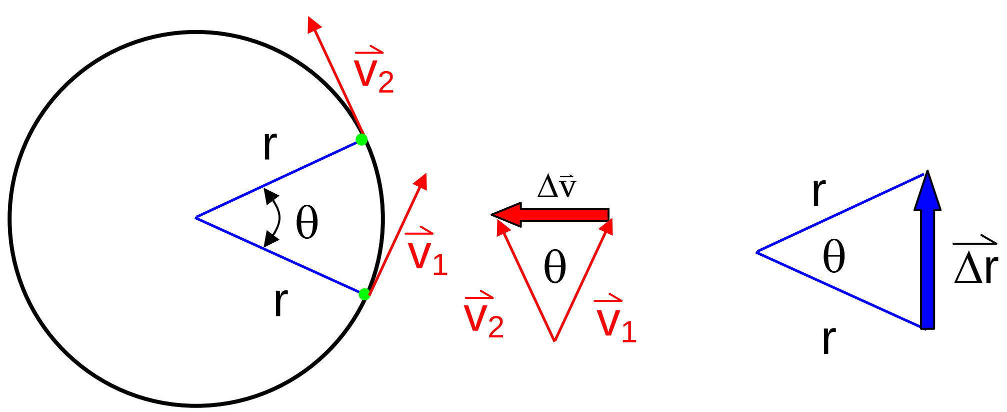
CENTREPITAL ACCELERATION a_c

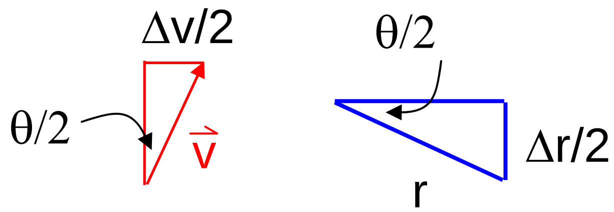
This is the specific term used for the acceleration experienced by an object in uniform circular motion (UCM)

In general: $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$, where for the case of UCM the <u>magnitude</u> of the velocity is *constant*, but its <u>direction</u> *constantly changes*.

a_c can be determined using a little geometry and trig. The figure shows a particle going around a circle (counterclockwise) at two times , t_1 and t_2, with respective velocities, $\vec{v}_1$ and $\vec{v}_2$. The positions correspond to an angular separation of θ. To find the difference $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$ we must assume that θ is small, since $\vec{v}$ is continuously changing its direction.



Since the velocity is everywhere tangent to the circular path, the velocity vector is everywhere perpendicular to the radius of the circle. Then the angle between the velocity vectors is also θ. If we form right triangles, then we have the following triangles:

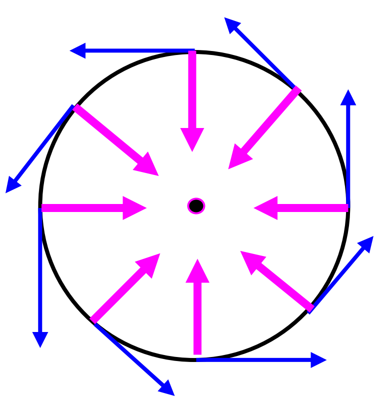


$$\sin\left(\frac{\theta}{2}\right) = \frac{\frac{1}{2}\Delta\text{r}}{\text{r}} = \frac{\frac{1}{2}\Delta\text{v}}{\text{v}} \Rightarrow \frac{\Delta\text{r}}{\text{r}} = \frac{\Delta\text{v}}{\text{v}} \Rightarrow \Delta\text{v} = \frac{\text{v}}{\text{r}}\Delta\text{r}$$

If the time difference between positions 1 and 2 is Δt, then the acceleration is

$$a_c = \frac{\Delta\text{v}}{\Delta t} = \frac{\text{v}}{\text{r}}\frac{\Delta\text{r}}{\Delta t}\text{, but } \frac{\Delta\text{r}}{\Delta t} \text{ is just v, so } \Rightarrow a_c = \frac{\text{v}}{\text{r}}\text{v} = \frac{\text{v}^2}{\text{r}}.$$

The direction of this acceleration is inward, pointing *toward the center of the circle*, this is the meaning of the word centripetal.



$$\boxed{a_c = \frac{\text{v}^2}{\text{r}}}$$