L-4 constant acceleration and free fall (M-3)

REVIEW

- Acceleration is the change in velocity with time
- Galileo showed that in the absence of air resistance, all objects, *regardless of their mass,* fall to earth with the same acceleration g
- **g ≅ 10 m/s²** → the speed of a falling object *increases* by 10 m/s every second
- Free fall is an example of motion with constant acceleration

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Motion with constant acceleration

- acceleration is the rate at which the velocity ***changes*** with time (increases or decreases)
- acceleration is measured in distance units divided by (time)^2, for example: m/s^2, cm/s^2, ft/s^2
- We will see how the velocity of an object changes when it experiences constant acceleration.
- First, we'll consider the simplest case where the acceleration is zero, so that the velocity is constant.

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Simplest case: *constant velocity* (a=0)

- If a = 0, then the velocity v is constant.
- In this case the distance x_f an object will travel in a certain amount of time t is given by **distance = velocity x time**

$$x_f = x_i + v\,t \quad (\text{for } a = 0)$$

- x_i is the starting (initial) position, and x_f is the final position.

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Example: constant velocity (a = 0)

A car moves with a constant velocity of 25 m/s. How far will it travel in 4 seconds?

Solution:

Suppose we take the starting point x_i as zero. Then,

$$x_f = 0 + vt = 0 + (25\text{ m/s})(4\text{ s}) = 100\text{ m}$$

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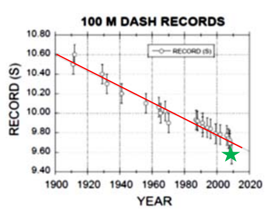
The 100 m dash

- Usain Bolt in 2009 set a new world record (★) in the 100 m dash at 9.58 s.
- Did he run with constant velocity, or was his motion accelerated?
- Initially at the starting line he was not moving (at rest), then he began moving when the gun went off, *so his motion was clearly accelerated*
- Although his average speed was about 100 m/10 s = 10 m/s, he did not maintain this speed during the entire race.



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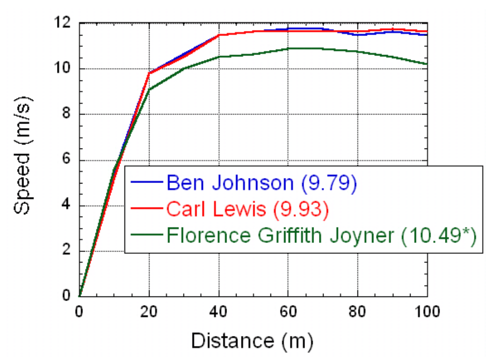
100 m dash Seoul 1988



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How to calculate acceleration

Example: Starting from rest, a car accelerates up to 50 m/s (112 mph) in 5 sec. Assuming that the acceleration was constant, compute the acceleration.

Solution: acceleration (a) = rate of change of velocity with time

$$a = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\text{final velocity} - \text{initial velocity}}{\text{final time} - \text{initial time}}$$

$$= \frac{50\,\text{m/s} - 0\,\text{m/s}}{5\,\text{s} - 0\,\text{s}} = \frac{50\,\text{m/s}}{5\,\text{s}} = 10\,\text{m/s}^2$$

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Motion with Constant acceleration

- Suppose an object moves with a constant acceleration a. If at t = 0 its initial velocity is **(v_i)**, then we want to know what its final velocity **(v_f)** be after a time t has passed.
- **final velocity = initial velocity + acceleration × time**

$v_f = v_i + a\,t$ (for constant acceleration)

- a t is the amount by which the velocity *increases* from v_i to v_f after a time t.
- *Note that if a = 0, $v_f = v_i$, i.e., velocity is constant.*

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Example: constant acceleration

A car moving initially at v_i = 3 m/s begins accelerating with a = 2 m/s². What is its velocity at t = 5 s?

Solution:

$$v_f = v_i + a \times t$$
$$= 3\,\text{m/s} + 2\,\text{m/s}^2 \times 5\,\text{s}$$
$$= 3\,\text{m/s} + 10\,\text{m/s}$$
$$= 13\,\text{m/s}$$

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Example – *deceleration – slowing down*

- Deceleration means that the acceleration is opposite in direction to the velocity
- Suppose you are moving at v_i =15 m/s and apply the brakes. The brakes provide a constant deceleration of **–5 m/s²**. How long will it take the car to stop?
- v_f = v_i + a t
- 0 = 15 m/s + (–5 m/s²) t
- 0 = 15 – 5t → 5t = 15 → t = 15/5 = 3 s

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Free Fall: Motion with constant acceleration

- According to Galileo, *in the absence of air resistance*, all objects fall to earth with a *constant* acceleration a = g ≅ 10 m/s²
- *g is the special symbol we use for the acceleration due gravity.*
- Since we know how to deal with constant acceleration, we can also solve problems involving free fall.

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Free fall – velocity and distance

time (s)	velocity (m/s)	distance y (m)
0	0	0 = ½ 10 (0)²
1	10	5 = ½ 10 (1)²
2	20	20 = ½ 10 (2)²
3	30	45 = ½ 10 (3)²
4	40	80 = ½ 10 (4)²
5	50	125 = ½ 10 (5)²

- If we observe an object falling from the top of a building we find that it gains speed as it falls
- Every second, its speed increases by 10 m/s.
- We also observe that it does not fall equal distances in equal time intervals. *The formula in the right column was discovered by Galileo*.

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Ball dropped from rest

- If the ball is dropped *from rest,* that means that its initial velocity is zero, $v_i = 0$
- Then its final velocity after a time t is **v_f = a t**, where $a = g \cong 10\ m/s^2$ so, **v_f = g t**
- Example: What is the velocity of a ball 5 sec. after it is dropped from rest from the top of the Sears Tower (*Willis Tower*)?

Solution: $v_f = g\ t = 10\ m/s^2 \times 5\ s = 50\ m/s$
(about 112 mph)

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Relationship between time and distance in free fall

- It would be useful to know how long it would take for an object, *dropped from rest,* to fall a certain distance
- For example, how long would it take an object to fall to the ground from the top of the Sears Tower, a distance of 442 m?
- Or, after a certain time, how far will an object, *dropped from rest*, have fallen?

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Falling distance

- Suppose an object *falls from rest* so its initial velocity $v_i = 0$.
- After a time t the ball will have fallen a distance: y_f = ½ acceleration • time^2
- $y_f = \frac{1}{2} g\ t^2$
- This is the formula Galileo discovered

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Falling from the Sears Tower

Example

How far would a ball dropped from rest at the top of the Sears Tower fall in 5 seconds?

Solution

$y_f = \frac{1}{2} \times 10\ m/s^2 \times (5\ s)^2 = 5\ m/s^2 \times 25\ s^2$

$= 125$ m (about 410 feet)

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Time to reach the ground

- Another interesting question, is how long it will take an object, *dropped from rest* from the top of the Sears Tower (442 m) take to reach the ground?
- To answer this question we need to solve the time-distance formula for t

$$y_f = \frac{1}{2} g t^2 \rightarrow 2y_f = g t^2 \rightarrow t^2 = \frac{2y_f}{g} \rightarrow \boxed{t = \sqrt{\frac{2y_f}{g}}}$$

$$\text{So: } t = \sqrt{\frac{2 \times 442}{10}} = 9.4\ \text{s.}$$

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Velocity as object hits the ground

- How fast will the object be moving when it hits the ground?
- We apply the velocity vs. time relation:
  - $v_f = v_i + g\ t$, with $v_i = 0$.
  - $v_f = g\ t = 10\ m/s^2 \times 9.4\ s = 94\ m/s$
  - or about 210 mph (neglecting air resistance)

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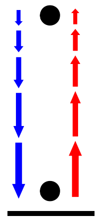
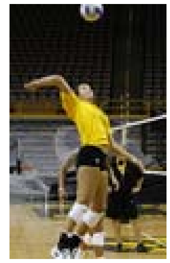
Time to go up

- Suppose a ball is thrown straight up with a speed v_i. *When does it reach its maximum height?*
- As it rises, it slows down (decelerates) because gravity is pulling it down.
- At its maximum height, it is instantaneously at rest, so that $v_f = 0$ at the top.
- **$v_f = v_i + a\,t$** applies whether an object is falling or rising. On the way down it speeds up, so $a_{down} = +g = 10 m/s^2$; on the way up, it slows down, so $a_{up} = -g = -10 m/s^2$
- Since $v_f = 0$ at the top, then we have: $v_f = 0 = v_i + (-g)\,t$, so **$t_{up} = v_i / g$** (time to max. height)



Example



A volleyball player can jump straight up at 5 m/s. *How long is she in the air?*

Solution:

total time in the air $= t_{total} = t_{up} + t_{down}$

- The time for her to get to the top $= t_{up} = v_i / g$, where v_i is her initial upward velocity, so

  $t_{up} = 5$ m/s / 10 m/s^2 = ½ sec.

- *It takes exactly the same amount of time to reach the top as it does to return to the ground, or $t_{up} = t_{down}$, so*

t_{total} = ½ s + ½ s = 1 s (This is the amount of time that she is in the air.)

Escape from planet earth

- To escape from the gravitational pull of the earth an object must be given a velocity larger than the so called *escape velocity*
- *For earth the escape velocity is 7 mi/sec or 11,000 m/s, 11 kilometers/sec or about 25,000 mph.*
- An object given at least this velocity on the earth's surface can escape from earth!
- The Voyager 2 spacecraft (*part of which was built in the UI Physics Dept.*) launched on Aug. 20, 1977, recently left the solar system and is the first human-made object to reach interstellar space.