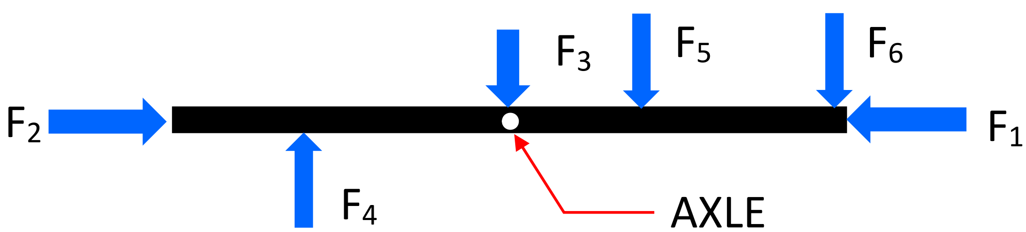
PHYS:1200 LECTURE 10 – MECHANICS (9)

This lecture deals with the topic of the rotational dynamics of a rigid body. It is basically an extension of Newton's 2nd law to the problem of the rotation of extended objects like a board,

wheel, or the airfoil of a wind powered electric generator. The basic question that we will deal with is what is necessary to start and keep an object rotating. The new concepts that will be introduced are torque, rotational inertia, and rotational momentum. One of the problems that will be discussed, for example, is why a

spinning ice skater spins very fast when she pulls her arms in close to her body.

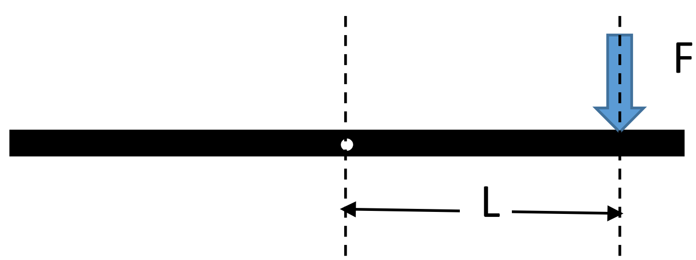
10-1. Torque on a Rigid Body.—A **rigid body** is an extended object, like a board or a rod, which cannot be bent, stretched or compressed. As a simple case, consider a rigid rod that has an axle through its center and can rotate either in the clockwise or counterclockwise sense about that axis, as shown below. The axle is also called the axis of rotation.



Various forces are applied to the rigid body at various places. Which forces will cause the rod to rotate, either clockwise or counterclockwise? Because of the locations of application of F_1, F_2, and F_3 none of these will make the rod rotate, regardless of the strength of these forces. F_4 will cause the rod to rotate in the counterclockwise sense and F_5 and F_6 will make it rotate in the clockwise sense. F_5 and F_6 have the same magnitude, but F_6 will be more effective in producing rotation than F_5 because it is applied farther from the pivot point. If you think about opening a door, you want to apply your hand at a point as far as possible from the hinge to make it open. It will require a much larger push if you put your hand on the side near the hinge.

Clearly force alone is not the only parameter that matters in producing rotation. Rotation depends on force and the point of application and orientation of the force relative to the axle. Even though F_1 and F_6 are applied at the same distance from the axle, F_6 causes rotation and F_1 does not cause rotation. The reason that F_1, F_2, and F_3 do not produce rotation is that if you imagine a line drawn through these forces, this *line of action* passes directly through the axle. **If the line of action of a force passes through the axis of rotation it cannot produce rotation about that axis.**

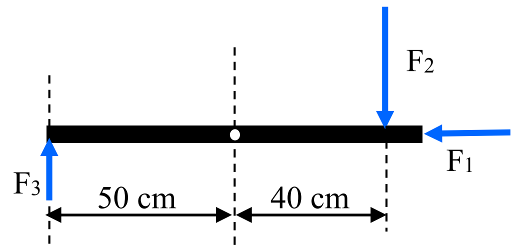
The combination of force and point of application is contained in a parameter known as **TORQUE**. **A net torque must be applied to a rigid body to make it rotate**. If a force applied to an object does not produce torque about the axis of rotation, it cannot cause the object to rotate about that axis. A force produces a torque about an axis of rotation only if the line of action of that force does not pass through the axis of rotation. Suppose we have a force F whose line of action does not pass through the axis of rotation. What torque (symbol tau, τ) is produced by a force that is applied at a distance L from the axis, as shown below?



TORQUE	$\tau = F \times L$	[1]

Torque is measured in units of N m. L is called the lever arm of the force, and the larger L is, the greater the torque that it produces. This is why it is easier to remove a bolt with a long wrench than with a short wrench, or why it is easier to pry open the lid of a crate with a long crowbar than with a short crowbar (**slide 4, 5, 6**).

EXAMPLE 10-1: A rigid body is 1 m long and can rotate about an axis through its center. Three forces are applied to it: F_1 = 10 N, F_2 = 20 N, and F_3 = 5 N, as shown in the figure. What is the net torque on the rigid body?

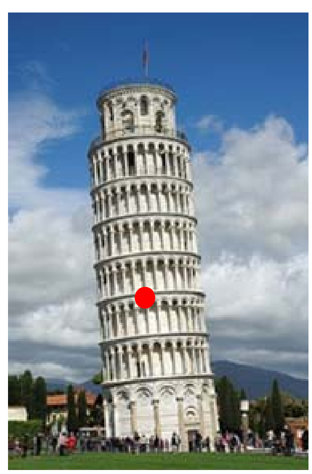


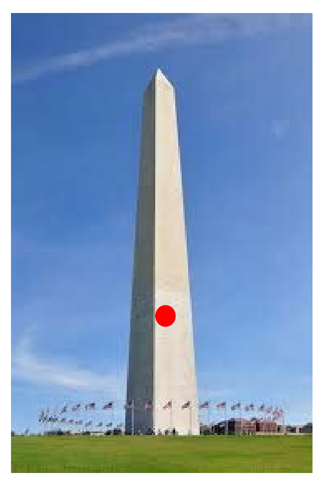
Solution- The net torque is the sum of thetorques produced by F_1, F_2, and F_3. Since the line of action of F_3 passes through the axis, it produces no torque. Both F_2 and F_3 produce torques that both cause clockwise rotation, so their net effect is additive.

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 = 0 + (20\,N)(0.4\,m) + (5\,\mathrm{N})(0.5\,m) = \left(8 + 2.5\right) Nm = 10.5\,N\,m.$$

Slides 7 and 8 show that if more than one force acts on an object, it is possible for the net force to be zero while the net torque is not zero, or why the net force might not be zero and yet the net torque is zero. **Slide 9** gives an example of how torques can be balanced to produce no net torque and no rotation. Torques can be thought of as having a direction, like forces, in the sense that one torque may produce clockwise rotation and another torque may produce counterclockwise rotation, and it is the net torque that determines the sense of rotation.

10-2. The Stability of a Rigid Body.—Stability is a property of an object that quantifies how difficult it is make it topple over. For example, the Leaning Tower of Pisa leans vertically at an angle of 4 degrees. What doesn't it fall over? Another example is the Washington Monument which is 169 meters tall—why is it stable?

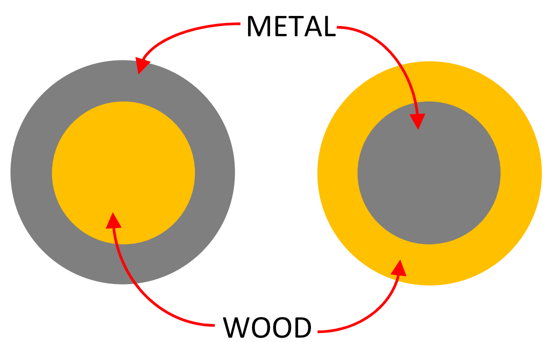




The stability of an object depends on the location of it **center of gravity**. The center of gravity **(CG)** of an object is the hypothetical point in the object where the force of gravity is considered to act. For any object that has a simple shape, like a rectangle, cube or a sphere, the CG is exactly at its center. For an object like the Tower of Pisa or the Washington Monument the

CG is somewhat below its center point because the base is wider that the top. (The red dot indicates roughly the location of the CG for the Washington Monument.) **The lower the CG of an object, the more stable it will be to being toppled over**. It is easier to balance a short wide object than a tall thin one. An object will be stable if the CG is above a point of support (**see slides 13, 15, 16, 17, 18**). The stability of an object depends on how it responds when a force is applied to try to tip it over (**slide 14**). If the object is tipped a bit and reverts back to its original position when released, then it is stable. As shown in **slide 14**, it all depends on where the CG is. If the line passing through the CG still remains within the point of contact, the object will rotate back to its original position.

10-3. Rotational Inertia.—A rigid body may have a complex shape and how it responds to an applied torque depends not only on its total mass, but also on how that mass is distributed in the body relative to the axis of rotation. The parameter which characterized the mass distribution in a rigid body relative to a specific axis of rotation is called its **rotational inertia**. We use the symbol I to represent the rotational inertia of a rigid body. **Slide 20** shows a rod of mass m and length L. The rotational inertia depends on the location of the axis of rotation, which is different for the axis through the center or through the end of the rod. An example of two rigid bodies having the same mass and same radius but different rotational inertias is shown below. Each object is a disk made of a combination of metal and wood, designed to have the same mass and radius. These wheels have different rotational inertias and thus require different torques to get them spinning. The wheel having the metal on the outside will have the larger value of the rotational inertia because metals are denser than wood and thus there will be more mass farther from the center than for the wheel with the wood on the outside.



The rotational inertia of a rigid body determines how it will respond to an applied torque. In other words, how long it will take to get it to spin up to a particular rotation rate for a given amount of torque applied to it. We will take this up in the next lecture.