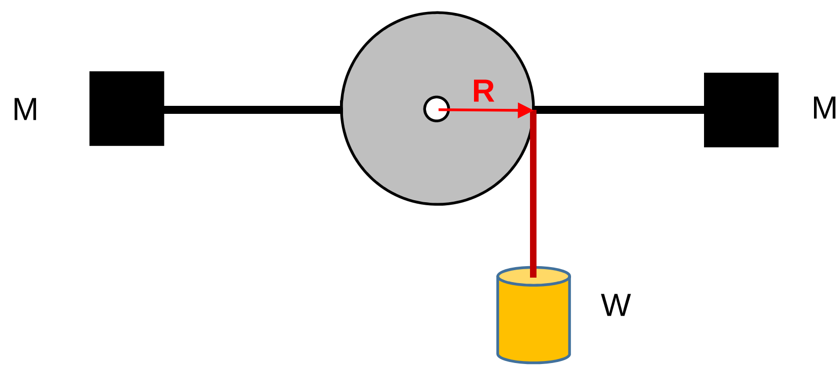
PHYS:1200 LECTURE 11 – MECHANICS (10)

This lecture continues the discussion of the dynamics of rigid bodies. We begin by reviewing the important concept of rotational inertia, which is the parameter that characterizes the physical characteristics of a rigid body in terms of its mass, dimensions, and mass distribution. We then discuss how the rotational speed of a rigid body is measured. Finally, we introduce another of the fundamental conservation laws of physics – the law of conservation of rotational momentum. This will enable us to understand why the spinning ice skater spins faster when she pulls her arms in close to her body.

11-1. Rotational Inertia.—The effect of the rotational inertia of a rigid body can be demonstrated using the device shown below. A wheel of radius R which can rotate about an axle through its center has a long rod attached to it with two masses that can be placed at various distances from the axle. A string is wrapped around the wheel and a weight W is attached to the string so that as the weight falls it makes the wheel spin. The rigid body consists of the wheel, the two masses and the connecting rod. The rotational inertia of the rigid body can be changed

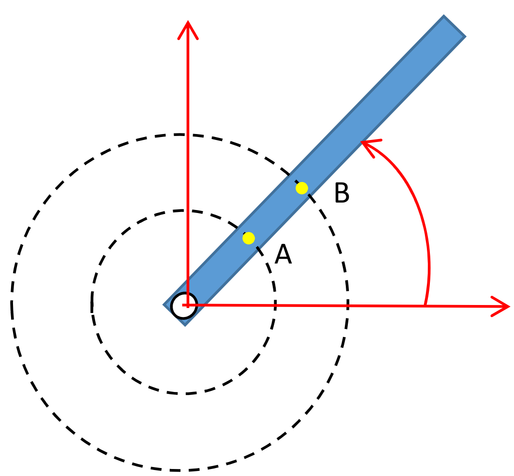


by changing the position of the two masses. The rigid body has its largest rotational inertia when the two masses are as far from the axle as possible. The hanging weight attached to the wheel provides the torque $\tau = FR$ which causes the rigid body to rotate. Two experiments are preformed: one with the two masses as far from the axle as possible (high rotational inertia), and one with the two masses closer to the axle (low rotational inertia). In both cases the rotation rate (the number of revolutions per second) of the rigid body is measured. The result is that the rotation speed is higher for the case in which the rotational inertia is smaller. Note that in both cases the torque on the rigid body is the same. **The experiment demonstrates that for a given**

torque applied to a rigid body, the rate of rotation depends inversely on the rotational inertia. Slide 7 describes another experiment that demonstrates the role of the rotational inertia.

11-2. Rotational Velocity.—In the experiment described above, we talked about rotation speed but never gave it a formal definition. How is rotation quantified? For an object going around and around in a circle, the rotation speed can be quantified by measuring the number of complete revolutions of the object each second, or in some applications the number of revolutions per minute, also known as rpm. For example, the earth rotates on its axis once per day, and revolves around the sun once per year. We might also keep track of the number of degrees a certain point on the rigid body rotates per second – this is the angular displacement per unit time. The direction of the angular speed is quantified by noting if the rotation is clockwise or counterclockwise.

What is the **relation between angular speed and linear speed?** Linear speed is the ordinary speed in m/s that an object has as it moves from one location to another location. Linear speed can also be used for a rotating object, but the linear speed of an extended object depends on which point on the object we are looking at. Think about a rod that is pivoted at one end and rotates in a circle, in the counterclockwise direction, as shown below. Look specifically about

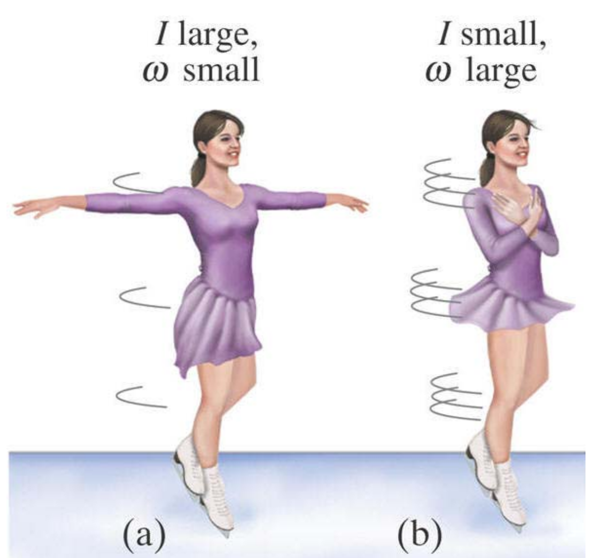


the two point on the rod labeled A and B. **First it is important to realize that as the rod rotates, all points on the rod rotate through the same angle on the same time interval—every point has**

the same angular speed. However the points farther from the axis of rotation (point B) move through an arc that is longer than that of a point closer to the axis of rotation (point A). In other words the circle that point B moves along has a larger circumference than the circle that point A moves along. (**slides 10 and 11**).

11-3. Conservation of Rotational (Angular) Momentum.—An object of mass m moving with a velocity v has a momentum p = mv. There is an analogous concept for rotating objects called rotational or angular momentum. The concept of angular momentum is very important in physics because like ordinary momentum, it obeys a conservation law – the **law of conservation of rotational or angular momentum**. It is a fundamental law of nature that **the angular momentum of an isolated system (one that does not have any external torques applied to it) is constant (i. e., is conserved).** The rotational or angular momentum of an object is a parameter that takes into account its rotational inertia and its angular velocity, in fact angular momentum is the product of rotational inertia and rotational velocity. A simple application of this law applies to a rotating rod. If the rod is spinning, it has angular momentum, and if nothing interferes with its spinning motion (like friction) it will keep spinning with the same value of angular momentum until some external torque is applied to slow it down.

The law of conservation of angular momentum is very useful for understanding what happens in situations where the rotational inertia of an object may change while it is rotating. Since the angular momentum is the product of rotational inertia and angular velocity, if the rotational inertia changes, the angular velocity must also change in a manner that keeps the angular momentum constant. This occurs when an ice skater executes a spinning motion. She starts spinning with her hands extended and as she is spinning she pulls her arms in close to her body, and as a result spins faster. When she pulls her arms in close to her body she makes her rotational inertia smaller and in order to keep her



angular momentum constant, she spins faster. See slides **16, 19, 20 and 21** for more examples.

A similar situation occurs when a diver pulls his arms and legs close to his body which causes him to spin very fast, then by extending his arms and legs he stops spinning before entering the pool.

Rotational or angular momentum also depends on **the position of the axis of rotat**ion of a spinning object. Slides **22 and 23** illustrate what occurs to a spinning object when the position of its axis of rotation is changed. To compensate for the reversal of the direction of spin of the bicycle wheel, the system rotates in the opposite direction.