**PHYS:1200 LECTURE 14 – FLUIDS (3)**

So far we have discussed the behavior of **fluids at rest or fluid statics**. Now we take up the interesting topic of the motion of fluids or **fluid dynamics**. The basic principles of fluid dynamics were developed in the $18^{th}$ century by a Swiss mathematician Daniel Bernoulli. Although it would take another 200 years of practical application, the principles he developed are the basis for how airplanes fly. What Bernoulli did was to apply Newton's laws of mechanics, developed in the $17^{th}$ century to fluids.

**14-1. The Measurement of Fluid Flow.**—How do you measure the rate at which water flows out of a faucet? One way of doing this *(slide 11)* is to time how long it takes to fill a bucket of known volume. This measures what is called the **volume flow rate**, the volume of fluid moving per unit time. Volume flow rate, designated by $\mathbf{Q_V}$ is measured in $m^3/s$, or $cm^3/s$, or liters/s, or gallons per minute (gpm), or cubic feet per minute (cfm). If the water (or other liquid) comes out of a pipe of cross-sectional area A (For a tube of circular cross section of diameter d, the area is $A = \pi d^2/4$), if the flow speed is u, then

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| **VOLUME FLOW RATE** | $Q_V = uA.$ | [1] |

By filling a container of known volume, we obtain $Q_V$, then from the size of the pipe, we can get the actual fluid speed, u.

**EXAMPLE 14-1**: A pipe of 25 cm diameter fills a tank of water at a rate of 0.25 $m^3/s$. What is the speed of the water coming out of the pipe?

Solution-

$$Q_V = uA = u\frac{\pi d^2}{4} \to 0.25 = u\frac{\pi\left(0.25\ m\right)^2}{4} \approx u \cdot 5\times10^{-2} m^2$$
$$\to u = \frac{0.25}{5\times10^{-2}} = 5\ m/s.$$

Another commonly used measure of fluid flow rate is the **mass** flow rate $\mathbf{Q_m}$ which gives the mass of the fluid coming out of an orifice per unit time. The volume flow rate and mass flow rate are clearly related, and we can show that

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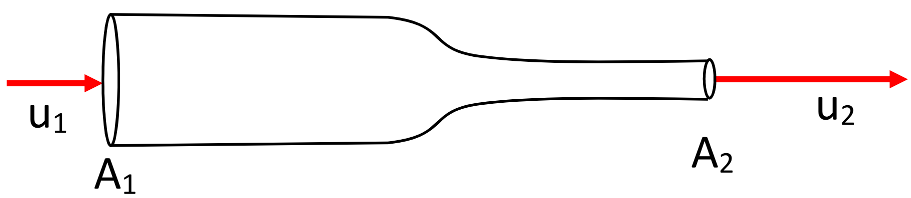
| **MASS FLOW RATE** | $Q_m = \rho Q_V = \rho u A$ | [2] |

where ρ is the mass density of the fluid measured in kg/s.

**What causes water to flow**? One obvious answer is gravity – water always flows downhill (slide 14). Suppose I want to get water to flow upward against gravity – that requires a pump. A water pump establishes a pressure difference between the ends of a pipe – a high pressure on the inlet side and a lower pressure on the outlet side. Even if the pipe is horizontal, the liquid flowing through it experiences a frictional drag with the walls and a **pressure difference** is required to keep it flowing.

**14-2. The Continuity Principle.—** Fluids obey a principle called the continuity equation which is a fancy way of saying that if there are no leaks in a pipe, whatever fluid flows in one end must

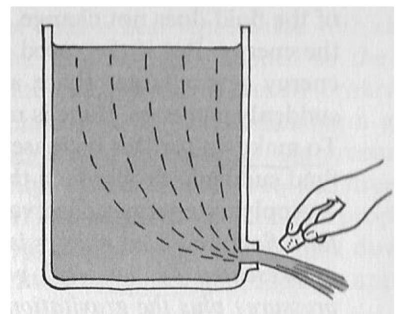



come out the other end. This is fairly obvious for a pipe of constant cross section, **but it also applies to a pipe that may contain a constriction** of the type shown in slide 16 and the diagram below. Since the volume flow rate $Q_V$ = u A, then if the cross sectional area changes from $A_1$ to $A_2$, the speed of the fluid must change to keep the flow continuous: $A_1\ u_1 = A_2\ u_2$, so that $u_2 = (A_1/A_2)\ u_1$. So if water enters a tube with a large cross section and then there is a constriction so that $A_1/A_2 > 1$, the fluid must come out at a higher speed, $u_2 > u_1$. This is the principle that is used to produce a long stream of water using either a nozzle on the end of a hose or simply placing you finger partially over its end (slide 18).

**Example 14-2:** Liquid enters a tube with a speed of 2 m/s. If the tube has a constriction such that the diameter is reduced to ½ of the value on the side that the liquid enters, what is the speed of the liquid as it leaves the smaller end?

Solution:
$$u_1 A_1 = u_2 A_2 \rightarrow u_2 = u_1\left(A_1/A_2\right),\ A_1 = \pi d_1^2/4,\ A_2 = \pi d_2^2/4$$
$$\rightarrow u_2 = u_1\left(d_1^2/d_2^2\right) = u_1\left(d_1/d_2\right)^2 \rightarrow d_1 = 2d_2 \rightarrow u_2 = u_1\left(2\right)^2 = 4u_1 = 4\left(2\,m/s\right) = 8\,m/s.$$

**14-3. Bernoulli's Principle.—**This is one of **the fundamental principles of fluid dynamics.** Bernoulli's principle is the reason why in a tornado sometimes the roofs of houses are blown off. It's not that the tornado tears the roof off, it's the air pressure inside the house that blows the roof off. The pressure of the swiftly moving air in the tornado is lower than that of the non-moving air inside the house. The point is that the speed of a fluid and the pressure within that fluid are entirely different parameters. **Bernoulli tells us that when the speed of a fluid increases, the internal pressure goes down proportionally.** This is true for liquids and gases.



Consider a liquid, e.g., water, initially at rest in a container. The pressure that the liquid exerts on the walls of the container depends on both the density of the liquid and the depth of the liquid. The pressure on the sides or bottom of the container is reduced if there is an opening for the liquid to get moving. The molecules of the liquid no longer press against the side of the container, but move toward the opening. **The pressure in the moving liquid is reduced. The faster the liquid moves, the lesser the pressure in that liquid**.

**Slide 20** illustrates an application of Bernoulli's principle. If you blow air over the top surface of a piece of paper, the paper rises. This is because, the pressure of the swift moving air stream above the paper is lower than the pressure of the non-moving air on the bottom side. The higher pressure on the bottom side lifts the paper. **Slide 21** illustrates this principle using a **Venturi tube** which is a tube with a constriction. Air flows into the left end of the tube, passes through the central constricted region and then exits the right end. A flexible tube filled with green colored water connects the left side with the center. We have already seen that the fluid in a constricted region must flow faster than in the larger region. Therefore the air pressure in the constricted region, according to Bernoulli's principle is lower than in the larger tube. The higher pressure in the larger tube then causes the green-colored water to be pushed downward. The difference if the levels of the green-colored water can be used to measure the flow speed. In the next lecture we will explore more applications of Bernoulli's principle.