Basic Math Review for PHYS 1200 - Physics of Everyday Experience

Basic Algebra

a) If $x = y + z$, then: $y = x - z$, and $z = x - y$

b) If $x = yz$, then: $y = \frac{x}{z}$, and $z = \frac{x}{y}$

Percentages

$N\%$ of x means $\frac{N}{100}$ times $x = \frac{N}{100} x = \frac{Nx}{100}$, where N is any number

Examples:

a) 50% of x = (50/100) x = 0.5 x = x/2

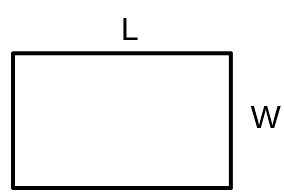
b) y is 25% larger than x. If x = 80 what is y?

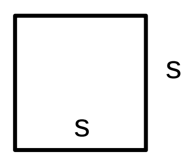
25% of x = 0.25 (80) = (1/4) 80 = 20, so y = 80 + 20 = 100, or y = 1.25x = 1.25(80) =100.

Geometry

a) *Rectangle*

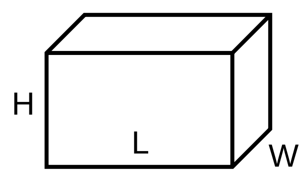
length L, width W, area: A = LW.
For a square of side s, area A = s^2

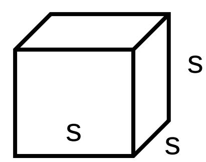




b) *Rectangular box*

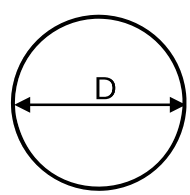
length L, width W, height H
volume: V = LWH, cube of side s, V = s^3





c) *Circle*

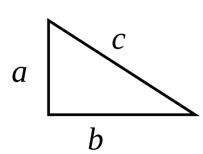
diameter D, radius R = ½ D, area A = $\pi R^2 = \pi D^2/4$, $\pi \approx 3.14$



d) *Right triangle*

sides *a, b*, hypotenuse *c*

$c^2 = a^2 + b^2$, so $c = \sqrt{a^2 + b^2}$, Area *A* = ½ *a b*



Exercises

(Try to do the exercises before looking at the solutions.)

1. $y = x + z$, if $y = 5$ and $x = 2$, find z.

2. $y = x + z$, if $y = 2$ and $x = 5$, find z.

3. $z = x\,y$
   (a) if $x = 2$ and $y = 7$, find z
   (b) if $z = 18$ and $x = 3$, find y.
4. $z = a(x + y)$. If $z = 25$, $x = 3$, and $y = 2$, find a.
5. Evaluate $\frac{1}{0.25}$.
6. The cost of an item is $15. If the cost increases by 20%, what is the new cost?

7. If $79 = 10\,x + 9$, find x.

8. A right triangle has a hypotenuse $c = 5$ and one side $a = 4$. Find the area.

Solutions

1. $z = y - x = 5 - 2 = 3$

2. $z = y - z = 2 - 5 = -3$

3. (a) $z = 2 \times 7 = 14$

   (b) $y = z/x = 18/3 = 6$

4. $25 = a(3 + 2) = a(5)$, $a = 25/5 = 5$

5. $\frac{1}{0.25} = \frac{1}{\left(\frac{25}{100}\right)} = \frac{100}{25} = 4.$ or, since $0.25 = \frac{1}{4}$, then $\frac{1}{0.25} = \frac{1}{\left(\frac{1}{4}\right)} = 4$

6. 20% of 15 = 1/5 x 15 = 3, so the new cost is $15 + $3 = $18.

7. $79 = 10x + 9$, so $79 - 9 = 10x$, $70 = 10x$, $x = 70/10 = 7$

8. $c^2 = a^2 + b^2 \rightarrow 5^2 = 4^2 + b^2 \rightarrow 25 = 16 + b^2 \rightarrow 25 - 16 = 9 = b^2$, so $b = 3$.
   $A = ½\, ab = ½\, 4 \times 3 = 6$

Scientific Notation

In physics we often deal with very small and very large numbers. For example the speed of light is approximately 300,000,000 m/s. The size of an atom is roughly 0.0000000001 m. Rather than writing out all of the zeros, we express these numbers in scientific notation – a decimal number (*the coefficient*) followed by the factor 10 raised to some power (*the exponent*).

In scientific notation, the speed of light is roughly 3×10^8 m/s (or more precisely 2.99×10^8 m/s). Small numbers (numbers less than 1) are handled using *negative exponents*. The size of the atom would be written as 1.00×10^{-10} m.

Big numbers (1 or greater than 1)

$10^0 = 1$ (*any number raised to the power 0 is just 1*)
$10 = 10^1$
$100 = 10 \text{ x } 10 = 10^2$ (hundred)
$1{,}000 = 10 \text{ x } 10 \text{ x } 10 = 10^3$ (thousand)
$1{,}000{,}000 = 10\text{x}10\text{x}10\text{x}10\text{x}10\text{x}10 = 10^6$ (million)
$1{,}000{,}000{,}000 = 10\text{x}10\text{x}10\text{x}10\text{x}10\text{x}10\text{ x}10\text{x}10\text{x}10 = 10^9$ (billion)
$1{,}000{,}000{,}000{,}000 = 10^{12}$ (trillion)
$2000 = 2 \text{ x } 1000 = 2 \text{ x } 10^3$
$2500 = 2.5 \text{ x } 1000 = 2.5 \text{ x } 10^3$

Small numbers (less than 1)

$$0.1 = \frac{1}{10} = 10^{-1} \qquad 0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$0.02 = \frac{2}{100} = \frac{2}{10^2} = 2 \times 10^{-2} \qquad 0.0057 = 5.7 \times 10^{-3}$$

Examples

$4{,}500{,}000 = 4.5 \times 10^6$, $0.000789 = 7.89 \times 10^{-4}$, $5.0001 \text{ x } 10^{-4} = 0.00050001$

Adding (or subtracting) numbers in scientific notation

Numbers in scientific notation can only be added (or subtracted) if they have the same power of 10. If the numbers do not have the same power of ten, one must be changed so that it has the same power of 10 as the other. For example:

Suppose: $x = 5.40 \times 10^5$ and $y = 7.3 \times 10^4$, find $x + y$.
First write y so that it is expressed with a power of 10^5 as $y = 0.73 \times 10^5$,
then $x + y = 5.4 \times 10^5 + 0.73 \times 10^5 = (5.4 + 0.73) \times 10^5 = 6.13 \times 10^5$.
Alternatively, change x so that its exponent is 4, as $x = 54.0 \times 10^4$, then
$x + y = 54.0 \times 10^4 + y = 7.3 \times 10^4 = 61.3 \times 10^4 = 6.13 \times 10^{5.}$

Multiplying and dividing numbers in scientific notation

Often we need to multiply and divide large or small numbers. For multiplication of numbers in scientific notation, first *multiply* the coefficients, and then *add* the exponents. Some examples:

$(7 \times 10^3) \times (3 \times 10^4) = 21 \times 10^7 = 2.1 \times 10^8$.

$(2.6 \times 10^6) \times (2.0 \times 10^{-4}) = 5.2 \times 10^{6-4} = 5.2 \times 10^2$

$(1.5 \times 10^{-5}) \times (3 \times 10^3) = 4.5 \times 10^{-2}$

$(2 \times 10^{-2}) \times (5 \times 10^{-8}) = 10 \times 10^{-2 +(--8)} = 10 \times 10^{-10} = 1.0 \times 10^{-9}$

When dividing two numbers in scientific notation, first divide the coefficients, and then the exponent is the difference between exponent of the numerator and the exponent of the denominator. Some examples:

$$\frac{8\times10^5}{2\times10^3} = \frac{8}{2}\times10^{5-3} = 4\times10^2$$

$$\frac{4\times10^7}{8\times10^{12}} = \frac{4}{8}\times10^{7-12} = 0.5\times10^{-5} = 5\times10^{-6}$$

$$\frac{3\times10^{-3}}{9\times10^{-7}} = \frac{3}{9}\times10^{-3-(-7)} = \frac{1}{3}\times10^{-3+7} = 0.333\times10^4 = 3.33\times10^3$$

$$\frac{7\times10^2}{2\times10^2} = \frac{7}{2}\times10^{2-2} = 3.5\times10^0 = 3.5$$

Exercises

1. Express the following numbers in scientific notation.

    (a) 2,530,000
    (b) 0.0000072
    (c) 859
    (d) 0.001
    (e) 300,000,000

2. Express the following in standard notation.

    (a) 7.35×10^4
    (b) 8.6×10^{-5}
    (c) 1.87×10^5
    (d) 6×10^8

3. Multiply.

    (a) $(4 \text{ x } 10^6) \text{ x } (3 \text{ x } 10^7)$
    (b) $(2 \text{ x } 10^{-7}) \text{ x } (5 \text{ x } 10^{-6})$
    (c) $(6 \text{ x } 10^{13}) \text{ x } (4 \text{ x } 10^{-15})$

4. Divide.
    (a) $(6 \text{ x } 10^2) / (3 \text{ x } 10^2)$
    (b) $(9 \text{ x } 10^{-2}) / (3 \text{ x } 10^{-5})$
    (c) $(1.6 \text{ x } 10^{14}) / (4 \text{ x } 10^{17})$

5. Add: $4\text{x}10^3 + 5\text{x}10^2$

Answers

1. (a) $2.53 \text{ x } 10^6$, (b) $7.2 \text{ x } 10^{-6}$, (c) $8.59 \text{ x}10^2$, (d) $1\text{x}10^{-3}$, (e) $3 \text{ x } 10^8$

2. (a) 73,500, (b) 0.000086, (c) 187000, (d) 600,000,000

3. (a) $1.2 \text{ x } 10^{14}$, (b) $1.0 \text{ x } 10^{-12}$, (c) 0.24

4. (a) 2 (b) 3000 (c) $4 \text{ x } 10^{-4}$

5. $4\text{x}10^3 + 5\text{x}10^2 = 40\text{x}10^2 + 5\text{x}10^2 = 45\text{x}10^2 = 4.5\text{x}10^3$, or $4\text{x}10^3 + 0.5\text{x}10^3 = 4.5\text{x}10^3$.

Unit conversions

Conversion factors

12 inches (in) = 1 foot (ft)
1 yard (yd) = 36 in = 3 ft
1 mile (mi) = 5280 ft = 1760 yd
1 cm = 0.01 m   1 m = 100 cm
1 mm = 0.001 m   1 m = 1000 mm
1 kilometer (km) = 1000 m = 0.62 miles (mi)
1 in = 2.54 cm = 25.4 mm
1 year (yr) = 365 days
1 day = 24 hours (hr)
1 hr = 60 minutes (min)
1 min = 60 s

Exercises on units and conversions

1. How many inches are there in 1 m?
2. How many yards are there in 1 m?
3. How many kilometers are there in 1 mile?
4. Convert 55 miles per hour (mph) to km/hr.
5. How many seconds are there in one year?
6. What is your age in seconds?
7. What is the average speed of a runner who runs a 4 minute mile?

Solutions

1. $1m = 1\,\cancel{m} \times \frac{100\ \cancel{cm}}{\cancel{m}} \times \frac{1in}{2.54\ \cancel{cm}} = \frac{100}{2.54} in = 39.4\,in$
2. $1m = 39.4\ \cancel{in} \times \frac{1\,yd}{36\ \cancel{in}} = 1.09\ yd$
3. $\frac{0.62\,mi}{km} \rightarrow \frac{1km}{0.62\,mi} = 1.61\frac{km}{mi}$
4. $55\frac{mi}{hr} = 55\frac{mi}{hr} \times \frac{1.61km}{mi} = 88.6\frac{km}{hr}$
5. $1\,yr = 1\,yr \times \frac{365\,days}{yr} \times \frac{24\,hr}{day} \times \frac{60\,\text{min}}{hr} \times \frac{60\,s}{\text{min}} = 365 \times 24 \times 60 \times 60\,s = 31,536,000\,s$
6. Suppose your age is N years. Your age in seconds = N x 31,536,000 s.
7. If you run 1 mile in 4 minutes, your average speed is ¼ mile/min.

   $4\ \text{min} = \frac{4}{60}\ hr = \frac{1}{15}\ hr \rightarrow \frac{1mi}{4\,\text{min}} = \frac{1\ mi}{\left(\frac{1}{15}hr\right)} = 15\ mph$