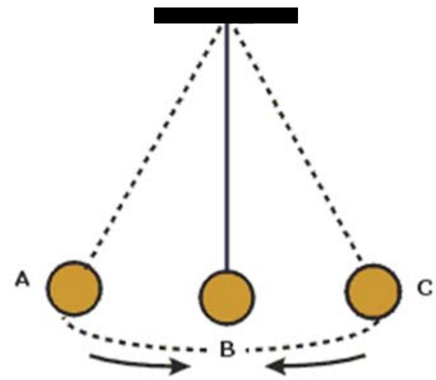
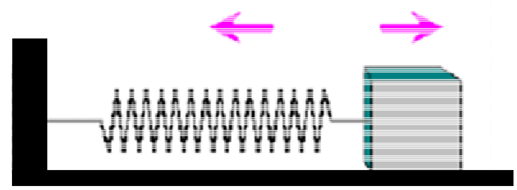
UNIT 4 VIBRATION, WAVES AND SOUND

PHYS:1200 LECTURE 20 — VIBRATION, WAVES, AND SOUND (1)

Lecture 20 begins the new unit on vibration, waves, and sound. We first study vibrating or oscillating systems, which are mechanical systems that execute some type of back and forth motion that repeats over and over again at regular time intervals. A simple example of a vibrating system is the pendulum – a mass hanging from the end of a suspended string. The mass is pulled aside to point A and let go; it then executes a back and forth motion A→B→C→B→A.



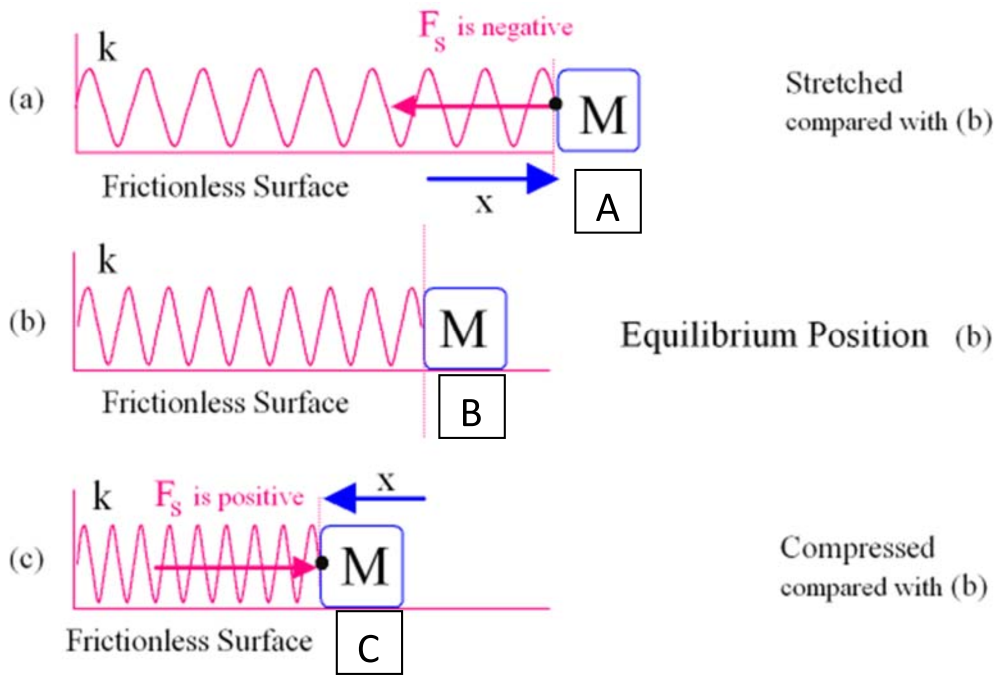
Another example of a vibrating system is a car attached by a spring to one end of the air track. The car is initially pulled to the right to stretch the spring and then when let go, oscillates back and forth horizontally. The motion of the pendulum is driven by gravity, while the car on the air track is driven by the force exerted by the spring. We will discuss the pendulum and the mass – spring system in more detail later in this lecture.



A final example is the tuning fork, which is a forked-shape metal object, usually made from aluminum or steel, which has two prongs. The prongs are set into vibration by hitting one with a rubber hammer. The prongs oscillate when excited due to the elastic properties of the material. The actual back-and-forth motion of the prongs is too small to see, however, the vibration of a prongs causes the air surrounding them to vibrate, and the vibration in the air travels away as a sound wave which we hear when it causes our eardrums to vibrate. Tuning forks are designed to produce a sound wave with a pure pitch (frequency). The tuning fork shown produces the musical note C at 512 vibrations per second.

20-1. General Characteristics of Vibrating Systems.—Two parameters are used to quantify a vibrating system: amplitude of oscillation and frequency. These quantities are best described in terms of a specific system, for example, the mass attached to a spring that moves back and forth on a horizontal surface.

*a. Amplitude of oscillation.—*To set the mass in motion, it is displaced from its equilibrium position (b) and let go. Before it is displaced the spring is in its relaxed condition, neither



stretched nor compressed. If the mass is pulled to the right as in (a) above, the spring pulls it back to equilibrium. The mass continues moving to the left, past its equilibrium position as in (c), and now compresses the spring. The spring then pushes it back to the right. **The amplitude of oscillation is the maximum distance (A) that the object is initially displaced from its equilibrium position.** The mass then oscillates back and forth from x = +A to x = –A.

b. Frequency.—**The frequency f of a vibrating system is the number of complete oscillations it completes per unit of time.** For the mass-spring system, a complete oscillation means from the starting point, say on the right side, to the far left position, and back to the starting point. A complete oscillation begins and ends at the same point. For the pendulum, a complete oscillation means A→B→C→B→A. Frequency is related to the time it takes for an oscillator to complete a full oscillation, this time is called the **period of oscillation** T measured in seconds. Clearly, the shorter this time is, the larger the number of oscillations every second, so period and frequency are related inversely:

FREQUENCY AND PERIOD	$f = \frac{1}{T}, \; and \; T = \frac{1}{f}.$	[1]

Frequency is measured then in units of number of complete vibrations (cycles) per second, or s^{-1}. **A frequency of 1 cycle per second is designated as 1 Hz (Hertz),** in honor of Gustav Hertz who discovered electromagnetic waves.

Example 20-1: What is the period of an oscillator that has a frequency of ½ Hz?

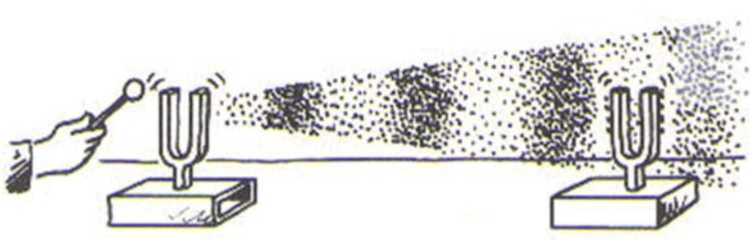
Solution- $T = 1/f = 1/(1/2) = 2s.$

Example 20-2: A pendulum is observed to take 2 seconds to swing from the right side to the left side. What is the frequency of this pendulum?

Solution- The period of a pendulum is the time to swing from one side to the other side and back again. Therefore the period of this pendulum is $T = 4\ s$. Then

$$f = \frac{1}{T} = \frac{1}{4s} = 0.25 s^{-1} = 0.25\ Hz.$$

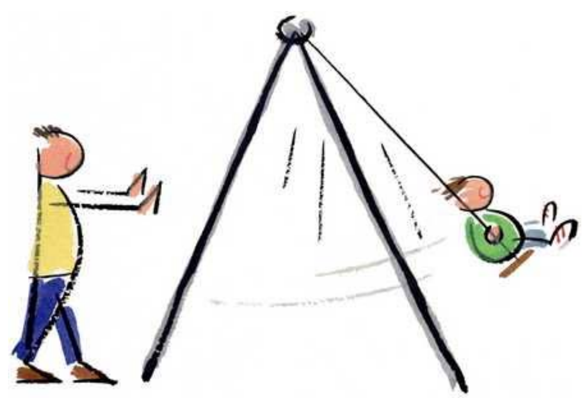
20-2. Resonance in Mechanical Systems.—One of the most interesting and important aspects of vibrating systems is the phenomenon of resonance. **Resonance occurs when the oscillations of one vibrating system are coupled into another system causing it also to vibrate.** Every physical system has certain "natural frequencies" at which it can vibrate. If some type of external vibration is coupled to one of the natural frequencies of another system, resonance can occur. When resonance occurs, the amplitude of oscillation of the second system builds up over time.



When this occurs, we say that the two systems are ***in resonance.*** A nice example of resonance occurs with two nearly identical tuning forks, as illustrated in the diagram above. One tuning fork is struck, and the sound waves that it excites causes a nearby tuning fork to vibrate. The resonance occurs because the two tuning forks are nearly identical and thus have the same vibration frequency. When two systems are coupled in this way, energy from one is very efficiently transferred to the other. If the second tuning fork were of a different frequency, the transfer of energy would not be sufficient to cause it to vibrate. This will be demonstrated in class.

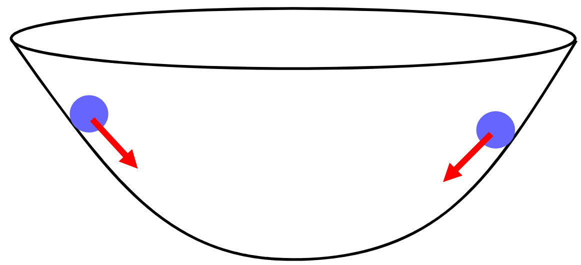
The phenomenon of resonance can be annoying, disruptive, or in extreme cases even destructive. For example, if the tires of your car are not properly balanced, then the vibration of the motor might couple into the rotation of the wheel causing it to vibrate. This occurs when a speed is reached where the frequency of the tire's vibration matches the frequency of the engines revolutions. A dramatic and tragic example of resonance occurred with the Tacoma Narrows Bridge in 1940. If you Google "Tacoma Narrows Bridge" you will find short video clips of this famous disaster (shown in class). The details of why this occurred are discussed on **slides 1, 2, and 3.**

Anyone who has ever pushed a child on a swing has used the phenomenon of resonance. If you push the child at the right frequency that matches the frequency of the swing's natural oscillation period, energy is efficiently transferred to the system and the oscillation amplitude grows – the child swings higher and higher.

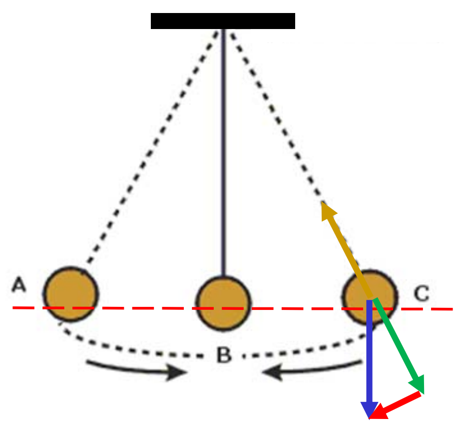


20-3. The Restoring Force and the Simple Harmonic Oscillator.—Next we will discuss the physics of the pendulum and mass/spring systems in more detail. Both of these systems are examples of what are referred to as **simple harmonic oscillators.** The adjective *harmonic* refers to the fact that the motion repeats itself at regular intervals, and the adjective *simple* indicates that the effects of friction or other drag forces are negligible and can be ignored. Friction and drag can never really be eliminated, but we imagine that they have been minimized, for example by using the air track. A pendulum is always subject to air resistance, which will eventually cause it to oscillate with smaller and smaller amplitude until eventually it comes to rest.

The essential feature of all oscillating systems is the presence of a **restoring force**. **A restoring force is a force that always acts in a direction to bring an object back to its equilibrium system if it is moved away from equilibrium.** For example, a marble in a bowl will be restored to equilibrium at the bottom of the bowl by gravity. When the ball is on the right side of the bowl, gravity pulls it to the left, and when it is on the right side, gravity pulls it to the right. In either case gravity acts as a force to "restore" the marble to its equilibrium position at the bottom of the bowl. Gravity is the restoring force for the pendulum and the spring provides the restoring force for the mass/spring system.



a. The simple pendulum.—The simple pendulum is a mass m hanging from the end of a suspended string of length L. The equilibrium position of the mass is at point B in the diagram. The weight of the mass w = mg acts downward. When the mass is pulled over to point A or point C, it is lifted up slightly relative to its equilibrium position at B and regardless of which side it is on, gravity causes it to fall back down to point B. Because m is held by a string, it is forced to move along a semicircular path by the centripetal force of the tension in the string. The force of gravity (the weight mg, indicated by the blue arrow) acts directly downward, but its effect is twofold. Part of the force of gravity acts to pull outward (the green arrow) on the string which gives rise to the inward tension force (brown arrow), and part of the force of gravity acts along the path of the motion (the red arrow). The part of gravity that acts along the path changes direction when the mass passes through point B, and always acts in the direction toward B – this is the restoring force.



Suppose the mass m is moved over to point A and released. Gravity pulls it down to point B. As it *falls* to point B its speed increases (as with any falling object), so that when it gets to B it has its maximum speed, and although the restoring force is instantaneously zero at B, *the mass has inertia* so it continues to move past B toward C. As it moves from B to C it is going uphill so it slows down, eventually coming instantaneously to rest at C. The conditions at C are the same as the conditions at A (except that the direction of the restoring force is reversed) so the pendulum falls back to B --- the motion repeats, etc, etc.

The motion of the pendulum can also be analyzed on the basis of the kinetic and potential energy of the mass. Since we are neglecting friction and air resistance, the total mechanical energy (KE + GPE) is conserved for the pendulum. At point A, the mass has gravitational potential energy, GPE and no kinetic energy, KE. As it moves from A→B its GPE decreases and its KE increases. At B its GPE is zero and its KE is maximum (its speed is also maximum there). From B→C, KE decreases and GPE increases. At C, KE = 0, and GPE is a maximum. At any point in the motion KE + GPE = constant. The value of this constant depends on the vertical position of point A relative to point B. The higher the pendulum starts relative to point B, the more total energy is has.

The period of a pendulum

A more detailed mathematical analysis of the pendulum using Newton's 2nd law leads to the following formula for the period:

$$T_{Pendulum} = 2\pi\sqrt{\frac{L}{g}}, \qquad [2]$$

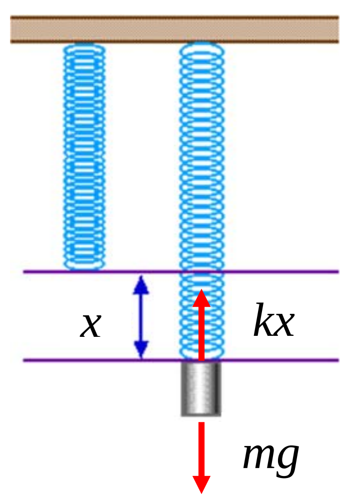
where L is the length of the string, and g is the acceleration due to gravity.

Notice that the period does not depend on the mass. To double the period, the length of the string must be increased to 4 times the original length. Also, since T depends on g, a pendulum on the moon will have a longer period than the same pendulum on earth.

b. Springs and Hooke's Law.—Before analyzing the mass/spring system it will be necessary to first discuss the properties of springs which are simple, interesting and useful devices. They come in a variety of sizes depending on the application. Springs can be stretched or compressed and a force must be applied to stretch it or compress it. **The force that a spring exerts depends on how much it is stretched or compressed.** This is quantified in **Hooke's Law** which states that the force that a spring exerts is directly proportional to the amount by which it is stretched or compressed relative to its relaxed state. If a spring is stretched by one unit of distance it exerts a force F. If it is stretched by 2 units it exerts a force = 2F. If we let x represent the amount of stretching (or compressing) measured with respect to its relaxed length, the magnitude of force it exerts is then $F_S \propto x$ (this means "F_S is proportional to x"). This rule is quantified by introducing a constant, k called the spring constant, which is a measure of the stiffness of that particular spring. Each spring, depending on its design, has its own k value. So the magnitude of the force exerted by a spring can be written as

Hooke's Law	$F_S = kx.$	[3]

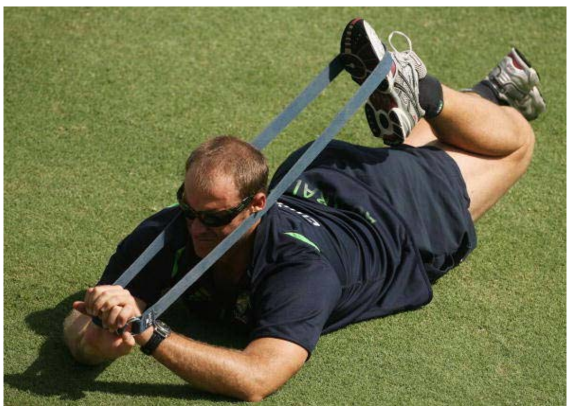
The spring constant for any spring can be easily measured by the following procedure. Hang the spring vertically and attach a known mass m to the end. Measure the distance that the spring has been stretched from its relaxed position. The spring will stretch to a length such that the spring force kx equals the weight mg of the hanging mass, thus $mg = kx,\ or\ k = \frac{mg}{x}$. By using a known mass m and measuring x, k can be measured. Heavy, stiff springs have large k values, and light, weak springs have small k values. k is measured in units of N/m.



Example 20-3: A mass of m = 2 kg is hung vertically from a spring having a spring constant k = 1000 N/m. By how much will be spring be stretched from its relaxed position?

Solution - $mg = kx \rightarrow x = \frac{mg}{k} = \frac{2\,kg \times 10\,m/s^2}{1000\,N/m} = \frac{20\,N}{1000\,N/m} = 0.02\ m = 2\ cm.$

A spring does not have to be a coil of stiff wire. Elastic bands and other devices are used to provide resistance to stretching or compression as exercise equipment.



*c. The mass-spring system.***—**The analysis of this system is simpler than that of the pendulum, because this motion is entirely horizontal so that gravity plays no role in the motion. **The restoring force is simply provided by the spring**. Referring back to the diagram on page 2, the system starts at point A where the spring has been stretched and thus pulls the mass back to point B. In moving from B→C, the mass now is compressing the spring, which pushes back on it. At point C, m is instantaneously at rest, the spring is compressed fully and pushes it back to point B. The spring is able to exert a restoring force because it pulls when stretched and pushes when compressed, so that the direction of its force changes depending on where m is. **If the effect of friction is negligible, the mechanical energy in the mass/spring system is also conserved.** In this case, the potential energy is the energy that the spring has when it is stretched or compressed.

The period of the mass-spring oscillator

A more detailed mathematical analysis of the mass/spring oscillator using Newton's 2^{nd} law leads to the following formula for the period:

$$T_{Mass/Spring} = 2\pi\sqrt{\frac{m}{k}}, \qquad [4]$$

where m is the mass and k is the spring constant.

Notice that the period is proportional to the mass and inversely proportional to the spring constant, k. For the same mass, the period is reduced by using a stiffer spring.

Exercise 20-4: A mass/spring oscillator has a period of 5 seconds. How can the mass or spring be changed so that the period is 10 seconds?

Solution- To double the period either use a mass that is 4 times larger, or use a spring that has a spring constant that is 4 times smaller.

d. The torsional oscillator.—Another system that executes harmonic motion is the **torsional oscillator**. A metal plate is attached to a wire at its center and the end of the wire is suspended. When the plate is rotated, the wire becomes twisted. A twisted wire always tries to untwist itself, thus providing a restoring force on the plate. The plate rotates as the wire untwists and keeps on rotating even after the wire is completely untwisted. As it continues to rotate, the wire now becomes twisted in the opposite direction, thus providing a mechanism to maintain an alternating clockwise and counterclockwise rotation.