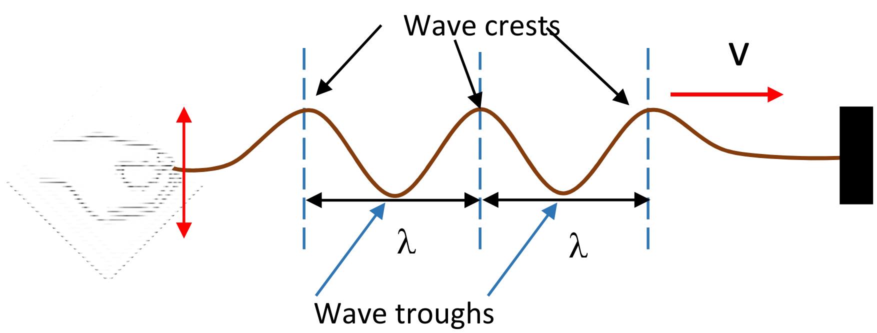
PHYS:1200 LECTURE 22 — VIBRATION, WAVES, AND SOUND (3)

The final lecture on vibrations, waves, and sound introduces a basic relation that applies to all periodic waves – the periodic wave relation. This is a relation between the three parameters which characterize a wave: wavelength, frequency, and wave speed. We will then discuss the production of sound waves in more detail, including some aspects of how musical instruments produce the pleasant sounds we call music. Finally, we will analyze wave interference, a unique property of waves, and how this effect leads to the phenomena of standing waves and beats.

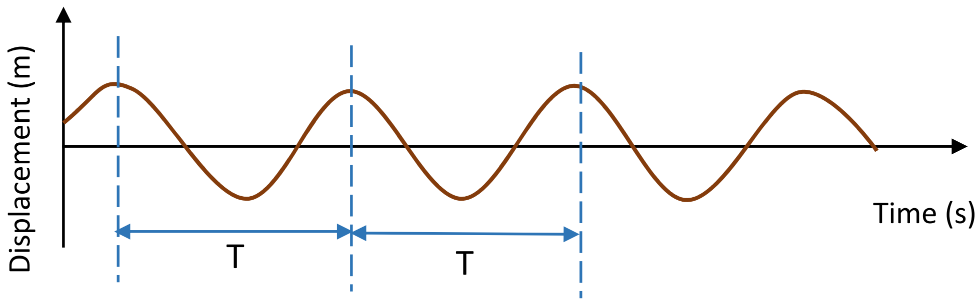
22-1. Basic Properties of Periodic Waves.—Waves are classified according to their spatial and temporal properties by their wavelength and frequency. The wavelength and frequency of a wave are related to the speed of propagation of the wave by the **periodic wave relation**.

*a. Wavelength.***—**Suppose a harmonic (repeating) wave is excited on a string by moving the free end up and down periodically (see figure below). If a photo is taken of the string at some



time, we get a snapshot of the spatial structure of the wave at that time. The wave consists of alternating wave crests and wave troughs that propagate to the right at a speed v. The distance between successive wave crests (or wave troughs) is the **wavelength (Greek letter lambda, λ)** of this wave. Wavelength is measured in distance units, m, cm, etc. The wavelength is the spatial distance over which the wave is repeated.

*b. Wave frequency.***—**Another characterization of the wave can be performed by an observer who is at a fixed location and focusses on the up and down motion of a small segment of the string that is directly in his or her sight. The observer records the position of the segment of the string relative to the undisturbed position of the string. A plot of the position of this segment in time is shown below. This plot is a temporal representation of the wave and is complementary to the spatial representation shown above. The time between successive peaks in this plot is the



period of the wave (T), and is measured in seconds. If instead of measuring the time between the appearance of successive peaks, the observer were to count the number of peaks that pass by in a given time interval, the number of peaks passing per second if the wave frequency (f). Frequency is measured in number/s or simply 1/s or s^{-1}. A frequency of 1 per second is called 1 Hertz (Hz), or $1\ s^{-1} = 1\ Hz$. Clearly the period and frequency are not independent quantities. For example, if the peaks pass by in intervals of 1 second, the frequency is 1 Hz. If the time between peaks passing is ½ s, then 2 peaks would pass by per second, and f = 2 Hz. So period and frequency are related inversely,

Frequency and Period	$f = \dfrac{1}{T}$ *and* $T = \dfrac{1}{f}$.	[1]

c. The periodic wave relation.—Wavelength, period (or frequency) and wave speed are related for a periodic wave. This is seen by application of the distance – speed relation (d = v t). Since the wave moves exactly one wavelength in one period, the wave speed is $\mathrm{v} = \lambda/T$. Since $f = 1/T$, this formula can be written as

Periodic Wave Relation	$\mathrm{v} = \lambda f$,	[2]

and is called the periodic wave relation. Formula [2] applies to all types of periodic waves, water waves, waves on strings, sound waves, and electromagnetic waves (light waves).

Example 22.1: A harmonic wave propagates on a string at a speed of 10 cm/s. If the wavelength is 2 cm, what are the frequency and period of this wave?

Solution - $f = \frac{v}{\lambda} = \frac{10\ cm/s}{2\,cm} = 5\,s^{-1} = 5\,Hz \ \rightarrow\ T = \frac{1}{f} = \frac{1}{5\,s^{-1}} = \frac{1}{5}s = 0.2\ s.$

Example 22.2: What is the frequency of yellow light having a wavelength of 600 nm?

Solution-

$1\ nm = 10^{-9}\ m \rightarrow \lambda = 600\ nm = 600\times10^{-9}m = 6\times10^{-7}\ m.$

$f = \frac{c}{\lambda}\ (\text{the speed of light is designated by the symbol c}), \textit{where}\ c = 3\times10^{8}\ m/s$

$f = \frac{3\times10^{8}\ m/s}{6\times10^{-7}\ m} = 0.5\times10^{8+7}\ Hz = 0.5\times10^{15}\ Hz = 5\times10^{14}\ Hz.$

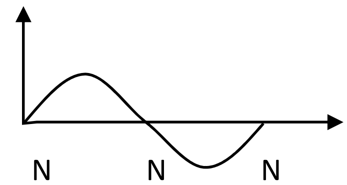
22-2. The Production of Sound.—Sound waves are generated by the rapid mechanical vibration of some object. The mechanical vibrations are coupled to the air molecules which then vibrate, and the vibrations of the air molecules propagates from the source as sound at approximately 340 m/s.

Musical instruments are devices which produce sound by plucking strings, hammering strings, bowing strings, or blowing air into variously shaped tubes. In **stringed** instruments, sound waves are produced either by plucking (guitar, bass, violin, harpsicord), bowing (violin, viola, cello, bass), or striking (piano). In stringed instruments, the strings are held fixed at both ends, and as we will discuss shortly, standing waves are produced. The sound wave that is produced has the same frequency (pitch) as the frequency of the wave on the string. The frequency of the wave on the string depends on the speed of the wave on the string and the wavelength. The wavelength depends on the length of the string and the speed depends on the diameter of the string (its thickness) and on the tension in the string. Stringed instruments have multiple strings of different thicknesses and the tension is adjustable. A musician tunes the instrument by adjusting the tension in the strings. By holding down the string at different points along its length, the musician is in effect changing the length of the string which changes the frequency (pitch) of the sound. The various stringed instruments (violin, viola, cello, bass) have strings of longer and longer lengths. Each of these instruments covers a different range of frequencies with some overlap. The violins produce the highest pitch sounds while the bass produces the lowest.

Pipe organs have many pipes of different lengths to produce sounds of different pitch (frequency). The generation of sound in a pipe is much like the process of blowing over the end of an empty pop bottle. Air is blown into the bottom of the pipe is caused to vibrate by a complicated aerodynamic mechanism, exciting various resonant modes. The short pipes produce the high notes and the long pipes produce the low notes.

22-3. Wave Interference.—Two (or more) waves can be launched on a string at the same time. Each wave affects the string exactly as if the other wave were absent. When both waves are present they produce a combined effect which is the sum of the effects that each would produce individually. This is called the **superposition of waves** and gives rise to the phenomenon of **wave interference.** If each wave individually would have the effect of causing a segment of a string to move up, then the combined effects add causing the string to move up by a larger amount --- this is **constructive interference**. If one wave tries to move a segment up while the other wave tries to move it down, the two effects partially or totally cancel each other – this is **destructive interference** (see the diagram on slide 14). Which effect occurs depends on how the waves are initially synchronized and on the position along the string.

a. Standing waves.—When a wave is launched from one end of a string that is fixed at the other end, the initial wave reflects off of the wall and emerges as shown on slide 16. The waves produced on the left can then interfere with the reflected waves and produce **standing waves**. At some locations along the string the waves interfere constructively, while at other locations they interfere destructively. At the points of destructive interference, the string does not move—these points are referred to as **nodes.** At the points where constructive interference occurs, the string moves with maximum amplitude—these points are referred to as **antinodes**. Depending on the frequency of excitation of the waves, various modes are possible, as shown on slide 17. Because the string is fixed at both ends, the ends must always be nodes and the eave amplitude must be zero there.



Now a sine wave has three points where its amplitude is zero, at the beginning, in the middle, and at the end. Since the wave modes must fit into the particular length of the string and always with nodes at the ends, it is only possible to fit waves along the string in multiples of ½ of a wavelength. Therefore the lowest frequency mode will correspond to ½ a wavelength, i.e., $\lambda = 2L$. This is called the fundamental mode because it has the lowest frequency $f_0 = \mathrm{v}/\lambda = \mathrm{v}/2L$, where v is the speed of the wave. The next mode is called the first harmonic, and consists of 2 half wavelengths (one full wavelength), so that $\lambda = L,\ and\ \ f_1 = \mathrm{v}/\lambda = \mathrm{v}/L$. Higher modes are formed by fitting in additional ½ wavelengths. You can see now how this applies to the sound pitch produced by a plucked string. Plucking a string excites the fundamental mode at f_0, which depends on the length of the string, the diameter of the string and the tension in the string. For a given string the diameter is fixed and the tension is adjusted when the string is tuned. Then the musician is able to produce different notes by moving his finger to different points along the string which effectively changes the length. The technique of changing the length of the strings with one's fingers is called "fingering". Using a bow to excite a string, as in a violin, produces a sound wave that contains many modes simultaneously. Musicians refer to this characteristic as the "richness" of the sound.

b. Beats.—Standing waves are the result of spatial interference of two waves. If two waves of slightly different frequency are produced at the same time, an interference effect in time, known as "beats" occurs. **Slide 20** illustrates what happens when two waves of slightly different frequency are added together. There are locations where constrictive interference occurs (waves reinforce) and other locations where destructive interference occurs (waves cancel). This produces a pattern, shown on the bottom plot on slide 20, of alternating high and low amplitude. If the two waves are sound waves, one hears a characteristic "beat" – alternating high and low intensity sounds. **Slides 21 and 22** discuss how the phenomenon of wave interference affects room acoustics, and how it is used to eliminate noise.