PHYS:1200 LECTURE 4 – MECHANICS (3)

This lecture covers the general case of **motion with constant acceleration** and free fall (which is one of the more important examples of motion with constant acceleration) in a more quantitative manner. The goal here is to discuss the formulas that can be used to calculate the position and velocity of an object at any time if we know the acceleration and the initial (beginning) speed of the object. The type of analysis that we will find is the same analysis that is used to reconstruct the facts of an automobile accident using some of the information, e.g., skid marks that might be available at the scene. We begin by reminding you that **acceleration is the rate of change of velocity**, $a = \Delta v / \Delta t$, where Δv means the change in velocity, and Δt is the time interval over which it changes. In the scientific system of units a is measured in meters per second squared, or m/s^2. We will begin by discussing the simpler situation of motion with constant velocity, or in other words, the case in which there is no acceleration, a = 0.

4-1. Motion with Constant Velocity.—Suppose an object moves with constant velocity v. Recall that "velocity" includes both speed and direction, so when we say that the **velocity is constant** that means that the object is moving with **constant speed in a straight line** – neither its speed or direction changes as it moves. To describe motion mathematically, we must first set up a coordinate system. For motion with constant velocity, we can take the motion to occur along the x-axis of a coordinate system. The other parameter we must track, of course, is the time. We can imaging that we have a video camera and we make a movie of the motion of the object. To analyze the motion we inspect the movie frame by frame and develop a table of where the object is on each frame of the movie. Suppose we call the first location of the particle x_i (we read this as x-i and it means the initial position). There is nothing particularly special about x_i, it is just where we decide to start the analysis, and it can also be taken as the time when we start the clock, i.e., t = 0. Suppose we know that the object's velocity is v. What we want is a formula which allows us to figure out, where the object will be at a later (or final) time t – call this position x_f (read x-f for x final). The formula for x_f in terms of x_i, v, and t is

Distance x_f travelled from initial position x_i in time t when moving with constant velocity v

$$x_f = x_i + vt. \qquad [1]$$

Notice that formula [1] makes sense if v = 0: $x_f = x_i$, the object stays at its initial position.

Now, in setting up the coordinate system, we can simplify things and say that the initial position is just $x_i = 0$. In this case, the above formula tells us, for example, that if you travel in a straight line at 75 mph for one hour, you will travel exactly 75 miles, which is the answer you would have arrived at using common sense.

4-2. Motion with Constant Acceleration.—Acceleration is the rate of change of velocity with time. Suppose that at some initial time, t_1 an object has a velocity v_1, then at some later time, t_2 its velocity is v_2, then over this time interval its acceleration is

Definition of acceleration:	$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$.	[2]

Example 4-1: The velocity of an object increases from 50 m/s to 75 m/s over an interval of 5 seconds. What is the acceleration?

Solution- $a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{75\ m/s - 50\ m/s}{5\ s} = \frac{25\ m/s}{5\ s} = 5\ \ m/s^2$.

Suppose an object moves with constant acceleration (a) and at t = 0, its velocity is v_i. Its velocity v at a later time t is computed by

Motion with constant acceleration	$v = v_i + a\,t$	[3]

In other words, to compute the velocity at a later time, we add to the initial velocity the quantity of a times t, so that a x t is the amount by which the velocity increases. An example of the application of formula [3] is given on **slide 10.**

Example 4-2: An object moving at 30 m/s begins accelerating at 5 m/s^2. When will its speed reach 50 m/s?

Solution- $v_f = v_i + at \Rightarrow 50\ m/s = 30\ \ m/s + \left(5\ m/s^2\right)t \Rightarrow 50 = 30 + 5t \Rightarrow 5t = 20 \Rightarrow t = 4\ s.$

Notice once again, that formula [3] simply tells us that if a = 0, v = v_i since the velocity does not change.

The velocity of an object can also decrease – this is also an example of accelerated motion, but in this case we use the special term **deceleration.** When you are driving and apply the brakes, you slow down or decelerate. Deceleration can be handled in the same way as acceleration, if we use a minus sign in front of a. For example a car may slow down if the brakes provide an acceleration (deceleration) a = -5 m/s^2. If you look at formula [3] when a is negative, the second term is negative, meaning that it subtracts from the first term so v will be less than v_i. This is illustrated by the example on **slide 11.**

Example 4-3: A car is moving at 150 m/s when the brakes are applied. If the brakes provide a deceleration of –30 m/s^2, how long will it take for the car to stop? (unrealistic numbers!)

Solution- $v_f = v_i + at \rightarrow 0 = 150 + (-30)\,\mathrm{t} \rightarrow 30\,\mathrm{t} = 150 \rightarrow \mathrm{t} = 5\ \mathrm{s}.$

4-3. Free-fall.—Free fall is an example of motion with constant acceleration – g. Formula [3] can be applied to free fall by setting a = g = 10 m/s^2. If an object falls from rest (you just let it go) so that v_i = 0, then its speed at a later time t is given by

Velocity after falling for a time t
starting from rest: $\mathrm{v} = \mathrm{gt} = 10\ \mathrm{t}.$ [4]

The table on **slide 13** gives the results of observing a falling object by giving its velocity at one second intervals and the distance that it has fallen. You can see that the measured velocities follow formula [4]. Each second, the velocity increases by 10 m/s. The right hand column of the table on slide 13 give the distances fallen each second. The first point to notice is that the object falls different distances in each successive second of free fall: 5 m for the *first* second, 15 additional m during the *second* second, 25 m during the *third* second, etc. **A freely falling object does not fall the same distance in each successive time interval.** The numbers in the right column are also expressed in a form that illustrates a pattern – the vertical distance travelled during each second appears to be given by ½ (10) x the square of the time. Galileo was the first to notice this pattern and was able to write down the formula for the distance (d) an object falls at any time, t assuming it was dropped from rest:

The distance y that an object falls in a time t, starting from rest:	$y = \frac{1}{2} g t^2$	[5]

An example of the use of formula [5] is given on slide 17. Formulas [4] and [5] can be used to figure out how long it would take for an object to fall a particular distance, and how fast it would be moving when it hit the ground. An example is given on slides 18 and 19.

Example 4-4: An object falls from rest from a great height. How fast will it be moving and how far will it have fallen in 10 seconds?

Solution- $v = gt = 10\ m/s^2\ (10\ s) = 100\ m/s;\ d = \frac{1}{2}gt^2 = \frac{1}{2}10\ m/s^2(10\ s)^2 = 5(100) = 500\ m.$

Formula [5] can also be used to analyze the problem of an object that is thrown straight up and then comes down. To throw an object up means that it is given an initial upward velocity, v_i. As it rises, gravity causes it velocity to decrease to zero at the top of its path, then gravity causes it to fall back down, increasing its velocity on the way down. In fact, in the absence of air resistance, it turns out that when the object falls back to its original position, it will have the same *speed* that it was given when it was tossed up. Note that it has the same *speed*, not the same *velocity*, because the direction of its velocity is not the same. This problem can be understood on the basis of equation [3]. Consider the first part of the motion where the object is thrown upward. If we consider the upward direction to be positive, then v_i is positive and the acceleration is negative ($a = -g = -10\ m/s^2$). At the very top of the object's path, v = 0, so equation [3] says that: $0 = v_i - g\,t$, or $v_i = g\,t$, or the time that it takes for an object thrown upward with a speed v_i to reach its highest point is

Time to reach highest point starting with v_i:	$t_{up} = \frac{v_i}{g}$.	[6]

Now, it takes exactly the same amount of time to rise to its highest point as it takes to fall back to the ground, So the total time the object is in the air is $t_{total} = t_{up} + t_{down}$, and since $t_{up} = t_{down}$ the total time the object is in the air is

Time for an object thrown up with speed v_i to return back down:

$$t_{total} = 2\,\frac{v_i}{g}. \qquad [8]$$

Example 4.5: Lebron James jumps straight up with a speed of 5 m/s to make a jump shot. How long is he in the air?

Solution- What we are looking for is the total time he is in the air, which is the time to jump up plus the time to fall back down. The time to jump up is $t_{up} = \frac{v_0}{g} = \frac{5\ m/s}{10\ m/s^2} = 0.5\ s.$ Since the time to jump up and the time to fall back down are the same, the total time he will be in the air is 1 s.