PHYS:1200 LECTURE 7 – MECHANICS (6)

We have now discussed Newton's 1st law (also known as Galileo's principle of inertia) and Newton's 2nd law (the law of dynamics). Both the first and second laws are contained in the mathematical statement: $F_{NET} = m\,a$. Newton recognized that it is impossible to have just an individual force. Rather, there is a mutual interaction between objects, and forces always occur in pairs. If you press on a wall with your hand, the wall also presses back on your hand. This lecture presents Newton's 3rd law and the concept of momentum. Consideration of the concept of momentum in connection with the third law leads to the important principle of the conservation of momentum.

7-1. Newton's Third Law.—Newton actually discussed three laws of mechanics and it is more likely that you have already heard about the third law:

> **NEWTON'S THIRD LAW**: *For every action, there is an equal and opposite reaction.*
> *(Newton used the word "action" for force.)*

In this lecture we will explain the meaning of the third law, why it is particularly useful for analyzing what happens in collisions, and how it leads to the concept of conservation of momentum. This will be the first discussion of the conservation laws of physics. Conservation laws are very important in physics because they tell us that in some sort of process that occurs, there is a physical quantity whose value remains the same (i.e., it is conserved) before and after the process. This may seem a bit vague now, but by the end of this lecture its meaning and application will be clear. Another important conservation law that we will take up in a future lecture is the law of conservation of energy.

Newton's 3rd law deals with forces that occur between two objects that are interacting, for example, a big hammer is used to break up a concrete surface. The law states that the force that when the hammer hits the concrete surface it exerts a force on the surface and the surface exerts an equal force on the hammer in the *opposite* direction. The third law always deals with two forces acting on different objects. The force of the hammer on the surface is the **action** and the force of the surface on the hammer is the **reaction** force (slide 3).

Example 7-1: if I push on the wall with my hand, **the wall pushes *back* on me with an equal strength** force. If I push harder the wall pushes harder back on me.
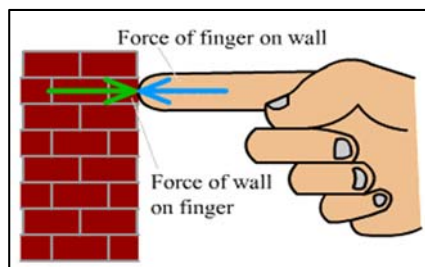
Example 7-2: A book rests on a table, and pushes down on the table with a force that equals its weight. The table pushes *upward* on the book with an equal force.
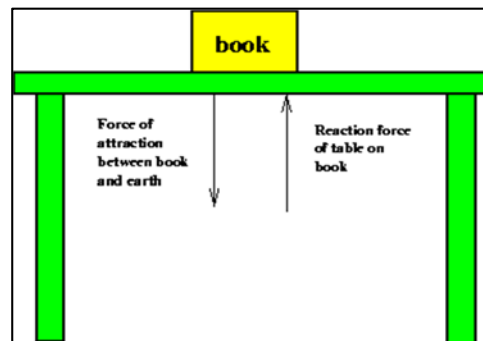
Note that the ability to exert a force depends not only on the agent which is exerting the force, but also on the agent which is supplying the reaction. You can only exert as much force on an object as the object is able to exert back on you. I can punch a wall with a large force, but I am not able to exert as large a force if I try to punch a sheet of paper.

The 3rd law also explains how a ball dropped onto the floor bounces back up. When the ball hits the floor (while it is actually in contact with the floor) it exerts a downward force on the floor. As a consequence of the 3rd law, the floor then exerts an equal but *upward* force on the ball causing it to rebound. We realize from Newton's 2nd law that a force must act on the ball to push it up – just before the ball hit the ground its velocity was upward, then just after it hits the ground it is moving upward --- so its velocity was changed --- it was accelerated and that requires a force be exerted on it.

Note that the action and reaction forces always act on *different* objects. Slide 8 presents an apparent paradox where a misapplication of Newton's 3rd law leads to nonsense.

a. The third law and gravity.—Newton's 3rd law also applies to the law of gravity (**slide 6).** The earth exerts a downward force on us – this is our weight. We then exert an equal upward force on the earth. This ir true whether we are on the ground or falling down. When an object is falling, the earth causes it to accelerate downward at a rate g. As the object is falling, it exerts an upward force on the earth. Then the earth must also accelerate--- however, even though the forces are equal (3rd Law) the acceleration of the object and the acceleration of the earth are NOT equal (2nd Law). The third law tells us that the magnitudes of the forces are equal: $F_{object} = F_{earth}$, where F_{object} is the force on the object by the earth (its weight) and F_{earth} is the force exerted on the earth by the object. Applying now the 2nd law since the forces are equal: $m_{object} a_{object} = m_{earth} a_{earth}$, where a_{object} is just g. Now the mass of the earth is much much larger than the mass of the object ($m_{earth} = 6 \times 10^{24}$ kg) so $a_{earth} <<$ (much much less than) a_{object}. [Mathematically, if we solve for a_{earth} we get $a_{earth} = (m_{object}/m_{earth})a_{object}$, but m_{object}/m_{earth} is a very very small number, so $a_{earth} << a_{object}$.]

b. The role of the third law in motion.—**Action/reaction forces are an integral part of everyday experience; they even play a role in our ability to walk or the ability of a car to move**. When we walk our feet push back on the ground—the ground simultaneously exerts a forward force on our feet and this forward force is what makes us move forward. The tires of a car are in contact with the road. To go forward, the tires push back on the road and the road then pushes forward on the tires – this is why the car moves forward. Note that if the ground is slippery (or if you try to walk or drive on ice) we are not able to push back on the ground so the ground does not push us forward. These examples are discussed on **slides 9 and 10.**

c. The bouncy and non-bouncy ball demonstration.—(**Slides11, 12, 13**) Two balls of identical mass fall to the ground from the same height. Both balls have the same velocity just as they reach the ground. One ball (ball A) hits the ground and bounces back up and the other ball (ball B) hits the ground and stops (it does not bounce back up). *Which ball exerted a larger force on the ground?* Now the 3rd law tells us that in either case, the force exerted by the ball on the ground must equal in magnitude (and be directed in the opposite direction) to the force exerted on the ball by the ground. If we think about the situation a bit, it should be obvious that the ground must exert a larger force on a ball that bounces back up than on a ball that stops. Why? Because the

ball that bounces back must have experienced a force that not only brought it to rest instantaneously but also propelled it upward. For the ball that just stopped the ground must only exert enough force to stop it. Since the force on the bouncing ball is larger than the force on the non-bouncing ball it must also be true, by the 3rd law that *the bouncing ball must have exerted a larger force on the ground than the ball that did not bounce.*

d. Stunt actors and air bags.— (Slides 14-17) A stunt actor leaps off of a building and by landing on a large cushion is not injured. You drive your car at high speed head-on into a steel barrier and you are not seriously injured because your air bag deployed. *What are the physics concepts that can be used to understand the outcomes of these situations?* This is illustrated in class by a demonstration in which a glass beaker is dropped onto either a cushion or a brick from the same height. Of course, the beaker is not damaged when it falls on the cushion, and is shattered when it falls on the brick. The first point to make is that the degree of damage (or injury) experienced by an object (or a person) depends on how much force is exerted on it: larger force → more damage or injury. From Newton's 2nd law we know that force is directly related to acceleration—higher accelerations require larger forces. So in analyzing the experiments with the beaker, we need to look at the accelerations involved when it hits either the cushion or the brick. Acceleration is the change in velocity with time, or $a = \Delta v / \Delta t$, where Δv is the change in velocity and Δt is the time interval over which the velocity changes. The interval in time, in this case, is the time over which the beaker's velocity changes. Since, in both cases, the beaker is dropped from the same height, it has the same speed just as it is about to hit the cushion or brick. Also, in both cases, the beaker comes to rest, in one case gently and in the other case dramatically. Therefore in both cases the change in the velocity of the beaker is the same. What is different about the two cases—the time during which the velocity changes. When the beaker hits the brick its velocity is changed suddenly, while in the case where it falls on the cushion, its velocity is changed over a longer interval of time. Since $a = \Delta v / \Delta t$ the acceleration is larger for the case in which the time is shorter, therefore the acceleration, and also the force, involved when the beaker hits the brick are much larger than for the case in which it hits the cushion. The same reasoning applies to the airbag. If your car hits a brick wall it stops, but because of your inertia you tend to continue moving forward. If you are not wearing your seatbelt, you might

even be thrown through the windshield. Even if you are wearing a seatbelt, you will tend to be thrust forward in the collision. If there is no airbag you may hit the steering wheel and be stopped abruptly with significant injury. However, if the airbag deploys, it allows you to come to rest over a longer time interval, reducing your acceleration and the force exerted on you, and as a result experience far less injury if any. For all of this to work, the airbag must be inflated very quickly; sometimes this can cause injury, especially to children.

7-2. Momentum.—A concept very closely related to Newton's 3rd law is momentum. Momentum is one of those terms that is used in everyday conversation but also has a very specific physics definition. We often hear people say – the team has momentum, or the team has lost its momentum. There is an implicit understanding that when you have the momentum, you can keep going, but one you lose it you may not get it back. There is actually some truth in this colloquial use of the term momentum because in physics, momentum is one of what we call a conserved quantity – in certain processes, the momentum of the objects involved before and after the process maintains the same value (i.e., is conserved). The physics definition of momentum is this: if an object of mass m is moving with a velocity v its momentum p is defined as

Momentum	$p = m\,v$

where m is in kg, v is in m/s and p is then given in the units kg m/s. At the moment, this is just a quantity we defined without trying to explain why it might be a useful quantity. So momentum depends on mass and velocity --- a small, fast moving object could have the same momentum as a large, slow moving object (slide 19). Am 18 wheel semi having a mass of 50,000 kg and moving at 1 m/s has the same momentum as a 5000 kg auto moving at 10 m/s.

7-3. The Law of Conservation of Momentum.—The concept of momentum is particularly useful when applied to the collision of two objects. When two objects collide, they exert very large forces on each other over a relatively short time. Often, collisions are so complicated that it may be impossible to know the details of these forces. But, fortunately Newton's 3rd law ensures that regardless of the complexity of the collision, **the two colliding objects exert equal forces on each other in opposite directions**. This fact, along with the definition of momentum

leads to the conclusion that in a collision, the total momentum of the two objects before the collision is the same as the total momentum of the two objects after the collision. (Before and after refer to immediately before they hit each other, and immediately after they hit each other.) **This is the law of conservation of momentum and it is one of the basic principles of physics.** (The law of conservation of momentum is a direct consequence of Newton's 3rd law. The detailed proof of this is given in an appendix to this lecture.)

   A very simple illustration of this principle can be shown using the air track with two identical mass cars. One car is initially at rest, and the other car is given a push to cause it to bang into the stationary car. If you look carefully at the result, you will observe that after the cars collide, the initial moving car stops and the second initially stationary one takes off with the same speed as the other one had before the collision. **Therefore, the total momentum before and after the collision has not changed --- it was conserved!** Momentum before $p_i = mv + 0 = mv$, momentum after $p_f = 0 + mv = mv \rightarrow p_i = p_f$ (the subscripts "i" and "f" refer to "initial" and "final"). Collisions will be discussed in more detail in the next lecture.
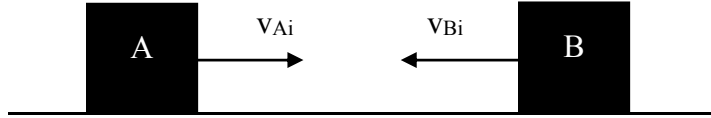
Before collision

m, v   m, v = 0

A   B

After collision   m, v = 0   m, v

A   B
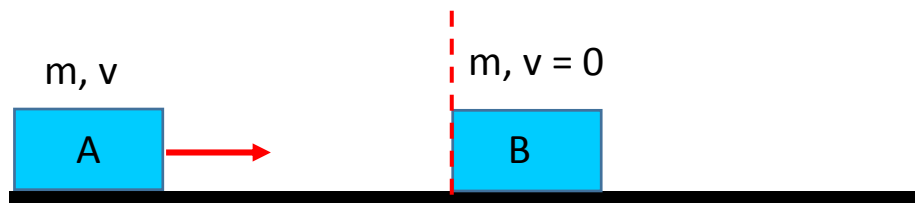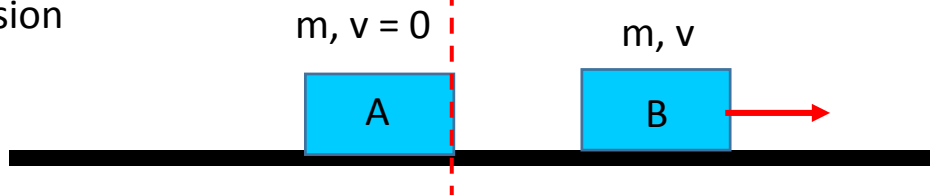
Appendix – Proof of the principle of conservation of momentum from Newton's 3rd law.

Consider two objects, A and B that are involved in a collision. Their initial velocities are v_{Ai} and v_{Bi}, respectively. After they collide, their final velocities are v_{Af} and v_{Bf}, respectively. From

Newton's 3rd law we know that the forces they exert are equal in magnitude and opposite in direction, so we can write: $F_{A \to B} = -F_{B \to A}$, where $F_{A \to B}$ is the force that A exerts on B, and $F_{A \to B}$ is the force that B exerts on A during the collision. Now according to Newton's 2nd law:

$F_{A \to B} = m_B a_B$ and $F_{B \to A} = m_A a_A$, where a_A and a_B are the accelerations experienced by A and B respectively during the collision. The accelerations can be written in terms of the change in velocity with time: $a_A = \Delta v_A / \Delta t$ and $a_B = \Delta v_B / \Delta t$, where $\Delta v_A = v_{Af} - v_{Ai}$ and $\Delta v_B = v_{Bf} - v_{Bi}$.

Then the fact that $F_{A \to B} = -F_{B \to A}$, tells us that $m_B \dfrac{\Delta v_B}{\Delta t} = -m_A \dfrac{\Delta v_A}{\Delta t}$. The Δt factors are the same because both objects are in contact for the same time, so Δt can be cancelled on both sides of the equation. Writing the changes in velocity in terms of the initial and final velocities we have that $m_B \left(v_{Bf} - v_{Bi} \right) = -m_A \left(v_{Af} - v_{Ai} \right)$, and rearranging the terms gives us that

$m_A v_{Af} + m_B v_{Bf} = m_A v_{Ai} + m_B v_{Bi}$ but, since $p = mv$, this just says that

$$p_{Af} + p_{Bf} = p_{Ai} + p_{Bi},$$

or, that the total momentum before and after the collision are the same → total momentum is conserved.