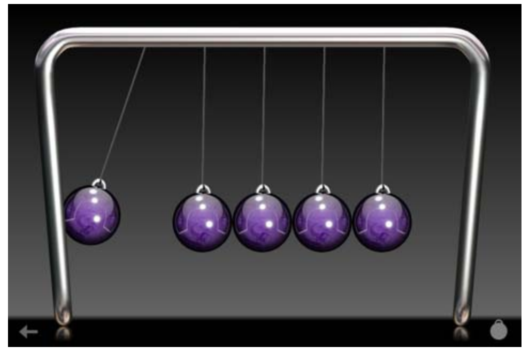
PHYS:1200 LECTURE 8 – MECHANICS (7)

This lecture will deal with two topics – **the law of conservation of momentum as applied to collisions** and **the law of conservation of energy**. Energy is another one of those concepts that we hear a good deal about in everyday usage- we may say that I have energy or I have lost my energy. As with all of these concepts, we will give a precise definition of this term and show how it is applied. Energy is related to another concept in everyday experience – work. We will also discuss the relationship between these terms. Energy is a very important concept because we believe that when we carefully keep track of it, we find in many situations that it remains constant (is conserved). There are various forms of energy and we find that it can be transformed from one form to another but it cannot be created or destroyed. **A generalization of our experiences leads us to believe that the total amount of energy in the universe is constant**.

8-1. Collisions and Momentum.—We began discussing collisions in the last lecture in terms of **the law of conservation of momentum**. An object of mass m moving with a velocity v has a momentum

p = mv. Collisions are usually very complicated interactions. For example, when two cars collide, both are usually damaged – they are **permanently deformed** in some way.

As we will shortly see, any object that is moving has energy, and when two objects collide and become deformed, some of the initial energy is used to cause the deformation. So typically, the energy involved in a collision is not the same before and after the collision. However, there are some very ideal situations where the objects are elastic and are deformed during the collision but spring back to their original shape after separating. For example, when an elastic ball (say a superball) is dropped onto the ground, it is deformed when it is in contact with the ground, but then it recovers its shape and moves upward. **The point is that in a collision, kinetic energy may or may not be conserved, but the total momentum of the objects immediately before the collision is the same as the total momentum of the objects immediately after the collisions**. Newton's cradle (RIGHT) is a nice illustration of this.



a. Conservation of momentum.—Conservation of momentum for two objects, A and B, undergoing a collision is expressed as:

Conservation of Momentum	$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$	[1]

where: $p_{Ai} = m_A v_{Ai}$ is the momentum of A before the collision, $p_{Bi} = m_B v_{Bi}$ is the momentum of B before the collision, $p_{Af} = m_A v_{Af}$ is the momentum of A after the collision, and $p_{Bf} = m_B v_{Bf}$ is the momentum of B after the collision.

b. Kinetic energy.—To discuss collisions in more detail it is necessary to distinguish between two extreme types of collisions – elastic and inelastic. The definitions are based on whether or not the kinetic energy of the objects before and after the collision is conserved or not conserved (remains the same or changes). **Kinetic energy (KE) is the energy that an object has because of its motion.** (We have not yet formally defined "energy", so just take this as a new parameter for the time being.) An object moving with a speed v and having a mass m has a <u>kinetic energy</u>

KINETIC ENERGY	$KE = \frac{1}{2} m \mathrm{v}^2$	[2]

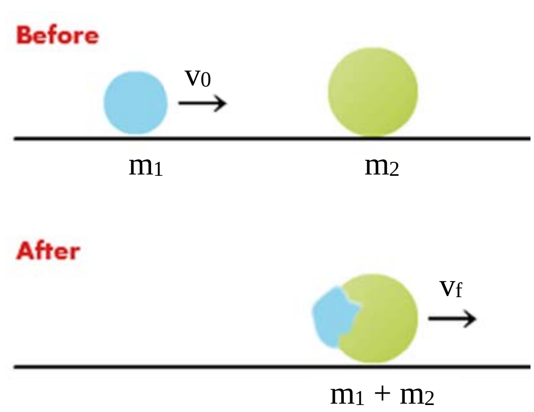
Notice that I used the term "speed" not "velocity" because the kinetic energy only depends on how fast something is moving and not on the direction. KE will be measured in the units of kg x $(m/s)^2$ or kg m^2/s^2. A special unit for kinetic energy was introduced to honor the scientist who clarified the concept, one kg m^2/s^2 is called one Joule (J).

Example 8-1, What is the kinetic energy of a 4 kg mass moving at 5 m/s?

solution: KE = ½ (4 kg) $(5\ m/s)^2$ = 50 J.

c. Elastic and Inelastic collisions.—Having introduced the concept of kinetic energy we can now define what is meant by elastic and inelastic collisions. An **elastic collision** is a collision in which the sum of the kinetic energy of the objects before they collide is equal to the kinetic energy of the objects after they collide, in other words a collision in which kinetic energy is conserved. An **inelastic collision** is one in which the total kinetic energy is not the same before and after the collision. Note that the concept of elastic collision is an ideal case, no collision are ever totally elastic, but in some instances it is still a good approximation. Collisions of cars on the air track are to a large extent elastic. The cars are fitted with springy metal bumpers that deform during a collision, but spring back after the collision. For the case (slide 9) in which two identical cars (same mass) collide with one initially stationary, we saw that after the collision, the first one stops and the one originally at rest takes off with the same speed as the other car had before the collision. Clearly, in this case, the collision is elastic: $KE_i = ½\ mv^2 + ½\ m\ (0)^2 = KE_f = ½\ m\ (0)^2 + ½\ mv^2$ (i and f refer to before and after the collision). A super ball bouncing off the floor is another example of an elastic collision (slide 8).

d. Completely inelastic collisions.—A special type of inelastic collision is the **completely inelastic collision in which the colliding objects stick together after they collide**, leaving just one body after the collision. This case is illustrated on **slide 10** and in the diagram on the right. Mass 1 moving with an initial velocity v_0 collides with mass 2 which is initially at rest. After the collision mass 1 sticks to mass 2 and the two go off with a speed v_f. Conservation of momentum requires that the total momentum before and after the collision is the same, so that $m_1 v_0 = (m_1 + m_2) v_f$. If the masses and

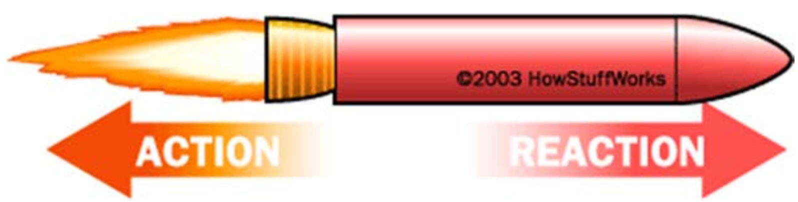


the initial velocity of mass 1 are known, the final velocity of the two connected masses can be found: $v_f = m_1 v_0 / (m_1 + m_2)$. Since $m_1 + m_2$ is larger than m_1, v_f must be less then v_0.

Not all "collisions" are violent. A non-violent collision example is given on **slide 12.**

e. Recoil.—One important consequence of the principle of conservation of momentum is the phenomenon of **recoil.** Anyone who has fired a hand gun or rifle has experienced the "kickback" when the weapon is fired (**slides 14, 15, 16**). Suppose the rifle has a mass M, and the bullet has a mass m. Before the rifle is fired, the system (rifle and bullet) are at rest so the initial momentum of the system is 0, $p_i = 0$. The law of conservation of momentum requires that the momentum of the system is conserved, so after the bullet is fired the sum of the momenta of the rifle and the bullet must be 0. Suppose the bullet's speed is v and the rifle's recoil speed is V, then conservation of momentum requires that: $0 = mv + MV$, or $V = -(m/M)v$. Thus the rifle kicks backward in the direction opposite to the bullet's velocity. Sine M is much larger than m, V is much smaller than v.

f. Rocket science.—**Recoil is what propels a rocket**. In a rocket engine fuel is ignited and the hot gases are ejected from the nozzle at very high speed. Since the rocket and fuel are initially at rest, the total momentum of the system is zero. Since momentum is conserved it must remain



zero after the engine is ignited and the hot gas exit. To keep the total momentum zero, the rocket is given a "kick" in the opposite direction which propels it forward.

8-2. Work and Energy.—Energy is the ability to do work. To perform some sort of physical activity you need energy. We get energy by consuming food and our body transforms this into energy through our metabolism. When we do something physical like lifting a large weight, we use some of our energy to do the work of lifting. Work

is another one of those concepts that has an everyday meaning, but has a very specific physics definition. If you were to hold a large object up in the air for an hour, you would probably think that it was a lot of work to do that. However, the everyday idea of work is different from the physics definition**. The physics definition of work involves both the exertion of a force but also motion in the direction of the force.** For example, when a weightlifter lifts a barbell over his head**, he is doing work on the weight only while he is actually lifting it**. He does no work, according to physics, if he just holds the weight over his head. Suppose you push a box across the floor over a distance s by exerting a force F, then the work (W) done on the box is

WORK	$W = F s$	[3]

The units for work are **Newtons x meters**, or N m, but since energy and work are related, one Nm is also one Joule (J), so **work is also measured in J.**

Example 8-2: How much work is required to lift a mass of 500 kg to a height of 20 m?

Solution: W = Fs. A force of at least the weight of the object is required to lift it, so

$F = mg = 500 \text{ kg} \times 10 \text{ m/s}^2 = 5000 \text{ N} \rightarrow W = 5000 \text{ N} \times 20 \text{ m} = 100{,}000 \text{ Nm} = 1\times10^5 \text{ Nm}$.